SPLIDA (S-PLUS Life Data Analysis)

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Wqm notes:
Dialog size 75; figure size 35
Textbox adv layout: move; allow

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1 Introduction

S-PLUS is a powerful, extendible, environment for data analysis and graphics. The "Standard Version" of S-PLUS provides access to the S-PLUS functionality through a point-and-click GUI. The "Professional Version" of S-PLUS includes, in addition, access to the S-PLUS command line and the capability to develop extensions to S-PLUS. Extensions to S-PLUS may also include additions to the S-PLUS GUI (and this is the way that the SPLIDA GUI was developed).

SPLIDA is a collection of S-PLUS extensions (both commands and GUI) for planning and analyzing the data from some reliability studies (e.g., laboratory life tests, accelerated life tests, warranty repair data, and other field data). SPLIDA has capabilities for the analysis of censored life data, as well as repeated measures degradation data, destructive degradation data, and recurrence data.

Most SPLIDA capabilities are available through the SPLIDA GUI (extensions to the S-PLUS GUI). Thus, SPLIDA can be used with either the Standard or the Professional versions of S-PLUS (but advanced users with the professional version have access to additional functionality and the ability to make further extensions to SPLIDA/S-PLUS).

As an example, **Figure 1** is a schematic diagram illustrating the steps that a user would follow in the analysis of a data set.
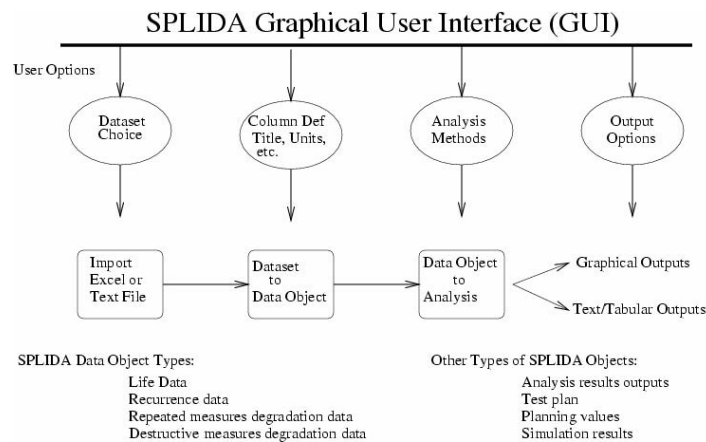
Figure 1 Schematic diagram of the SPLIDA GUI

An analysis starts with a data set, usually imported from an Excel file. This is read into S-PLUS/SPLIDA, and used to create a data object. The data object can then be conveniently analyzed in many different ways to create various graphical and tabular outputs. There is an extensive set of user options to control the analyses and outputs. Whenever it is possible, these choices have sensible defaults to reduce the amount of effort that is needed to do a sequence of analyses. SPLIDA dialogs have been designed carefully to guide users through an analysis.

The main part of this document uses a sequence of examples to describe and illustrate the SPLIDA graphical user interface (GUI) for data analysis for many different kinds of data and statistical models. A similar sequence of examples is used to illustrate the corresponding test planning methods. The methods and analyses parallel closely the methods and examples used in Meeker and Escobar (1998). There are, however, new methods in SPLIDA, described here, that are not available in that book. The items in the SPLIDA menu correspond approximately to the chapters in the Meeker and Escobar (1998) textbook.

The Appendix contains an outline of the SPLIDA menu structure.

There appears to be a non-serious bug in the S-PLUS GUI that causes the "white" part of a multi-select dialog from a back page of a dialog to appear sporadically when a dialog first appears. By paging across the different pages in the dialog, this will fix itself without further difficulty.

2   Getting Started with SPLIDA

All SPLIDA data analyses require a data object. Life data analyses use a SPLIDA "life data object." A life data object contains, for a given data set, available information about failure times (e.g., failure times, running times, intervals in which failures are known to have occurred, etc.), time units (e.g., hours or days), explanatory variables (if any), and a data-set title. Having all of this information in a life data object makes it much easier to do various analyses on a particular data set. There are similar (but different) kinds of data objects for recurrence data analysis and for degradation data analysis, as described in subsequent sections of this document.

2.1   Example data sets and a simple example

For convenience of illustrating features of SPLIDA and for purposes of teaching courses, SPLIDA has a large number of built-in data objects. Data objects are included for most of the examples and many of the exercises in Meeker and Escobar (1998). The appendix contains an index of and references for the original sources for these data sets.

To see how easy it is to do an analysis with SPLIDA, click **SPLIDA ► Single distribution analysis ► Probability plot with nonparametric confidence bands**, to bring the dialog in Figure 2. Choose from the list of SPLIDA life data objects (e.g., `BearingCage.ld`, for the Bearing Cage data from Abernathy et al. 1983, also analyzed in Example 8.16 of Meeker and Escobar, 1998). Now choose a distribution (e.g., Weibull), click "Apply" and a probability plot, like that in Figure 3, will appear in the graphics window.

Figure 2 Dialog to request a probability plot of the bearing cage data.
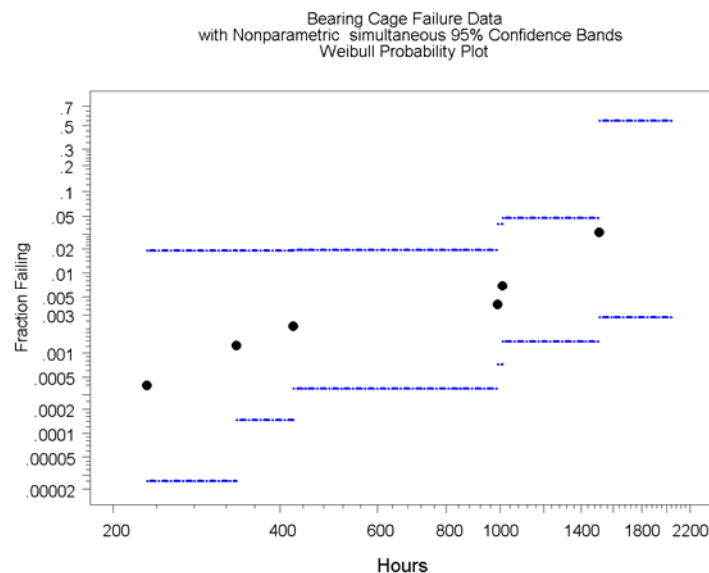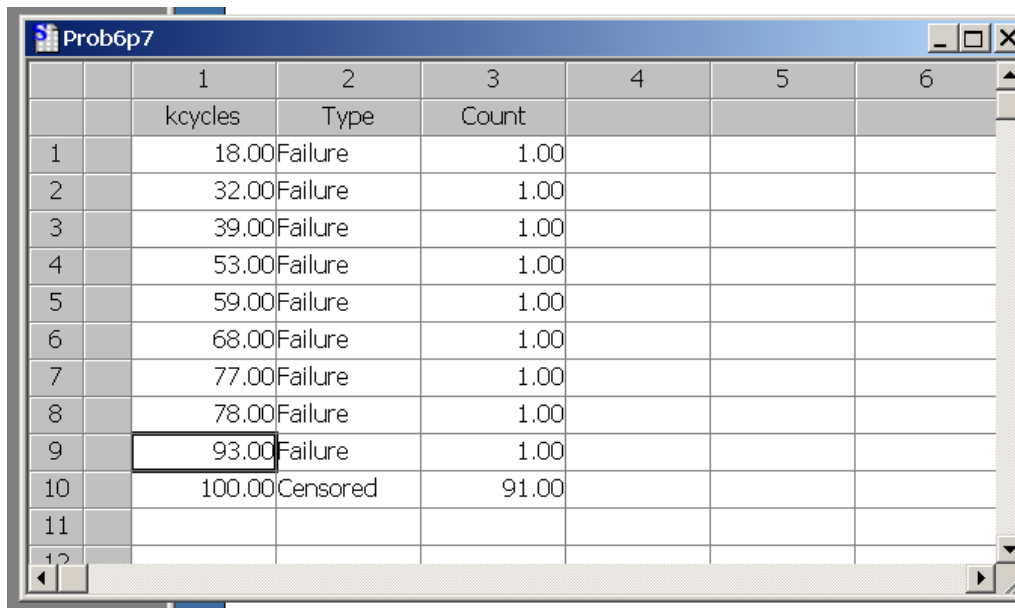
Figure 3 Weibull probability plot of the bearing cage data.

2.2   Inputting data directly into S-PLUS: Creating a data set

In order to analyze your own data, you first have to enter the data into S-PLUS and then create a SPLIDA life data object. It is easy to import from an existing file (e.g., and Excel file or a text file), as described in Section 2.4. For a small data set, however, one can simply enter data directly to make a S-PLUS data set and then create a SPLIDA data object.

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Here we use the titanium crack initiation times described in Problem 6.7 of Meeker and Escobar (1998). A sample of 100 specimens was tested until failure or 100 thousand cycles, whichever came first. Among these, there were nine crack initiations (failures). Choose **Data ► Select Data**. Then choose the "New Data" option in the dialog, enter a name for the data set (e.g. Prob6p7 for Problem 6.7), and click "OK." A blank spreadsheet will appear. Type the failure times "18, 32,…, 93" into the first column. In the second column, type the word "Failure" in each row. Then add a last row with time 100 and the word "Censored." We also need a column of "Weights" to indicate observation multiplicity. Put "1" in all rows except the last, where you should put "91" for the 91 censored units. The S-PLUS default names on the columns are V1, V2, and V3. You can use these, but it is better to change them to something more meaningful. Right-click on a column, choose properties, change the name, and click "OK". Repeat for other columns. After doing this, your spreadsheet (actually an S-PLUS data set) should look something like Figure 4.

	1 kcycles	2 Type	3 Count	4	5	6
1	18.00	Failure	1.00			
2	32.00	Failure	1.00			
3	39.00	Failure	1.00			
4	53.00	Failure	1.00			
5	59.00	Failure	1.00			
6	68.00	Failure	1.00			
7	77.00	Failure	1.00			
8	78.00	Failure	1.00			
9	93.00	Failure	1.00			
10	100.00	Censored	91.00			
11						
12						

Figure 4 Worksheet for inputting or editing data by hand.

Users can also easily import a data set from an Excel worksheet or from a text file (as shown later).

Before doing life data analyses (e.g., when the response is time to failure) with SPLIDA, one needs to make a SPLIDA life data object *(in effect, defining the purpose of the*

columns in the data and adding other important information like a title and time units). Once constructed, the life data object can be analyzed using different SPLIDA methods (without having to re-specify all of the detail for each analysis).

Note the distinction between S-PLUS data sets (like the one just created) and SPLIDA data objects (to be created from the S-PLUS data set in the next section).

2.3   Creating a life data object

A SPLIDA data object is constructed by choosing an S-PLUS data set containing the raw data, specifying which column is the response and, as needed or desired, specifying a title for the data, units of the response, and making other choices. Once a data object is created, the corresponding data can be analyzed repeatedly without having to repeat the often-complicated characteristics of the data.

From the S-PLUS menu bar, choose **SPLIDA ► Make/summary/view/edit data object ► Make life data object** to bring up the dialog shown in Figure 5.

Figure 5 Blank (initial) dialog for making a SPLIDA life data object.

Clicking on the "Choose an S-PLUS data set" pull-down arrow will show S-PLUS data sets in the SPLIDA database and in your working database. Data sets for most of the examples in Meeker and Escobar (1998) and many other examples from the reliability

data analysis literature are included on the SPLIDA database. ASCII text data sets are also given in one of the SPLIDA folders (SPLIDA_text_data). Choose the data set Prob6p7 from the list. After one chooses a data set, SPLIDA will use the column names in that data set to make lists of variables for other dialog inputs. Choose the response column (named "kcycles" in the Prob6p7 titanium crack initiation data set). Then choose the response (kcycles), the censoring indicator (Type), and weights (Count). Then click "Apply" (or "OK"). In the commands or report window, you will get a brief summary of the data.

The default name for the life data object is Prob6p7.ld. The Prob6p7 data set also has a "Count" column indicating observation multiplicity. The Prob6p7 data has no failure mode column, explanatory variables, or truncation. A note can be added to the life data object. There is no limit to the length of this note; the note is printed when a summary of the data object is printed.

In order to simplify use, SPLIDA chooses, when possible, sensible defaults for inputs, but allows the user to easily change these if desired. The final dialog for mapping the Prob6p7 data set into a life data object Prob6p7.ld is shown in Figure 6.

Figure 6 Dialog to make a life data object.

