Exercises on Financial Engineering

MASTER IN STATISTICAL TECHNIQUES

Course 2009 – 10

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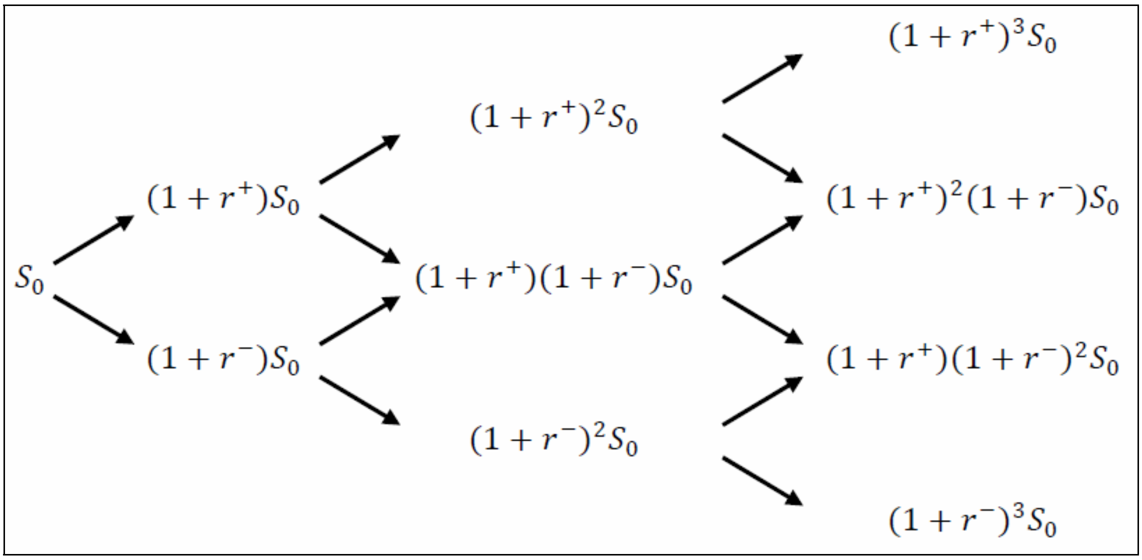
Exercise 1:

Compute the S and the C-tree for

n = 3 S_0 = 7 r^+ = 0.05 r^- = −0.01 r = 0.03 K = 7

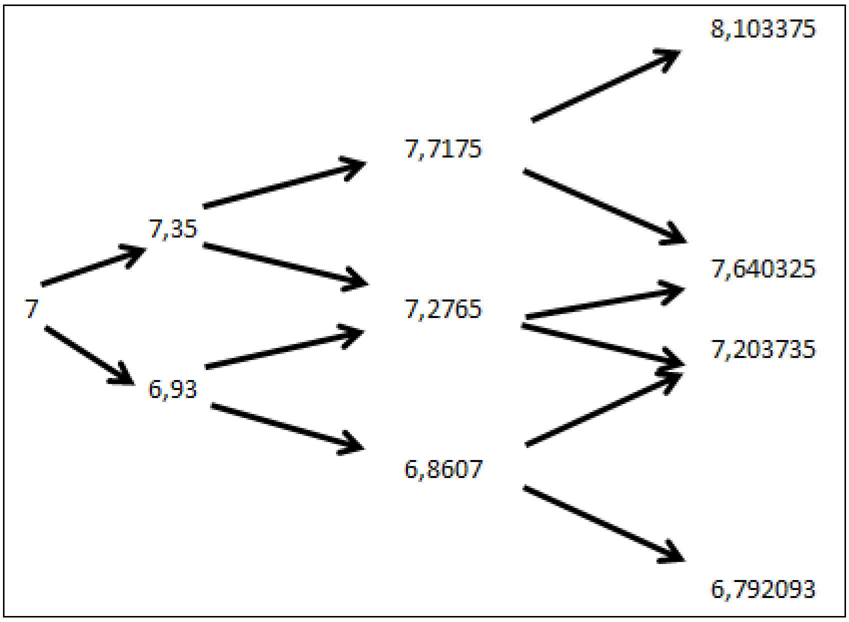
Solution:

To compensate the risk of the writer of an option, some premium needs to be paid when the contract is signed. Since this premium will depend on the price process of the underlying. A three-period binomial (Cox-Ross-Rubinstein) model for the stock price movements is:



Ilustration 1: three-period binomial (Cox-Ross-Rubinstein)

Replacing n = 3, S_0 = 7, r^+ = 0.05, r^- = −0.01, r = 0.03 and K = 7 in the tree, we obtain:



Ilustration 2: S-tree

C is a convex combination of C_1^+, C_1^-. C_0 is function of r, q, $C_1^+ and\ C_1^-$.

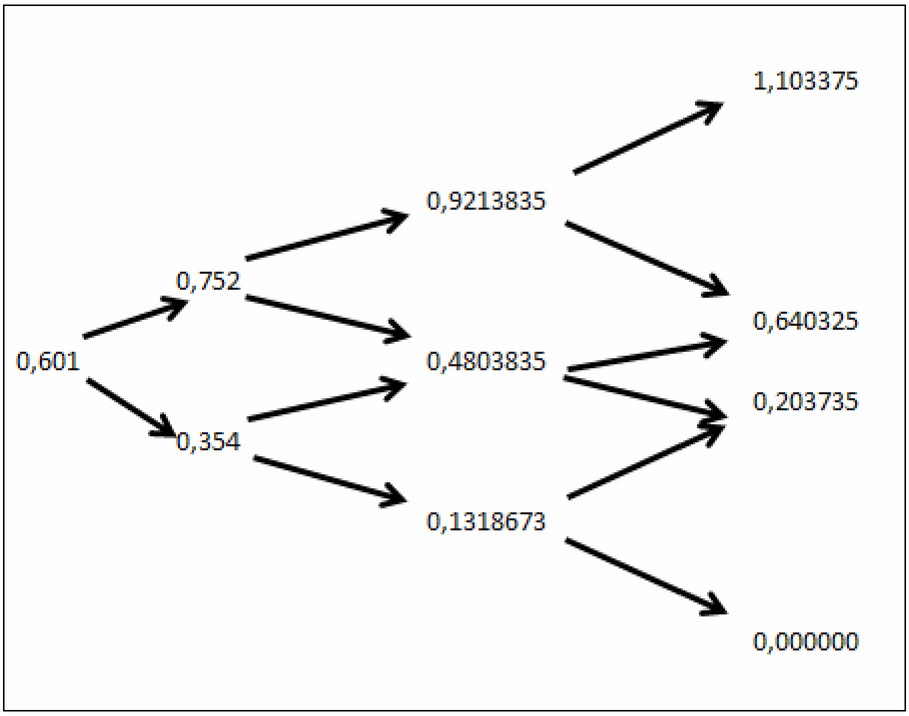
$$q=\frac{r-r^-}{r^+-r^-}=\frac{0.03+0.01}{0.05+0.01}=\frac{0.04}{0.06}=0.667$$

$$p=1-q=1-0.337=0.333$$

And the value of the call at each node can be obtained for i = 0, 1, 2, 3 with the formula of LEMMA3 (page 28):

$$C_i(s)=\frac{1}{(1+r)^{n-i}}\sum_{k=0}^{n-i}\binom{n-i}{k}q^k(1-q)^{n-i-k}\max\left\{s(1+r^+)^k(1+r^-)^{n-i-k}-K,0\right\}$$

Then the C-tree is:



Ilustration 3: C tree

For obtain these trees we have executed this code in R:

```
#DATA:
n=3;K=7;r=0.03;s=7;rp=0.05;rm=-0.01
sTree=matrix(nrow=7,ncol=4) # Matrix for the S-tree
sTree[4,1]=s
sTree[3,2]=(1+rp)*s
sTree[5,2]=(1+rm)*s
sTree[2,3]=(1+rp)^2*s
sTree[4,3]=(1+rp)*(1+rm)*s
sTree[6,3]=(1+rm)^2*s
sTree[1,4]=(1+rp)^3*s
sTree[3,4]=(1+rp)^2*(1+rm)*s
sTree[5,4]=(1+rp)*(1+rm)^2*s
sTree[7,4]=(1+rm)^3*s
sTree #Play, here is de the S-tree
cTree=matrix(nrow=7,ncol=4) # Matrix for the C-tree
cTree[4,1]=cTreeFunc(0,sTree[4,1],n,r,rp,rm,K)
cTree[3,2]=cTreeFunc(1,sTree[3,2],n,r,rp,rm,K)
cTree[5,2]=cTreeFunc(1,sTree[5,2],n,r,rp,rm,K)
cTree[2,3]=cTreeFunc(2,sTree[2,3],n,r,rp,rm,K)
cTree[4,3]=cTreeFunc(2,sTree[4,3],n,r,rp,rm,K)
cTree[6,3]=cTreeFunc(2,sTree[6,3],n,r,rp,rm,K)
cTree[1,4]=cTreeFunc(3,sTree[1,4],n,r,rp,rm,K)
cTree[3,4]=cTreeFunc(3,sTree[3,4],n,r,rp,rm,K)
cTree[5,4]=cTreeFunc(3,sTree[5,4],n,r,rp,rm,K)
cTree[7,4]=cTreeFunc(3,sTree[7,4],n,r,rp,rm,K)

cTree #Play, , here is de the C-tree
# ##################################
```

```
# Function for the C-tree
# #################################
cTreeFunc=function(i,s,n,r,rp,rm,K){
	q=(r-rm)/(rp-rm)
	summa=0
	for (k in 0:(n-i)){
		summa=summa+(factorial(n-i)/(factorial(k)*factorial(n-i-
k))*(q^k)*((1-q)^(n-i-k))*
		max(s*(1+rp)^k*(1+rm)^(n-i-k)-K,0))
	}
	summa=(1/((1+r)^(n-i)))*summa
	summa
}
```

Exercise 2:

Apply the Black-Scholes formula in the following case and compute the Call-price:

T − t = 3 months S_0 = 12 K = 13 r = 0.01 σ = 0.3

<u>Solution:</u>

The price of a European Plan-Vanilla Call wih Maturity T and Strike K equals, at time t:

$$C_t(s) = s\Phi(d_1) - Ke^{-r(T-t)}\Phi(d_2)$$

for the THEOREM 1: Black-Scholes formula (page 47).

With:

$$d_1 = \sqrt{T-t\,\sigma/2}\ and\ d_2 = -\sqrt{T-t\,\sigma/2}$$

$$\Phi = \frac{1}{2\pi}\int_{-\infty}^{x} e^{-u^2/2}\,du$$

In our problem have T − t = 3 months, S_0 = 12, K = 13, r = 0.01 and σ = 0.3.

Replacing in the form:

$$C_t(s) = s\Phi(d_1) - Ke^{-r(T-t)}\Phi(d_2)$$

$$= 12\Phi(d_1) - 13\Phi(d_2)$$

$$T = \frac{3}{12} = 0.25 years$$

$$d_1 = \frac{\ln\frac{12}{13} + 0.25\left(0.01 + \frac{0.3^2}{2}\right)}{0.3\sqrt{0.25}} \cong -0.441$$

$$d_2 = d_1 - 0.3\sqrt{0.25} \cong -0.591$$

Hence:

$$C_0(s) = s\Phi(-0.441) - Ke^{-r(T-t)}\Phi(-0.591)$$

$$= 12 \cdot 0.329 - 13 \cdot e^{-0.01 \cdot 0-25} \cdot 0.272$$

$$= 0.359895$$

Therefore the Call-price is 0.3598952

For obtain the Call-price in R:

```
# Data:
s=12;K=13;t=0;T=0.25;r=0.01;sigma=0.3

d1=(log(s/K)+(T-t)*(r+(sigma^2)/2))/(sigma*sqrt(T-t))
d2=d1-sigma*sqrt(T-t)

phi.d1=pnorm(d1)
phi.d2=pnorm(d2)
C=s*phi.d1-K*exp(-r*(T-t))*phi.d2
```