Exercises on Financial Engineering.

Master in Statistical Techniques. Curse 2010-2011

Leyenda Rodríguez, María

Índice

1. Exercise 1

Compute the S- and the C-tree for
n=3 K=180 r=0.01 r^+=0.1 r^-=-0.1 S_0=180
Solution

- S-tree

  A three-period binomial model of Cox-Box-Rubinstein for the stock price movements is:

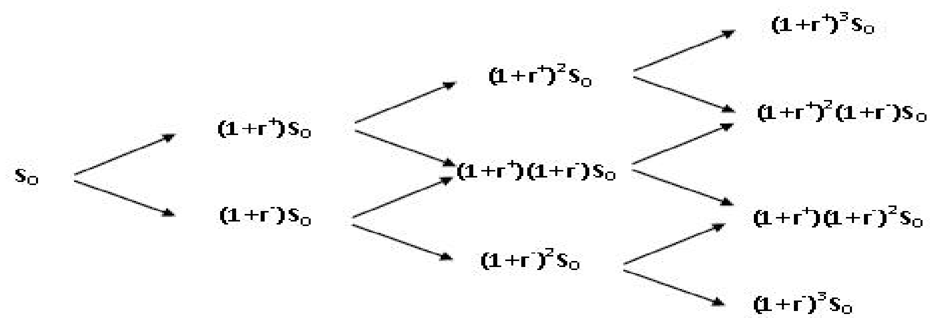

Figura 1: three-period binomial model of Cox-Box-Rubinstein

In this particular case, we have n=3, K=180, r=0.01, r^+=0.1, r^-=-0.1 and S_0=180. Then we obtain

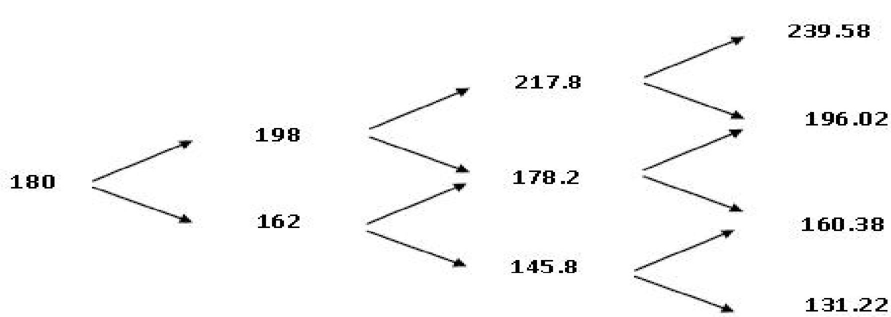

Figura 2: S-tree

- C-tree The value of the call at each node can be obtained with the formula (1). **Lema** For $i = 0, \ldots, n$ we have

$$C_i(s) = \frac{1}{(1+r)^{n-i}} \sum_{k}^{n-i} \binom{n-i}{k} q^k (1-q)^{n-i-k} max \left\{ s(1+r^+)^k (1+r^-)^{n-i-k} - K, 0 \right\} \quad (1)$$

Note $q = \frac{r-r^-}{r^+-r^-} = \frac{0{,}01-(-0{,}1)}{0{,}1-(-0{,}1)} = 0{,}55$