Now when you launch the analysis dialog by using **SPLIDA ► Single distribution analysis ► Probability plot with nonparametric confidence intervals** (see Figure 7) Prob6p7.ld will appear in the list of life data objects (actually, if it was just created, SPLIDA will remember it and chooses it as the default in this list). Now choose a

distribution (e.g., Weibull), click "Apply" and the probability plot shown in Figure 8 will come up in the graphics window.
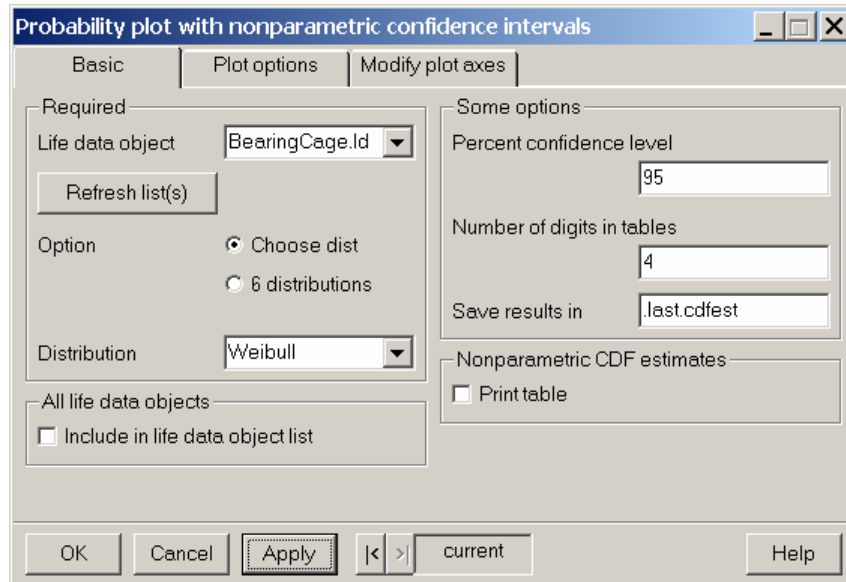

Figure 7 Dialog to request a probability plot of the titanium crack initiation data.
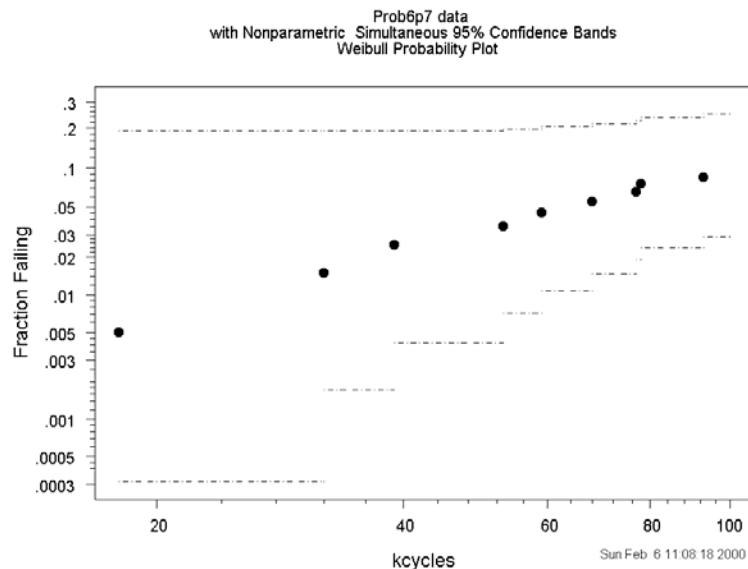
Figure 8 Weibull probability plot of the titanium crack initiation data.

In this and the other SPLIDA dialoges, there are other options on the "back" pages of the dialog, but we do not need to use them at this time. Generally, the required and frequently-used options are on the front page, with less frequently needed options on the back pages. In this documentation, the focus will be on the required and other most important inputs for SPLIDA. Users are encouraged to experiment with the options on the back pages. These options are, for the most part, self explanatory and, as much as possible, their operation is consistent across different SPLIDA dialogs.

2.4 Importing data from Excel and text files

S-PLUS has powerful tools for importing rectangular data sets (in which rows are cases and columns are variables) into a data set. For example, to import data from an Excel worksheet, into a data set, use the following:

(a) From the S-PLUS File menu, choose **Import Data ► From File...** This will bring up the "Import Data" dialog.
(b) Under "File format:," select "Excel Worksheet (xl?)."
(c) Choose the desired file, using the `Browse` feature to switch folders, if `needed`.
(d) The name of the data set can be changed from the default name (aaaa if the Excel file is aaaa.xls).
(e) Click "Apply" to import the file into the S-PLUS data set object.

Importing from a text (or ASCII) file is similar, except that one needs to specify ".txt" files (or "All Files"), choose the appropriate column delimiters, and specify whether the first line in the file contains column names.

See the S-PLUS documentation for more details and other options.

2.5 Graphics options and color schemes in SPLIDA

S-PLUS allows the user to have control over various graphics options. Although it is not necessary for the beginner, frequent users of S-PLUS or SPLIDA will want to become familiar with the available options. Some modifications of the default options have been implemented in the .Prefs file that comes with SPLIDA.

The default color scheme in SPLIDA has been changed from "Standard" to "Cyan Magenta." This option provides a much better (sharper and less washed-out looking) set of colors for SPLIDA graphics. The standard or other build-in options can be chosen with **Options ► Graph Style► Color** and changing the color option box.

Another useful option is to use **Options ► Graph Options** and change "Graph Style" to black and white. The SPLIDA default for the black and white graph style (which differs from the S-PLUS default) uses just black on white, with no grey scales. The S-PLUS standard black and white Graph option can be restored with **Options ► Graph Styles ► Black and White Style**. And choosing `Standard` under `User Colors`.

Under **Options ► Graph Options**, the SPLIDA default for Auto Pages is "Every Page" so that new pages automatically are added to a graphsheet as new plots are created. This makes it easy to compare plots from different analyses. Delete unwanted pages by right-clicking on the page number to be deleted.

2.6 Controling and locking SPLIDA plot axes

Many Splida dialogs have a Modify plot axes page like the one shown in Figure 9.
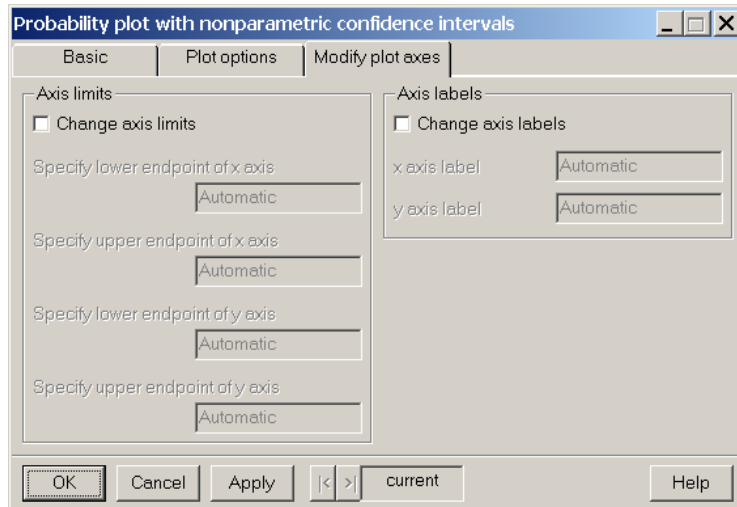
Figure 9 A typical Modify plot axes page of a SPLIDA dialog

This dialog allows the user take choose one of the four limits on the axes or to change the axis labels.

Another SPLIDA feature allows the user to lock the axes of a sequence of similar plots. The locking is turned on by clicking on the ▉ (for Lock) icon on the tool bar. To turn off locking, click on the ▉ (for Unlock) icon. "Similar plots" are defined as plots that have identical x scale axes , identical y scale axes (e.g., linear, log, or other transformation), and the same titles on both the x-axis and the y-axis. This feature is useful when you want to make direct comparisons between two or more plots and is much easier to do than the method of specifying the axis limits directly in the a Modify plot axes page.

3   Changing SPLIDA Default Options

Under certain circumstances, users may want to change SPLIDA defaults. Many defaults can be changed in individual dialogs (e.g. confidence levels in analysis routines). In addition, there is a special dialog for making global changes to certain defaults and controlling defaults cannot be controlled in individual dialogs.

3.1   Basic, plot, and print SPLIDA options

Using **SPLIDA ► Change SPLIDA default options** will bring up the dialog shown in Figure 10, allowing one to make global changes to the several of the SPLIDA defaults options. Changes are effected immediately after chosen on the dialog pages (no need to click on "OK" or "Apply"). Changes made when the "Save changes across sessions" box is checked remain in effect in future SPLIDA sessions. To restore all of the original SPLIDA defaults, click on the "Restore defaults" box.
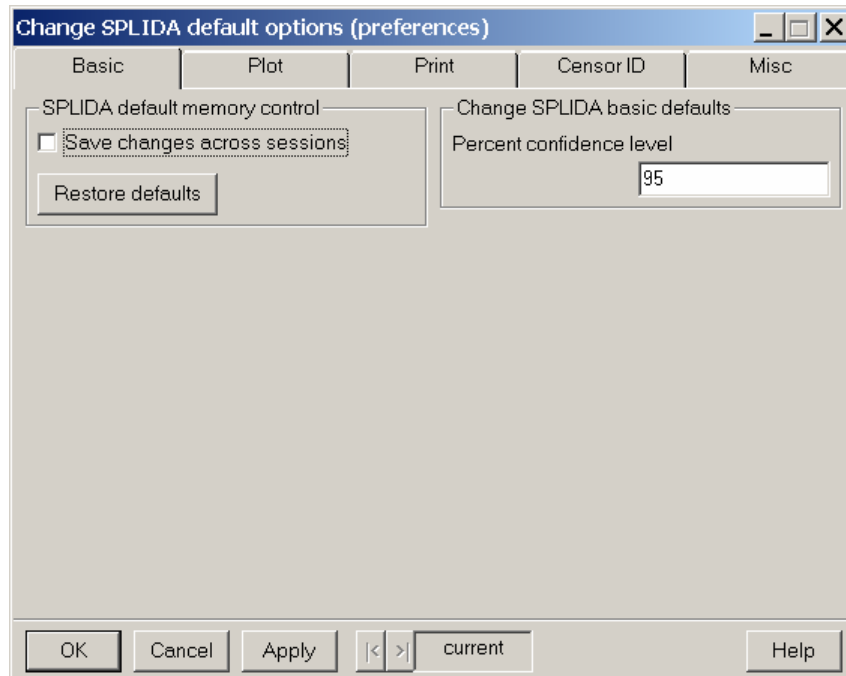
Figure 10 Basic page of the dialog for changing SPLIDA operation defaults.

The "Plot" page of the SPLIDA options dialog, illustrated in Figure 11, allows the user to choose whether to date-stamp each plot (the default) or not. The "label or name on plot" option allows some personalization in which the user can insert a company or personal name on the plot.

Figure 12 shows the "Print" page of the dialog, which allows changing the default number of digits used in SPLIDA tabular output, the default list of quantiles to be estimated, and an option to print out the variance-covariance matrix of ML estimates when such estimation is performed.
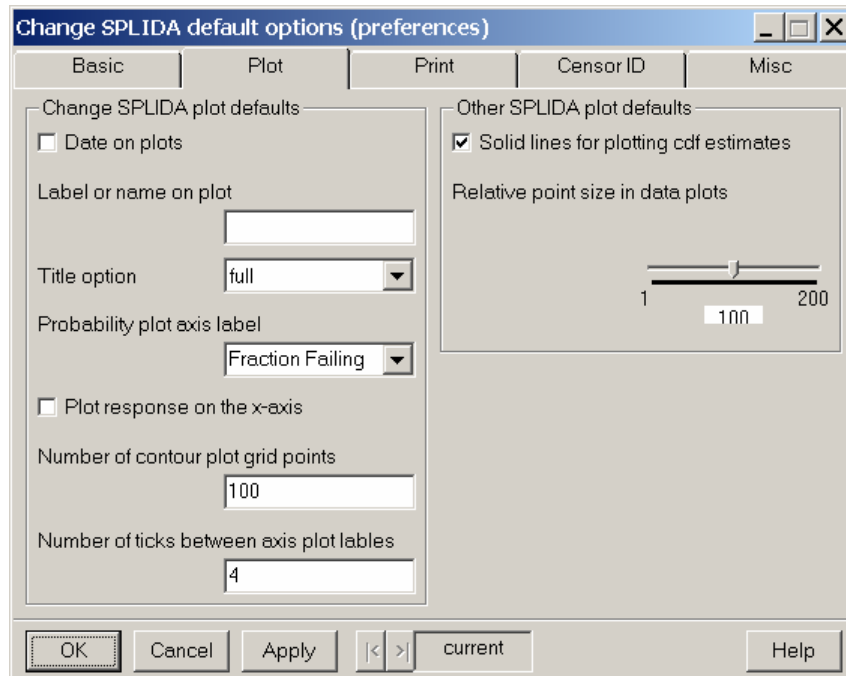
Figure 11 Plot page of the dialog for changing SPLIDA default options.
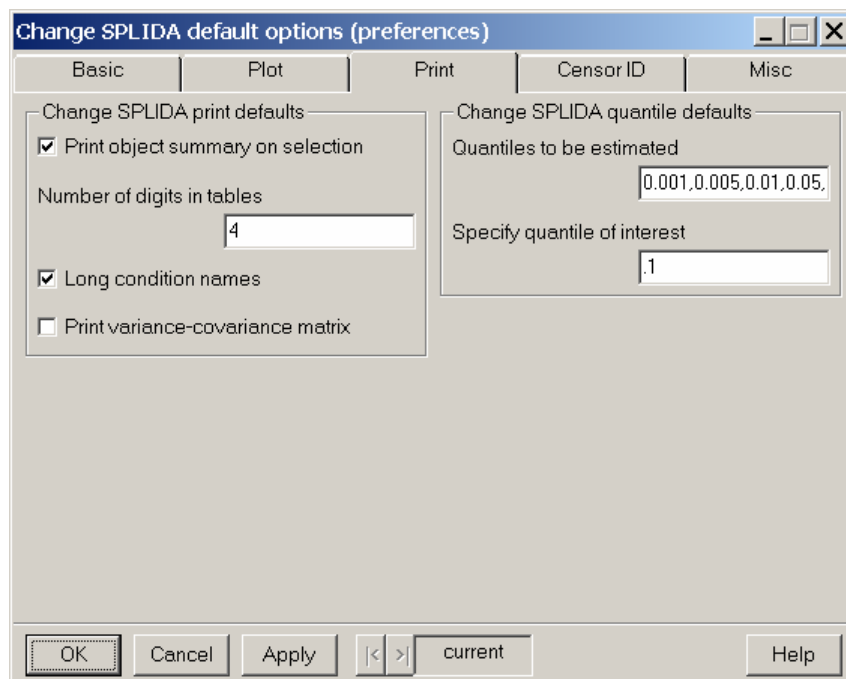
Figure 12 Print option page of the dialog for changing SPLIDA operation defaults.

3.2   SPLIDA data-type aliases

SPLIDA maintains internal lists of names or aliases that can be used to identify "exact failures" and three different kinds of censored observations: right censored, left censored, and interval censored. These lists are used in creating a data object

when there is a "Status" or "Censoring" identifer column (e.g., used to differentiate between failures and censored observations). The default aliases for different kinds of observations are:

- Exact failures: Fail, Failed, Failure, Dead, Died, Exact, E, F, Report, or the number 1.
- Right censored: Alive, Censored, Censor, C, Noreport, R-Censored, Removed, Right, R, S, Survived, Survive, Suspend, Suspended, Saturated, or the number 2.
- Left censored: L-Censored, Left, L, L-censored, left, Miss, or the number 3.
- Interval censored: Bin, Interval, I, or the number 4.

These aliases are not case sensitive (i.e., you can use "fail" or "Fail").
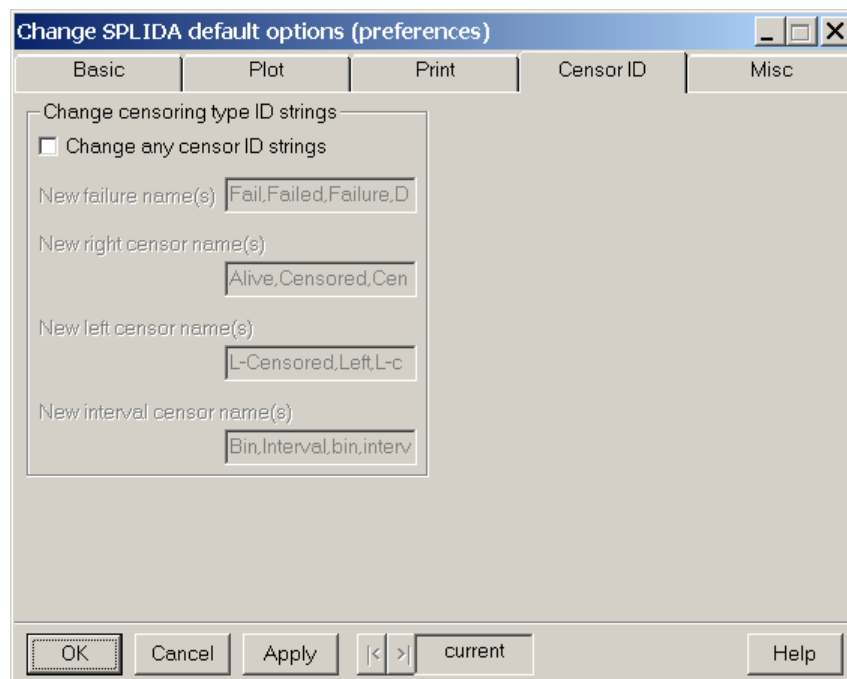
Figure 13 Dialog page allowing changes in the default aliases for observation identification.

It is possible to add to or modify the default alias lists by using the "Censor ID" page of the "Make life data object" dialog when creating a particular data object. Alternatively, it is possible to make a persistent change (either for the current session or across sessions) by using the **SPLIDA ► Change SPLIDA default options (preferences)** menu item and going to the "Censor ID" page. In either case, the dialog page looks like Figure 13. To edit an entry, check the "Change any censor ID strings" box and then right-click on the list to be changed and choose zoom, to allow easy editing.

3.3   Other SPLIDA options
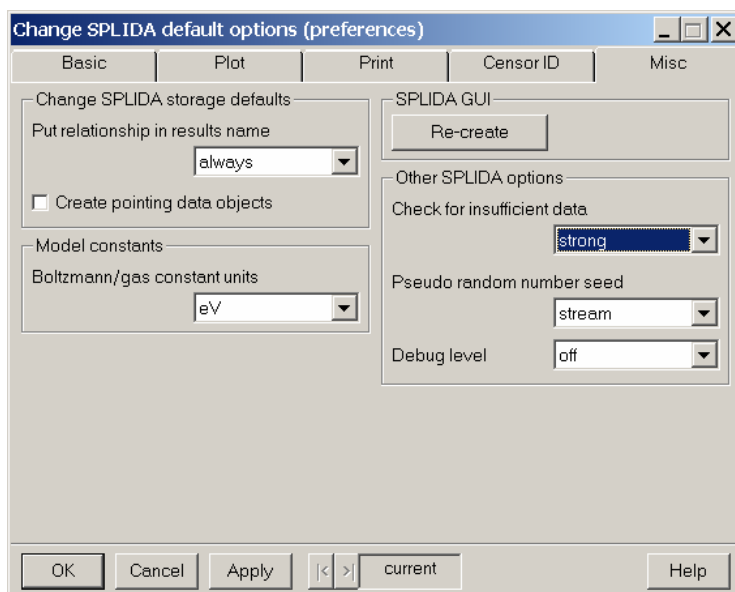
Figure 14 Misc page of the dialog for changing SPLIDA operation defaults.

The "Misc" page, shown in Figure 14, allows the user to choose other options. In particular

- Whether to include the regression relationship(s) as part of the name of ML estimation results objects (the choices are: always, never, and multiple regression only).

- "Create pointing data objects" for which the main part of the data object is a character string giving the name of the data set to use. By default, the main part of the data object is the data set itself. Usually, the character string version will use less memory, but require more execution time.

- Recreate the SPLIDA GUI. This is most useful when a new version of SPLIDA has been installed on your system, and for some reason, the automatic update feature did not work. Clicking the Recreate button will modify the S-PLUS Prefs directory in the current working folder.

- Change the units for Boltzmann's constant (or gas constant) used in the Arrhenius relationship.

- Choose the level of checking for data that are sufficient to provide unique ML estimates. The decision rules used here are not perfect and are set, by default, to "strong." If you get a message that your data were not sufficient, you can try a lower level of checking, but should watch the results carefully to make sure that proper convergence was achieved.

- Generally, the debug level should be set to "off." Otherwise, SPLIDA will print some level of extra information that is used only in debugging.

4 Single Distribution Life Data Analyses

Analysis of data for the purpose of estimating a *single* underlying failure-time distribution is the most common type of reliability data analysis**.** This chapter describes methods for such analyses. Even when there are several different identifiable groups, it is often desired to focus on one group or another or (when appropriate) to pool data from different groups for analysis**.** Choosing **SPLIDA ► Single distribution life data analyses** shows the SPLIDA single distribution analysis options. For this series of examples, we will use the turbin fan failure data (based on the diesel generator fan failure data described on page 133 of Nelson 1982 and reanalyzed in Example 7.12 of Meeker and Escobar 1998) , for which a life data object Fan.ld, is built into SPLIDA.

Figure 15 *is a snapshot of the* ***SPLIDA ► Single distribution life data analyses*** *part of the SPLIDA menu. The SPLIDA menu items (shown on the left) correspond to different tasks/analyses (e.g., plan a life test, analyze single-distribution life test data, compare two different populations, etc.). The SPLIDA submenu items (shown on the right) are organized according to the order in which a complete analysis would be done, typically starting with simple graphical methods and then progressing to model fitting, graphical display of model-fitting results, and sensitivity analyses.*
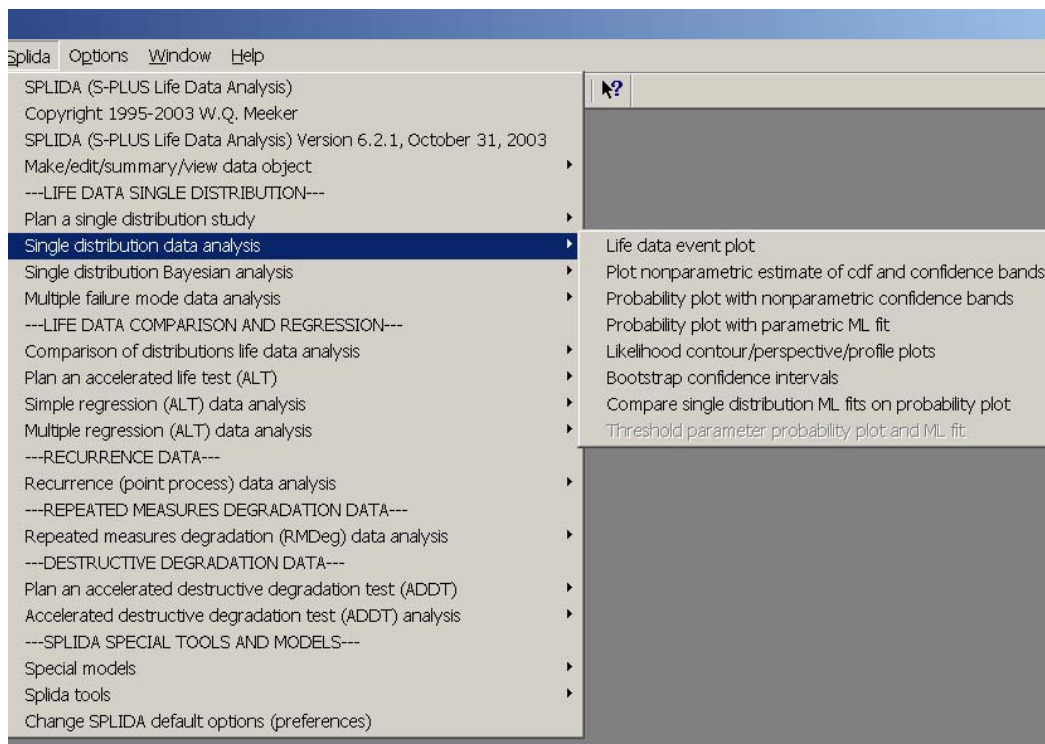
Figure 15 View of the SPLIDA menu showing the options for single distribution analyses.

4.1   Event plot

For some data sets, an event plot provides a useful visualization of censoring patterns in the data. Using **SPLIDA ► Single distribution life data analyses ► Life data event plot** brings up the dialog shown in Figure 16.

Figure 16 Dialog requesting a life data event plot for the fan data.

After choosing the Fan.ld life data object from the pull-down list, click "Apply" or "OK" to see the event plot shown in Figure 17. Lines in Figure 17 ending in an * indicate failures. Lines with a → at the end are units that had not failed at the time that the data were analyzed. The numbers indicate the observation multiplicity for those observations that have counts greater than one. In the case of the fan data, there is a complicated pattern of reported failure times and right-censored observations resulting from the different amounts of operating time for the systems in which the fans had been used.
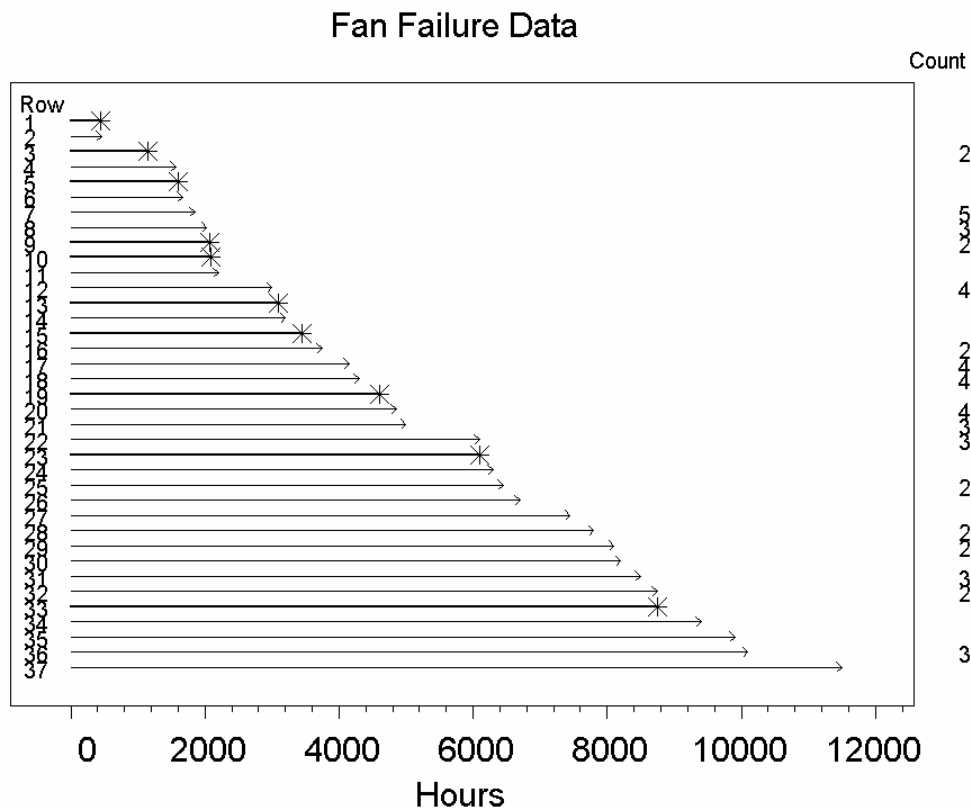
Figure 17 Life data event plot for the fan data.