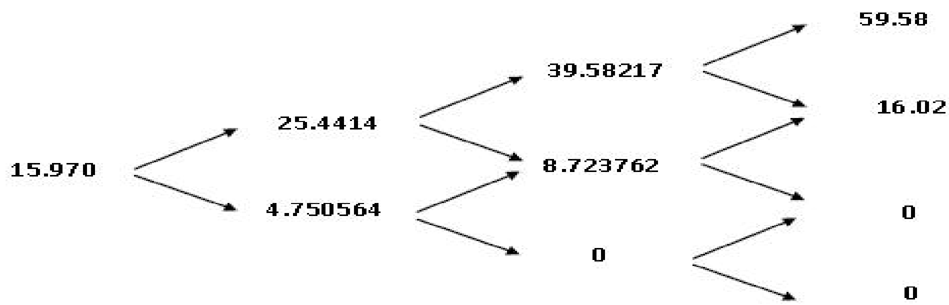

Figura 3: C-tree

1.1. Code executed to obtein S-tree and C-tree

```
### Data
rplus=0.1
rminus=-0.1
S0=180
r=0.01
n=3
K=180


### S-tree
STree<-matrix(0,7,4)
STree[1,1]<-S0
STree[1,2]<-(1+rplus)*S0
STree[1,3]<-(1+rplus)^2*S0
STree[1,4]<-(1+rplus)^3*S0
STree[2,4]<-(1+rplus)^2*(1+rminus)*S0
STree[3,3]<-(1+rplus)*(1+rminus)*S0
STree[4,4]<-(1+rplus)*(1+rminus)^2*S0
STree[5,2]<-(1+rminus)*S0
STree[6,3]<-(1+rminus)^2*S0
STree[7,4]<-(1+rminus)^3*S0
STree

### C-Tree
q=(r-rminus)/(rplus-rminus)
q # 0.55

CTree_general<-function(i,r,n,q,s,K){
sum=0
for(k in 0:(n-i)){sum=sum+
(factorial(n-i)/(factorial(k)*factorial(n-i-k))
*(q^k)*((1-q)^(n-i-k))
*max(s*(1+rplus)^k*(1+rminus)^(n-i-k)-K,0))}
sum=sum/((1+r)^(n-i))
sum}
```

```
CTree<-matrix(0,7,4)
CTree[1,1]<-CTree_general(0,r,n,q,STree[1,1],K)
CTree[1,2]<-CTree_general(1,r,n,q,STree[1,2],K)
CTree[1,3]<-CTree_general(2,r,n,q,STree[1,3],K)
CTree[1,4]<-CTree_general(3,r,n,q,STree[1,4],K)
CTree[2,4]<-CTree_general(3,r,n,q,STree[2,4],K)
CTree[3,3]<-CTree_general(2,r,n,q,STree[3,3],K)
CTree[4,4]<-CTree_general(3,r,n,q,STree[4,4],K)
CTree[5,2]<-CTree_general(1,r,n,q,STree[5,2],K)
CTree[6,3]<-CTree_general(2,r,n,q,STree[6,3],K)
CTree[7,4]<-CTree_general(3,r,n,q,STree[7,4],K)
CTree
```

2. Exercise 2

Compute the Black-Scholes formula prices $C_t(s)$ and $P_t(s)$ for
K=100 s=100 σ=0.1 T-t=3 months$\equiv$ 0,25$years$ r=0.01
What are the inner and the time value?
Solution

- Compute the Black-Scholes formula prices $C_t(s)$ and $P_t(s)$ **Theorem** The price of a European Plan-Vanilla Call with Maturity T and Strike K equals, at time t:

$$C_t(s) = s\phi(d_1) - Ke^{-r(T-t)}\phi(d_2) \tag{2}$$

  Here s equals the present (t) price of the stock, $S_t = s$. Note s=k, we are at the money, then $ln(\frac{s}{K}) = 0$.

  Note

$$d_1 = \frac{ln(\frac{s}{K}) + (T-t)[r + \frac{\sigma^2}{2}]}{\sigma\sqrt{T-t}} \tag{3}$$

  and

$$d_2 = d_1 - \sigma\sqrt{T-t} \tag{4}$$

  Futhermore, in this case, s=K=100, then ln(s/K)=0.

  Then, remplacing K=100 s=100 σ=0.1 T-t=3 months$\equiv$ 0,25$years$ r=0.01 in equation (2),(3),(4) we obtain $C_t(s) = 2{,}119346$,$d_1 = 0{,}075$ and $d_2 = 0{,}025$.

  On the other hand, the put price may be computed by means of the Put-Call paruity

$$P_t(s) = C_t(s) - s + Ke^{-r(T-t)} \tag{5}$$

  Then, remplacing K=100 s=100 σ=0.1 T-t=3 months$\equiv$ 0,25$years$ r=0.01 in equation (5) we obtain $P_t(s) = 1{,}869659$.

- What are the inner and the time value?

  It's know that $C_t = I_t + T_t$ where I_t is called intrinsic value (inner) and T_t is called time value. Futhermore, of the equation (5) follows that $C_t(s) = P_t(s) + s - Ke^{-r(T-t)} = 2{,}119346$. Then $I_t = s - Ke^{-r(T-t)} = 0{,}2496878$ and $T_t = P_t = 1{,}869659$.

2.1. Code executed to resolve Exercise 2

```
### Data
K=100
s=100
sigma=0.1
TM=3/12 # TM=(T-t) time to maturity
t=0
r=0.01

### Black-Scholes price
d1=(TM)*(r+(sigma^2)/2)/(sigma*sqrt(TM))
d1
d2=d1-sigma*sqrt(TM)
d2
C=s*pnorm(d1)-K*exp(-r*TM)*pnorm(d2)
C

### The Put price
P=C-s+K*exp(-r*TM)
P
### Time value and inner
Time_value=P
Time_value
Inner=s-K*exp(-r*TM)
Inner
```