4.2   Nonparametric estimation of the failure-time cdf

Using **SPLIDA ► Single distribution life data analyses ► Plot nonparametric estimate of cdf and confidence bands** brings up the dialog in Figure 18.
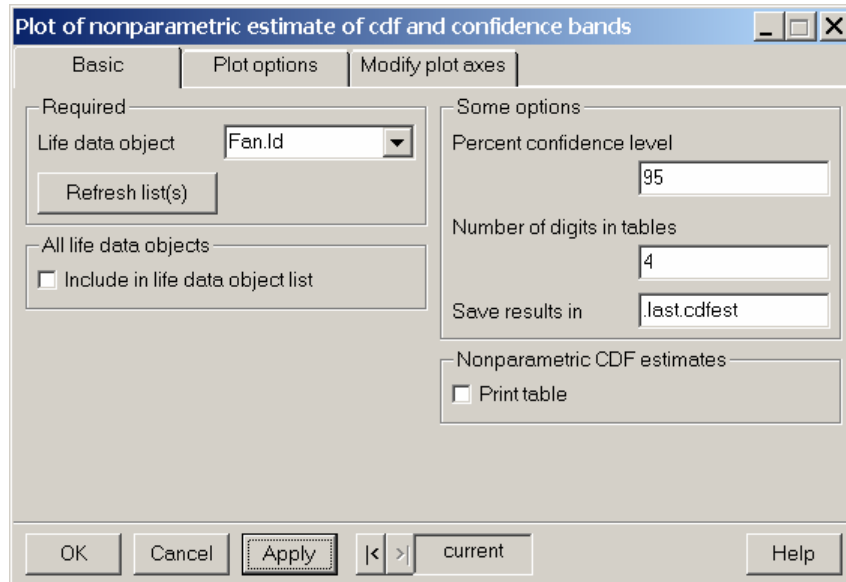
Figure 18 Dialog for requesting a plot and table of the estimated fraction failing as a function of time.

Then choose life data object `Fan.ld` (the only required input), put a check in "Print table" and click "Apply." This will produce a plot like that in Figure 19 and a table of the nonparametric estimate and confidence intervals for the fraction failing as a function of time. The estimates are computed using the Kaplan-Meier estimator. When data have complicated censoring pattterns such as interval-censored observations with overalpping intervals, the Peto/Turnbull estimator (a generalizationof the Kaplan-Meier estimator), is used instead (as described in Section 3.10 of Meeker and Escobar 1998). Chapter 3 of Meeker and Escobar (1998) describes the methods for computing nonparametric estimates and corresponding confidence intervals/bands from censored data (including complicated arbitrary censoring) and provides references and examples corresponding to these SPLIDA capabilities.

Fan Failure Data
Nonparametric CDF Estimate
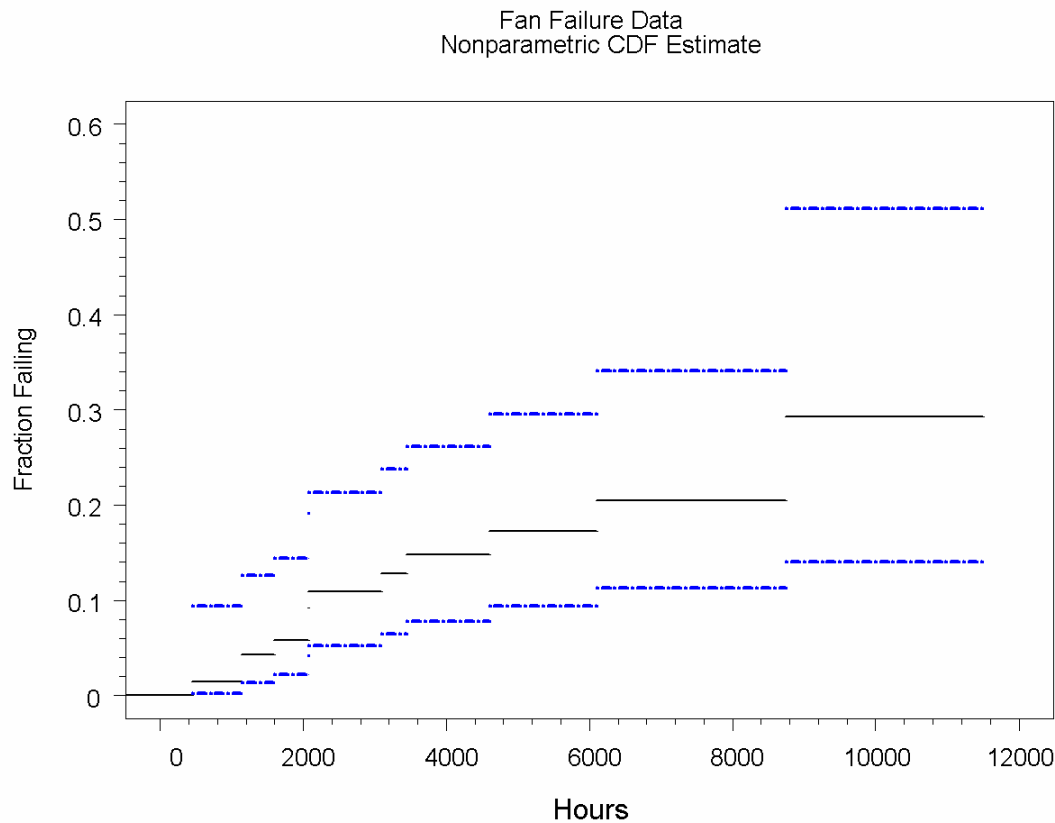
Figure 19 Plot of estimated fraction failing as a function of time for the turbine fans.

Table 1 provides a nonparametric estimate from the turbine fan data, along with standard errors and approximate 95% pointwise confidence intervals.

Table 1 Kaplan-Meier nonparametric estimate of the fraction failing as a function of time for the turbine fans.

Nonparametric estimates from Fan Failure Data
with approximate 95% pointwise confidence intervals

	Hours-lower	Hours-upper	Fhat	SE_Fhat	95% Lower	95% Upper
1	0	450	0.00000	0.00000	NA	NA
2	450	1150	0.01429	0.01418	0.002009	0.09449
3	1150	1600	0.04328	0.02444	0.014024	0.12577
4	1600	2070	0.05800	0.02815	0.021934	0.14459
5	2070	2080	0.09225	0.03607	0.041857	0.19121
6	2080	3100	0.10938	0.03925	0.052811	0.21291
7	3100	3450	0.12833	0.04274	0.065085	0.23742
8	3450	4600	0.14770	0.04597	0.078091	0.26173
9	4600	6100	0.17277	0.05100	0.094016	0.29593
10	6100	8750	0.20458	0.05812	0.113253	0.34122
11	8750	11500	0.29296	0.09804	0.140791	0.51166

When using SPLIDA, it is often better to click on "Apply" rather than on "OK" to request an analysis. Then minimize the dialog to better see the results. This allows one to modify the analysis easily; restore the dialog, make desired changes, click "Apply," and

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again minimize the dialog. When completely done with the dialog, click "Cancel." If one uses "OK" instead, it will be more difficult to modify previous analyses, as certain steps in the analysis specification would need to be retraced. S-PLUS allows one to choose a previous dialog state by clicking on the buttons at the bottom of the dialog. When doing this, however, S-PLUS does not properly restore whether entries in the dialog are active or inactive

4.3   Probability plots and distribution assessment

Probability plots are one of the most important tools for reliability data analysis. The ideas and concepts underlying probability plots, along with a large number of examples, is given in Chapter 6 of Meeker and Escobar (1998). The basic idea is to plot a representation of a nonparametric estimate of the fraction failing as a function of time (traditionally, this estimate has been a set of points) on special distribution-dependent nonlinear plotting axes on which the specified theoretical distribution would plot as a straight line.

Use **SPLIDA ► Single distribution life data analyses ► Probability plot with nonparametric confidence bands** to bring up the basic probability plot dialog, illustrated in Figure 20. Again, choose the life data object Fan.ld, choose "6 distributions," click "Apply," and minimize the dialog. This will make a six-distribution probability plot like the one in Figure 21, from which one can visually compare several different distributions, including the popular Weibull and lognormal distributions.
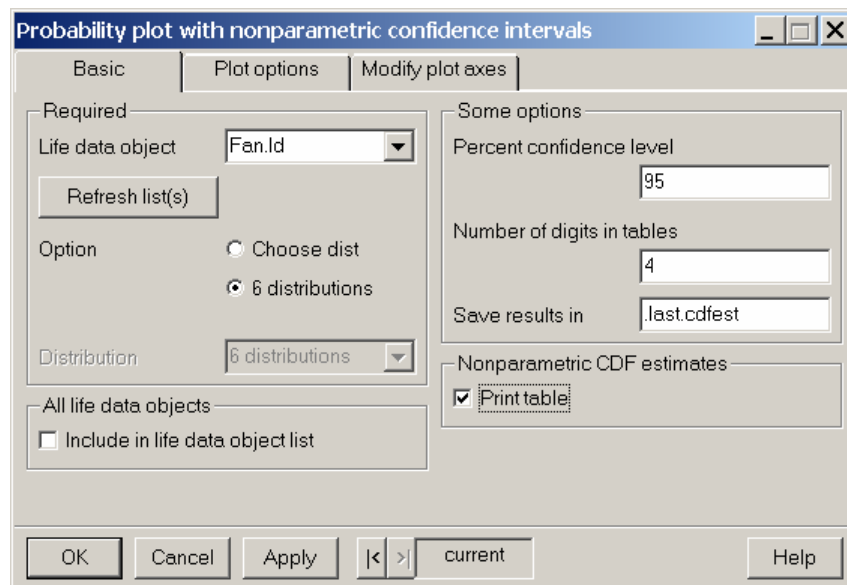
Figure 20 Dialog requesting the 6-distribution probability plots with (default) simultaneous confidence bands.
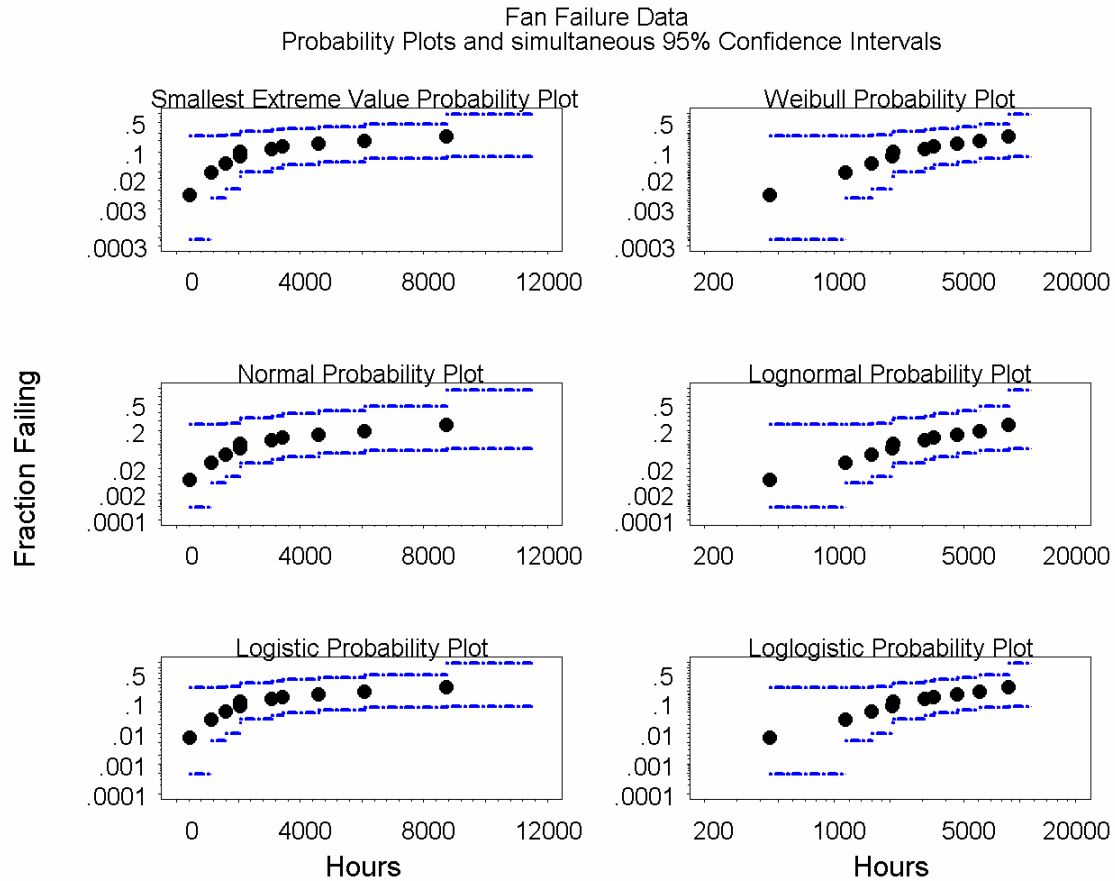
Figure 21 Six-distribution probability plots with simultaneous confidence bands.

Now, restore the dialog in Figure 20, change to "Choose dist," and choose a particular distribution (e.g., lognormal), and click "Apply." This will create the probability plot, shown in Figure 22, that will allow one to focus the reasonableness of the specified distribution as a model for the data. By default, simultaneous confidence bands are also provided. See Sections 3.8 and 6.3 of Meeker and Escobar (1998) for technical details, references, and other examples of these simultaneous confidence bands.

Fan Failure Data
with Nonparametric simultaneous 95% Confidence Bands
Lognormal Probability Plot
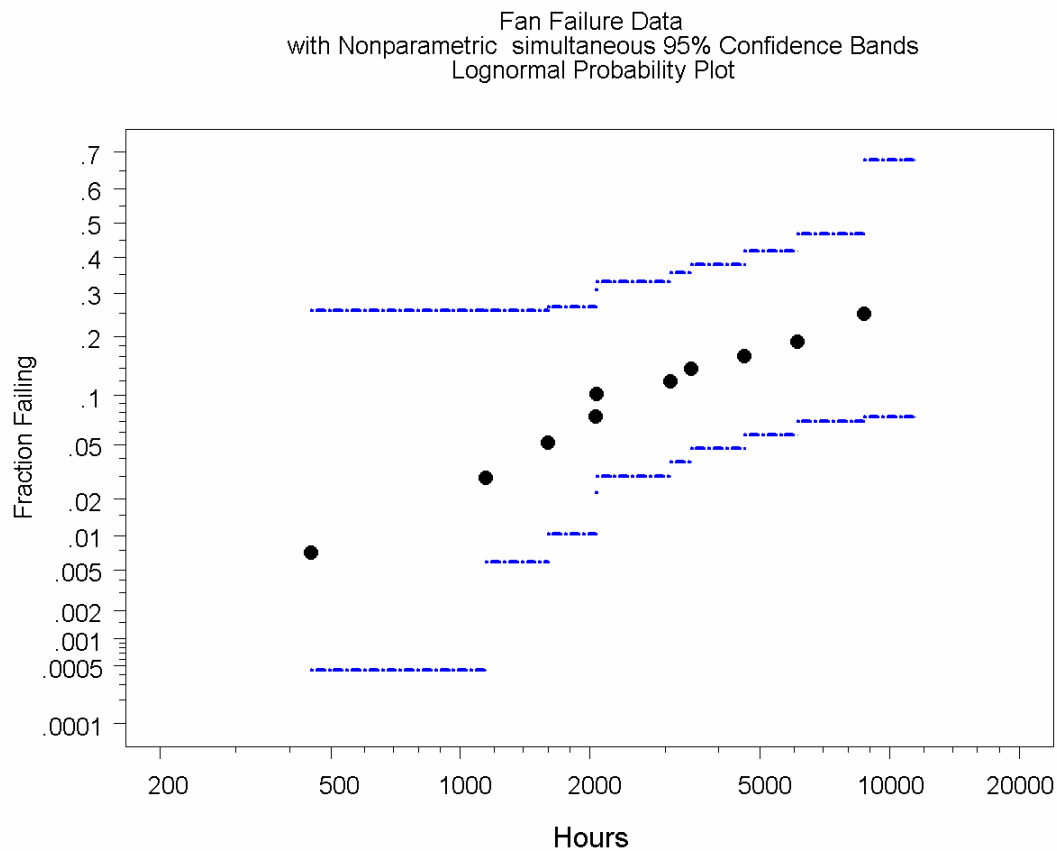
Figure 22 Lognormal probability plot with simultaneous confidence bands.

Chapter 6 of Meeker and Escobar (1998) provides theory and a detailed description of methods for interpreting these probability plots. If one can draw a straight line through the simultaneous confidence band, for the chosen distribution (lognormal in Figure 22), then there are lognormal distributions that are consistent with the data. Then it is not possible to rule out the possibility that the data were generated by a lognormal distribution. One should, of course, try other distributions and compare the results.

4.4   Maximum likelihood estimation of the failure-time cdf

Using **SPLIDA ► Single distribution life data analyses ► Probability plot with parametric ML fit** produces the dialog in Figure 23. This dialog allows the user to request a probability plot showing the ML estimate for the chosen distribution (plotted as a straight line) along with the data points usually plotted in the probability plot.
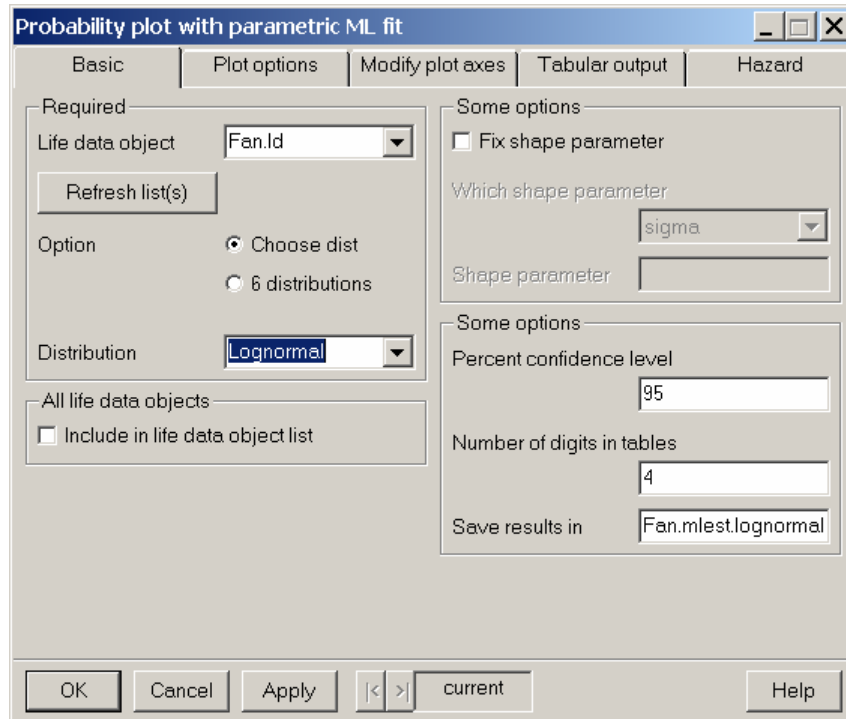
Figure 23 Dialog for getting a probability plot with an ML fit.

By default, SPLIDA also plots a set of normal-approximation 95% pointwise confidence intervals. These intervals allow one to obtain a visualization of estimates of both failure probabilities and distribution quantiles and the associated statistical uncertainty due to limited sample size. The Hazard page of the dialog in Figure 23, shown in Figure 24, allows the user also to obtain a plot of the corresponding parametric ML estimate of the hazard function versus time. A table of the hazard function estimates and corresponding confidence intervals can also be requested. The resulting probability plot with ML estimates is shown in Figure 25 and the plot of the hazard function is given in Figure 26. Chapters 2, 7 and 8 of Meeker and Escobar (1998) give the theory and methods for understanding these functions and using the corresponding analyses.
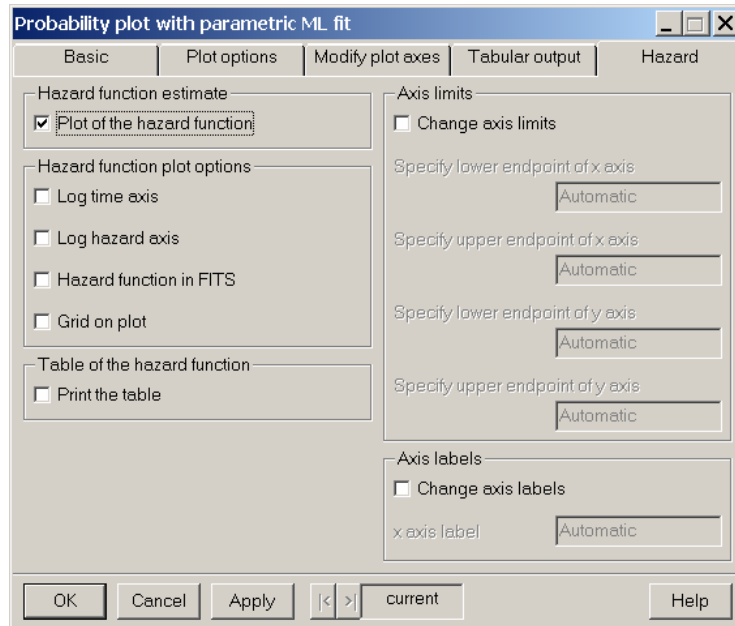
Figure 24 The Hazard page of the probability plot with ML fit dialog, allows the user to request and customize a plot of the hazard function ML estimate.

Table 2 gives tabular output from fitting the lognormal distribution to the fan data. The rows with the Greek letters mu (μ) and sigma (σ) are, respectively, ML estimates for the mean and standard deviation of the distribution of logarithm of fan life. Standard errors and 95% normal-approximation confidence intervals are also given.

Table 2 Lognormal ML estimation results for the turbine fan data.

```
Fan Failure Data
Maximum likelihood estimation results:
Response units: Hours
Lognormal Distribution
Log likelihood at maximum point: -134.5

Parameter                    Approx Conf. Interval
         MLE Std.Err.   95% lower 95% upper
    mu 10.14    0.5211       9.122     11.165
sigma  1.68    0.3893       1.066      2.645
```

Fan Failure Data
with Lognormal ML Estimate and Pointwise 95% Confidence Intervals
Lognormal Probability Plot
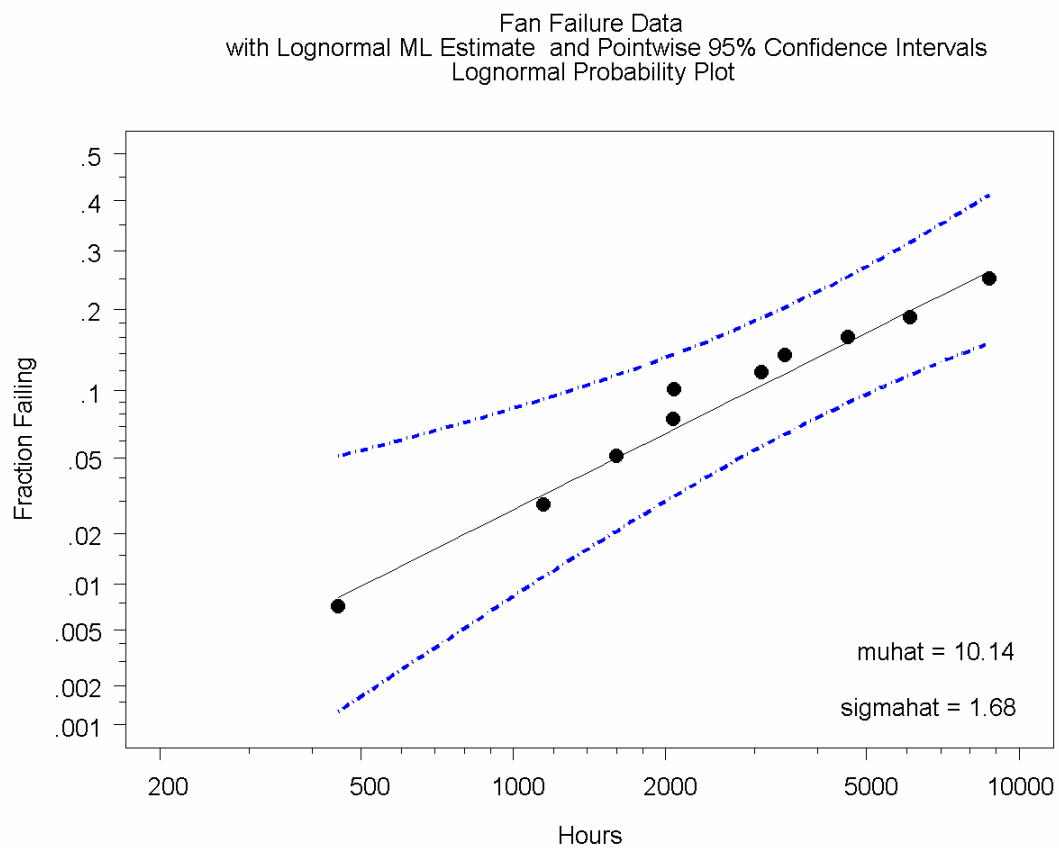
Figure 25 Lognormal probability plots of the fan data showing the ML estimate and parametric *pointwise* confidence intervals.

Fan Failure Data Lognormal Distribution Hazard Function ML Estimate
and Pointwise 95% Confidence Intervals

muhat = 10.14

sigmahat = 1.68
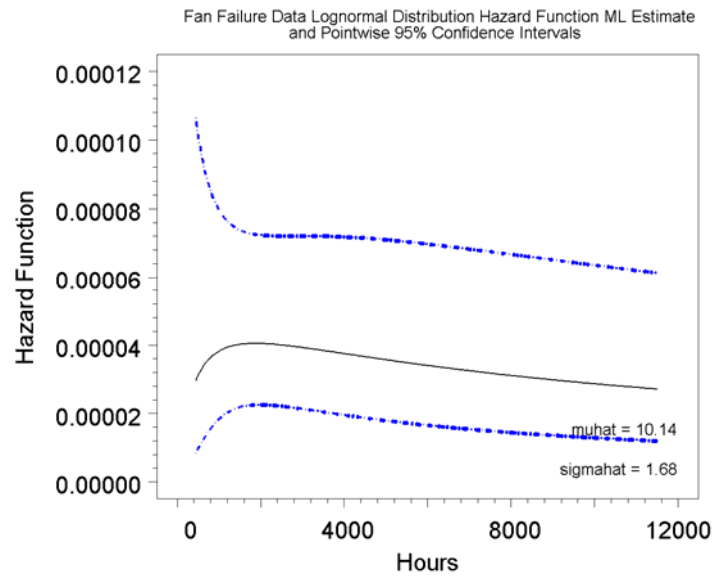
Figure 26 Lognormal hazard function estimate and parametric *pointwise* confidence intervals for the fan data.

4.5   Comparison of ML estimates from different distributions

The dialog produced by **Single distribution life data analyses ► Compare distribution ML fits on probability plot**, shown in Figure 27, allows the comparison of ML estimates for a chosen baseline distribution with one or more other distributions. The required inputs are the life data object (for the following example we continue to use the Fan.ld life data object), the baseline distribution, and the comparison distribution(s).
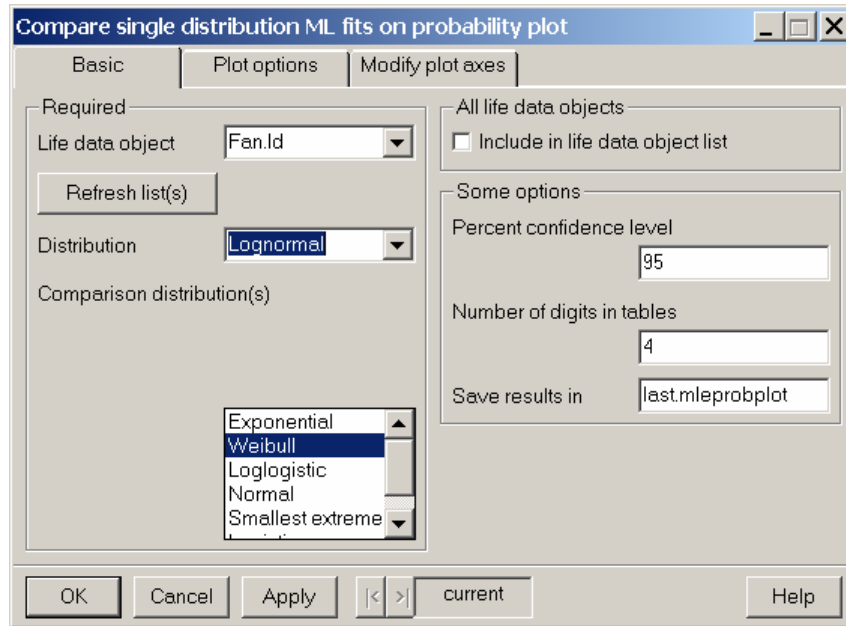
Figure 27 Dialog requesting a comparison of the lognormal and Weibull ML estimates on lognormal probability paper.

Suppose that the objective of the analysis of the fan data was to estimate life out to 50,000 hours. In order to do this, select the "Choose cdf evaluation range" in the Plot Options page, shown in Figure 28, of the "Compare single distribution ML fits on probability plot" to request extrapolation outside of the range of the data . Clicking on "OK" or "Apply" will then produce the desired comparison probability plot shown in Figure 29 (changing the range using the options in the Modify plot axes page of the dialog will change only the axis range and not the range of cdf evaluation).
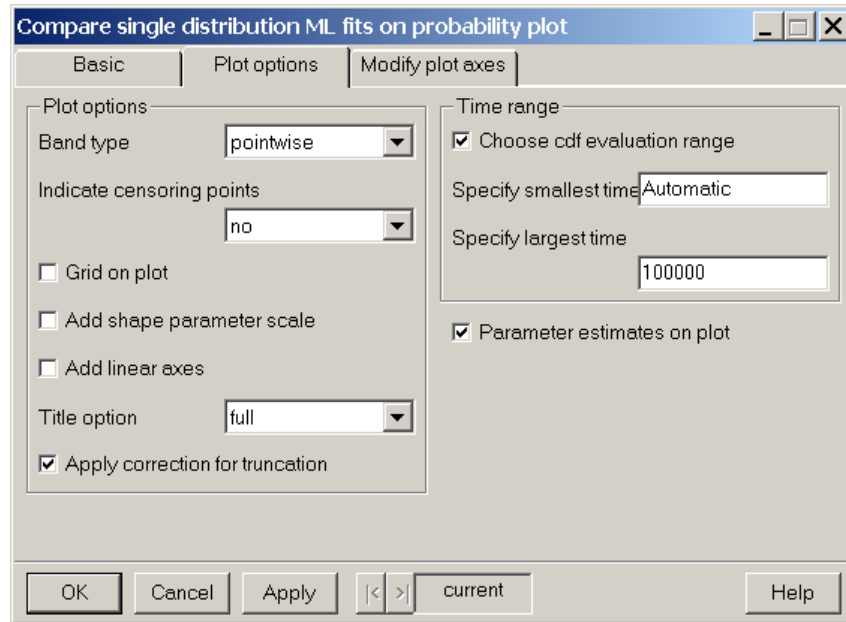
Figure 28 Plot Options page of the compare ML fits dialog showing how to extend the range of cdf evaluation
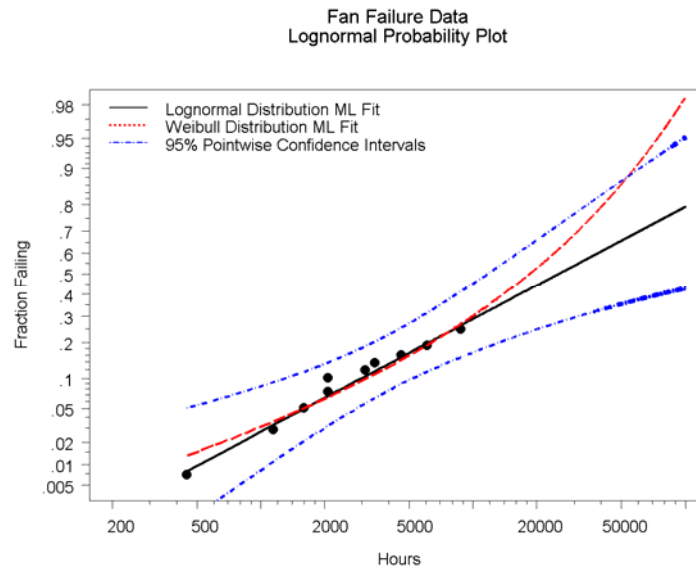
Figure 29 Lognormal probability plot comparing the lognormal and Weibull ML estimates for the fan data.

There is little difference between the Weibull and lognormal probability estimates up to 1000 hours. Note, however, the large difference between the estimates of fraction failing outside the range of the data.

4.6 Likelihood contour/perspective plot

Using **SPLIDA ► Single distribution life data analyses ► Likelihood contour/perspective plot** produces the dialog shown in Figure 30. This dialog allows the user to request either a contour or a perspective plot of the relative likelihoood function or the corresponding joint confidence region (described and compared in Sections 8.2 and 8.3 of Meeker and Escobar, 1998) for a given set of data. Alternatively, one can request a perspective (or wire-frame) plot of the relative likelihood function. The dialog has an option to indicate the position of the ML estimates, to request correcponding profile likelihood plots, as well as the usual options to control plot axes.

The likelihood can be computed as a function of the underlying location and scale (scale and shape) parameters of the location-scale (log-location-scale) distributions. Alternatively one can the scale (shape) parameter and a specified quantile of the distribution. We will illustrate both.

We illustrate the use of this dialog with the bearing cage data (life data object `BearingCage.ld`) from Abernathy et al. (1983). This data object is built into SPLIDA. Figure 31 shows the perspective plot of the Weibull relative likelihood, followed by the contour plot of the joint confidence region in Figure 32.

Figure 30 Dialog for requesting likelihood plots of the bearing cage likelihood function.
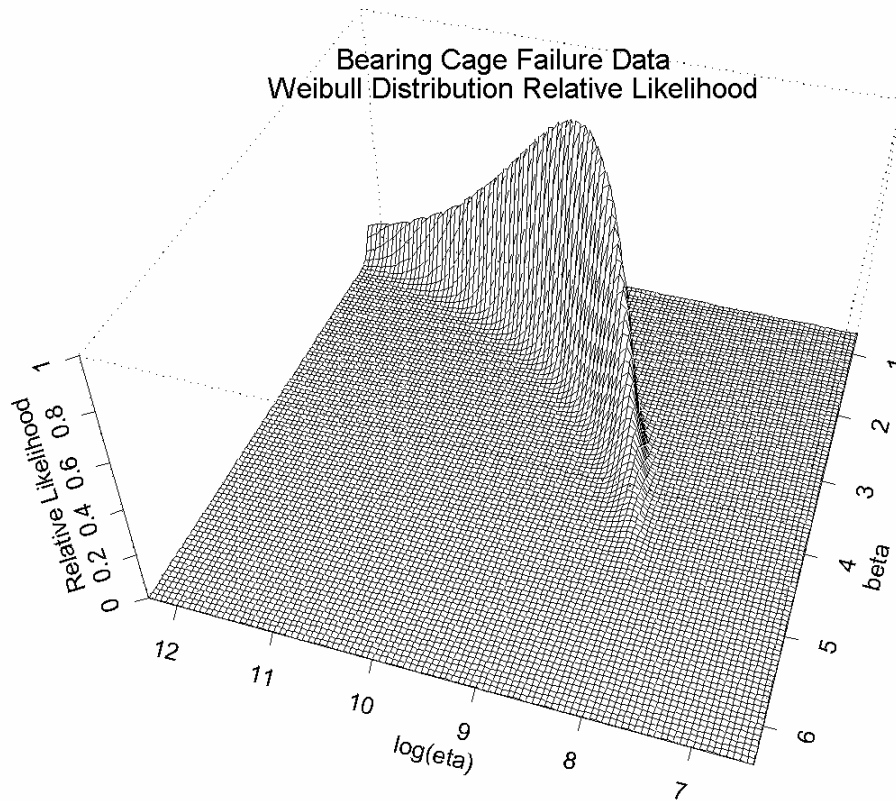
Figure 31 Perspective plot of the relative likelihood function for Weibull parameters based on the bearing cage data.

Notice the "shark-fin" shape of the relative likelihood in Figure 31. This reflects the strong positive correlation between the ML estimators of the Weibull parameters; this correlation is the result of the heavy right censoring in this data set (the estimate of the fraction failing is less than .06 at the largest failure time). The contour plot of the joint confidence region for the Weibull parameters shown in Figure 32 is easier to interpret and shows (by option) the ML estimates.

Bearing Cage Failure Data
Weibull Distribution Joint Confidence Region
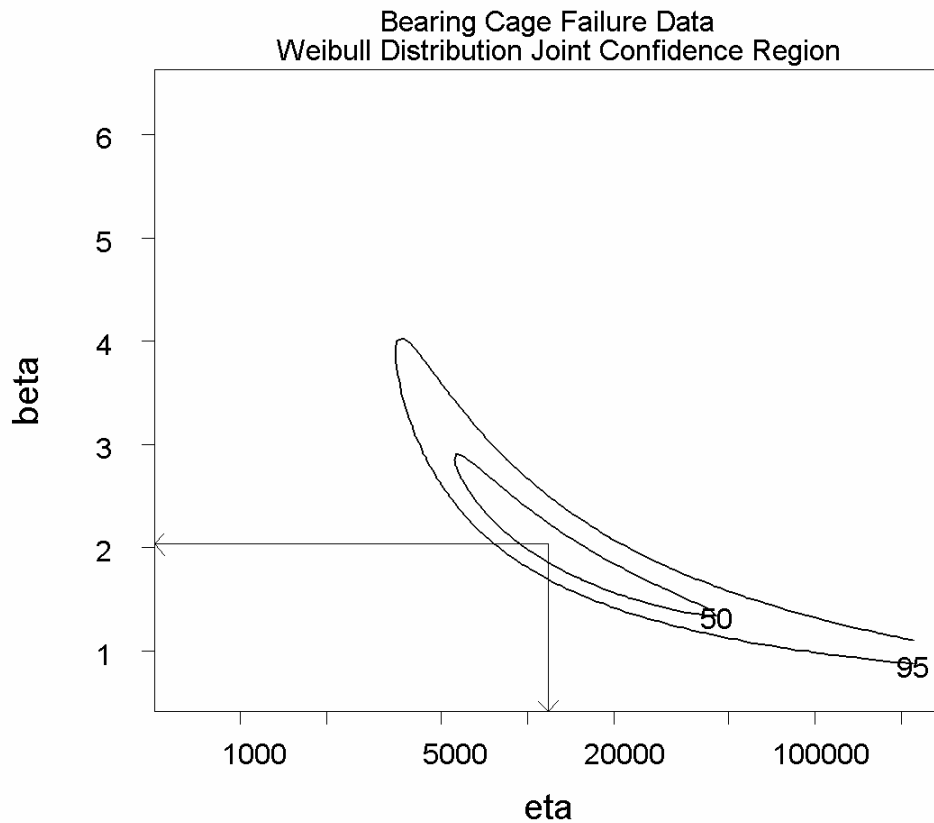
Figure 32 Joint confidence region for Weibull parameters based on the relative likelihood function for the bearing cage data.

Figure 33 shows a similar contour plot for the 0.1 quantile and σ for the Bearing Cage data with the Weibull distribution and **Figure 34** shows the corresponding profile likelihood for the 0.1 quantile. The upper endpoint of the confidence interval for the .1 quantile is NA because the profile likelihood had not dropped far enough to cross the 95% line in **Figure 34**.

There were problems finding the confidence interval from the computed likelihood profile

The approximate 95% likelihood confidence interval for 0.1 Quantile Hours is: 2082   NA

The approximate 95% likelihood confidence interval for beta is: 0.9697 3.5713

Table 3 Output From the profile likelihood for β and the 0.1 quantile of the Bearing Cage data.
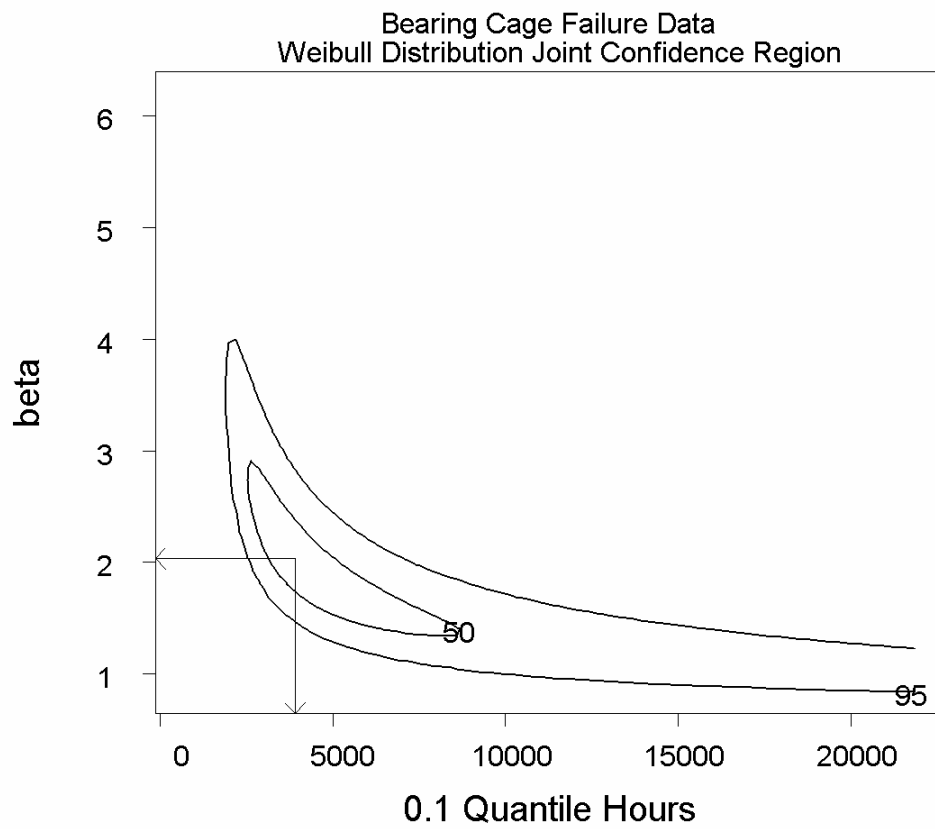
Figure 33 Joint confidence region for the Weibull 0.1 quantile and β based on the relative likelihood function for the bearing cage data.

Bearing Cage Failure Data
Profile Likelihood and 95% Confidence Interval
for 0.1 Quantile Hours from the Weibull Distribution
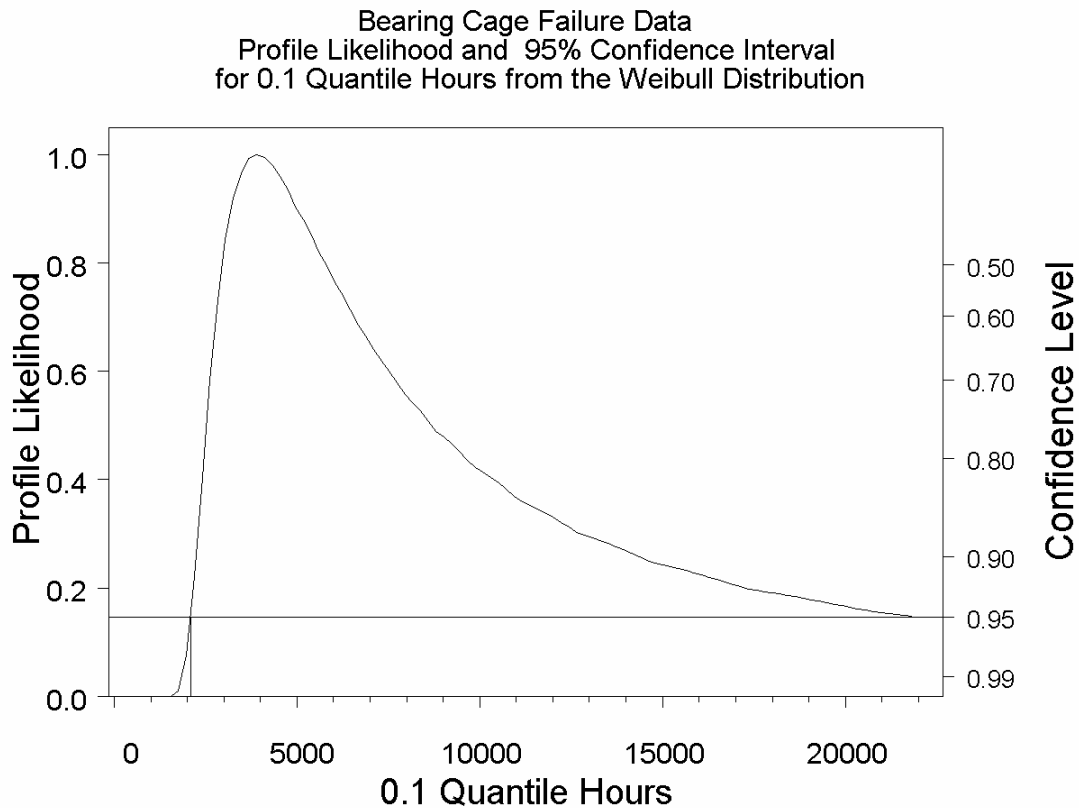
Figure 34 Profile likelihood for the Weibull 0.1 quantile from the bearing cage data.

4.7   Bootstrap confidence intervals

SPLIDA has some simple tools for doing parametric bootstrap for a single distribution (there are plans to extend these tools to more complicated models such as regression). To do such an analysis of the Bearing Cage data, choose use **SPLIDA ► Single distribution life data analyses ► Bootstrap confidence intervals** produces the dialog in Figure 35. Choose the Bearing Cage results object `BearingCage.mlest.weibull.out` and check both boxes under summary plots. Then clicking on Apply will result in summary plots like those in Figure 36 and Figure 37.
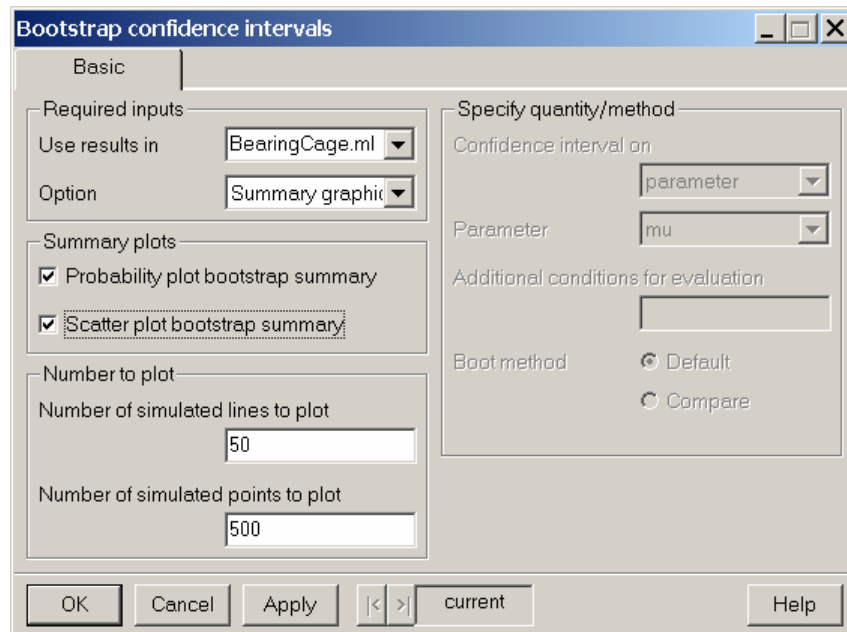
Figure 35 Dialog to request the computation of bootstrap samples and summary plots.

Weibull Distribution ML Estimate and Bootstrap Results for
Bearing Cage Failure Data
Weibull Probability Plot
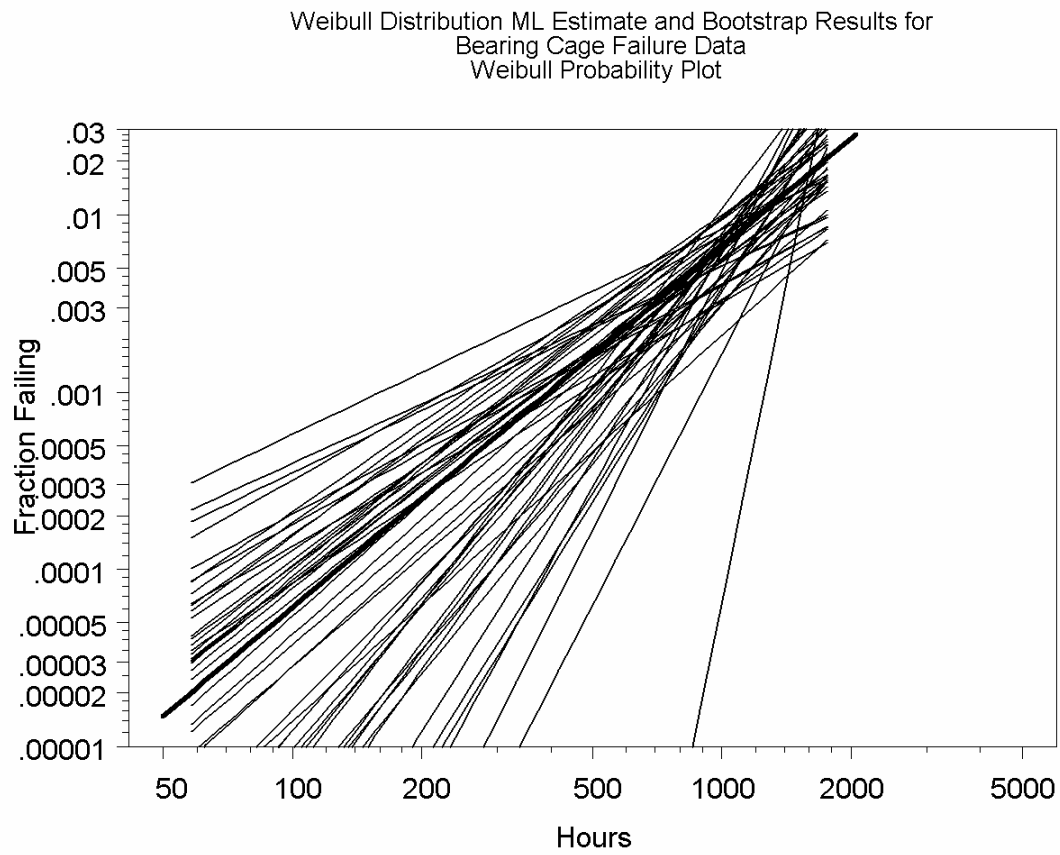
Figure 36 Probability plot showing the ML estimates for 50 of the 2000 bootstrap samples for the Bearing Cage data.
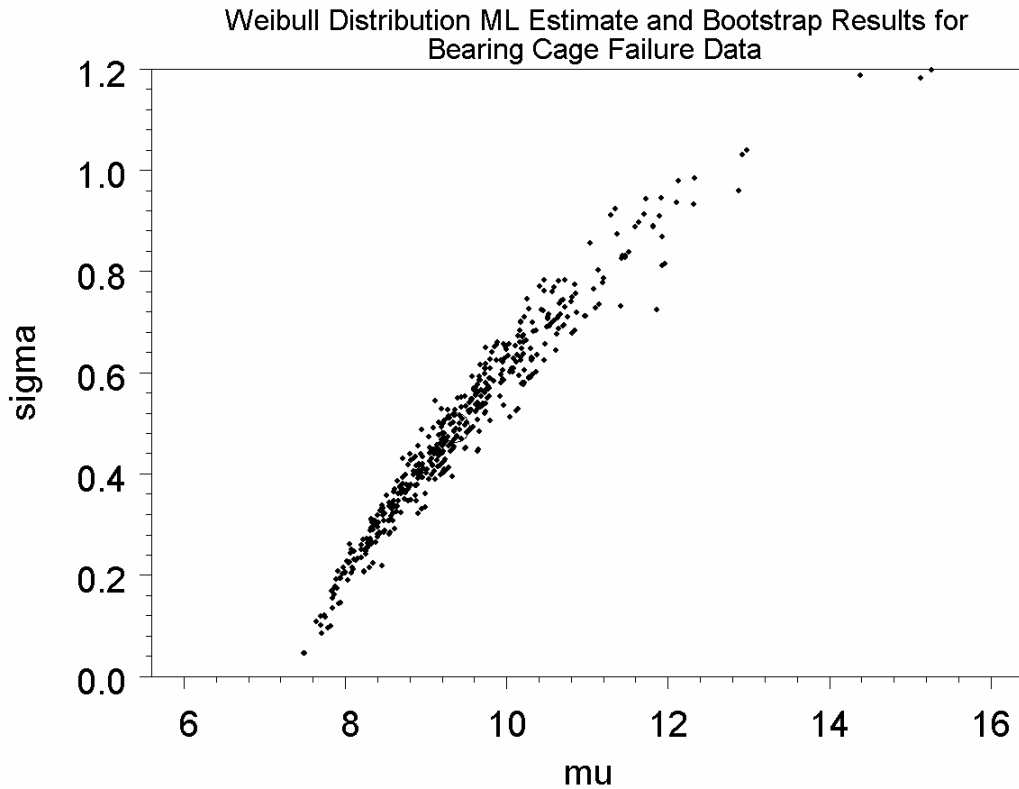
Figure 37 Scatter plot showing the ML estimates of 500 of the 2000 bootstrap samples for the Bearing Cage data.

Restore the bootstrap confidence interval dialog and under Option, choose "Conf interval." On the right-hand side of the dialog, choose "Quantile" under "Confidence interval on" and enter 0.1 in the quantile box. Then click Apply. This will give the text output shown in Table 4, and the histogram of the bootstrap distribution of the t-like statistics in Figure 39. The text output also provides diagnostics about the number of troublesome bootstrap samples. In particular, some samples resulted on no failures.
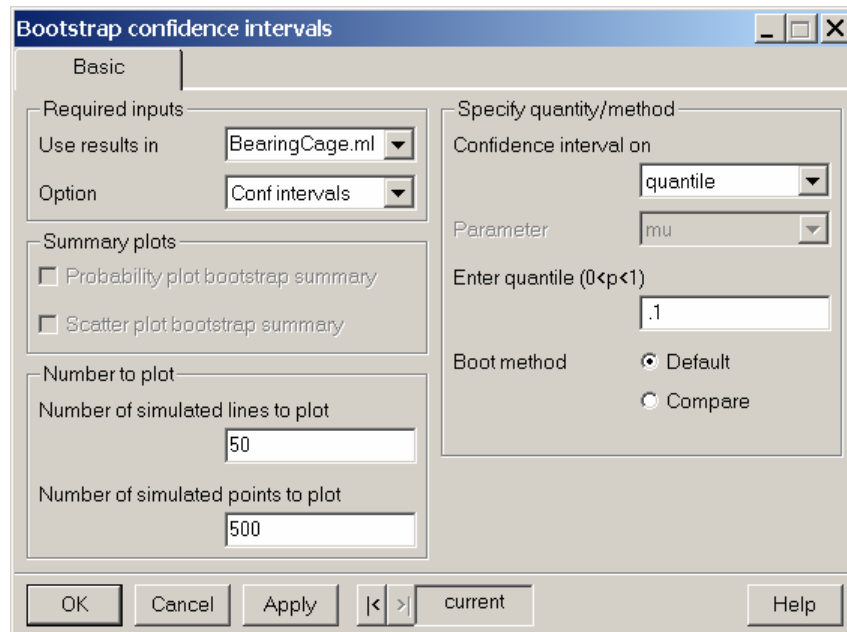
Figure 38 Dialog requesting bootstrap confidence interval for the 0.1 quantile of the Bearing Cage life.
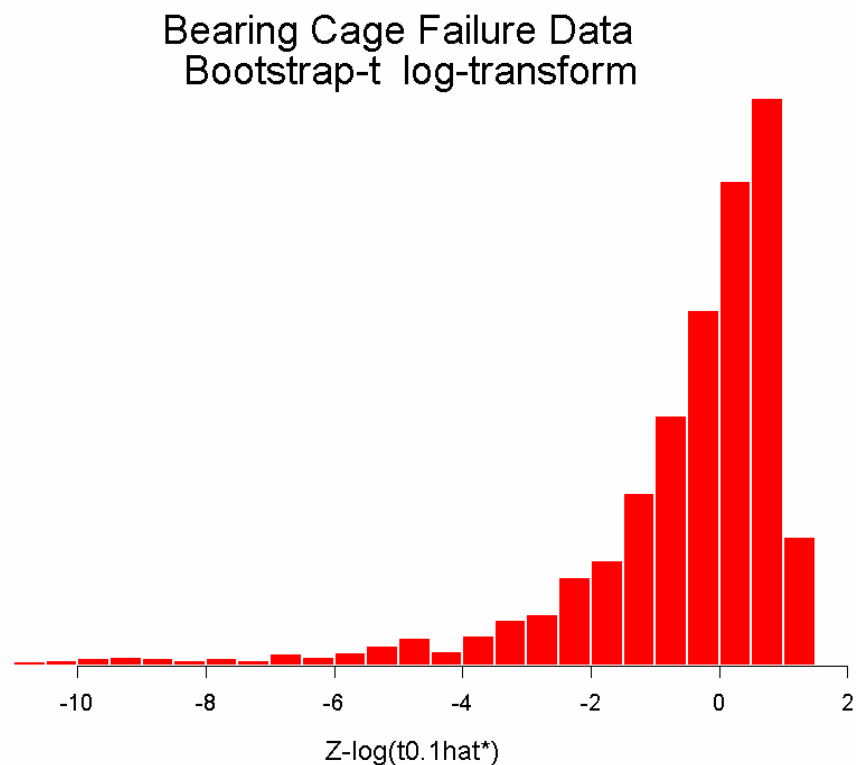
Figure 39 Histogram of the bootstrap-t distribution for the log 0.1 quantile of the Bearing Cage data.

```
There were  1999 unique bootstrap samples out of
 2000 total bootstrap samples

 A total of 5 of 2000 (0.25%) bad bootstrap samples
 were removed because of convergence difficulties.
 These were probably samples with 0 failures
 or no variability in the response.

Using the bootstrap t log transformation method,
an approximate 95 percent confidence interval
for t_{0.1} is [2256.1,   146649] .
```

Table 4 Text output from the bootstrap confidence interval procedure.

4.8 Fitting a log-location-scale (location-scale) distribution with a given shape (scale/slope) parameter

In some applications, especially when the amount of data is limited, it may be useful to fit a distribution with a given shape parameter. Although it is rare that one would know such a shape parameter exactly, such evaluations are useful if one repeats the analysis using different values of the given parameter within given ranges, perhaps based on previous experience with a similar failure mode. More details are given in Section 8.5 of Meeker and Escobar (1998).

Fitting a location scale or log location scale distribution with a fixed shape (slope) parameter is easy in SPLIDA. To illustrate this, we will continue with the bearing cage example. For the bearing cage data, the wide confidence intervals for $F(t)$ outside of the range of the data in Figure 29 are due, in large part, to the fact that the Weibull shape parameter is unknown and that there is only a small amount of data available. Important improvement in precision can be obtained by specifying a value for the shape parameter. Using **SPLIDA ► Single distribution life data analyses ► Probability plot with parametric ML fit** produces the dialog in Figure 40.

Figure 40 Dialog showing how to fit a Weibull distribution with a fixed shape (slope) parameter.

The resulting probability plot is given in Figure 41.

Bearing Cage Failure Data
with Weibull ML Estimate and Pointwise 95% Confidence Intervals
Weibull Probability Plot

etahat = 12320

betahat = 2
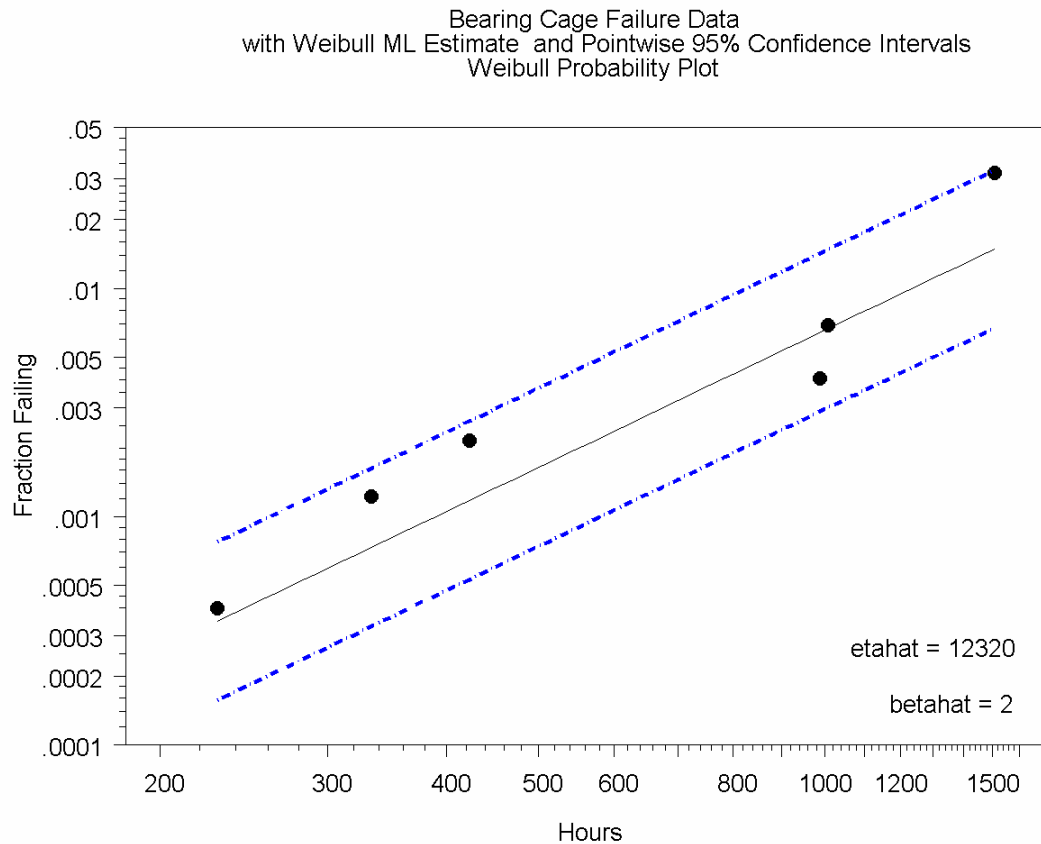
Figure 41 Bearing cage Weibull probability plot with the fixed $\beta=2$ ML estimate line.

Notice that the confidence interval lines in the Weibull probability plot are parallel. This implies that the intervals will be narrower outside of the range of the data. Of course, this would provide a false sense of precision if the Weibull shape parameter (not to mention the assumption of a Weibull distribution itself) does not provide an adequate approximation to the actual bearing cage distribution.

5 Single Distribution Bayes Analysis

This section describes some simple, but useful, procedures for Bayesian estimation with censored data for log-location-scale distributions. The procedures provide bayesian "crediability intervals" for quantities of interest such as distribution parameters, quantiles, and failure probabilities. Graphical outputs of the results from the procedures provide insight into how the Bayesian methods combine prior information (prior distribution) with the data (likelihood).

We will continue the use of the Bearing Cage data used in Section 4.8, where it was assumed that the Weibull shape parameter was given as β=2. The example in this chapter will assume that there is some information about β, described by a prior distribution.

5.1   Specifying prior information

The first step in the use of these procedures to do a Bayesian analysis is to specify the available prior information by creating a prior distribution object. This is done with the dialog obtained by using **SPLIDA ► Single distribution Bayesian analyses ► Specify a prior distribution** produces the dialog in Figure 42. In this dialog, the Model distribution referrs to the distribution of the response in the data object to be used in the analysis, Weibull in this case. In the "Prior for" parts of the dialog, one needs to specify a distribution for each of the Weibull "parameters."

Figure 42 Dialog to specify the prior distributions for the Weibull parameters for the Bearing Cage data.

As described in Section 14.3 of Meeker and Escobar (1998), the quantile parameter should be chosen such that its prior distribution is approximately independent of the prior distribution for the shape/spread parameter. In the Bearing Cage example, the 0.1 quantile was chosen because previous experience with such components had seen, ultimately, somewhat more than 10% failing while in service. The prior distribution is chosen as log-Uniform between 1000 and 20000 (i.e., a distribution that is uniform on the log scale between log(1000) and log(20000)). The prior distribution for this parameter has a very broad range, and was chosen to reflect the small amount of knowledge available for this quantity for this specific bearing cage. Such a prior is known as a "diffuse prior distribution."
More precise information is available for the Weibull shape parameter β. In this case, the prior distribution is specified to be lognormal with 99% of its probability between 1.5

and 3 (using such a range for a prior distribution is more convenient than using the traditional lognormal parameters that have little or no meaning for engineers from which such information is elicited). By default, the name of the resulting prior distribution object is `last.distribution.prior`, where `distribution` is automatically replaced by the chosen distribution. Often it is useful to specify the name of this object, so that it can be recognized later. `BearingCage.weibull.prior` was used in the example. Clicking on OK or Apply will create the prior object (with a summary being printed).

5.2   Making a posterior distribution

Using **SPLIDA ► Single distribution Bayesian analyses ► Make a posterior distribution** brings up the dialog in Figure 43. This simple dialog is used to make a posterior distribution (actually what is generated is a sample from the posterior using the simple method described in Section 14.4.2 of Meeker and Escobar 1998 and Smith and Gelfand and 1992). All that one needs to specify is the prior object and the life data object. The default number of points in the posterior is 6000, and this is sufficiently large for most purposes. There is an option to change the name of the resulting posterior object, but the default (obtained by pasting together the names of the input object names) is almost always adequate.
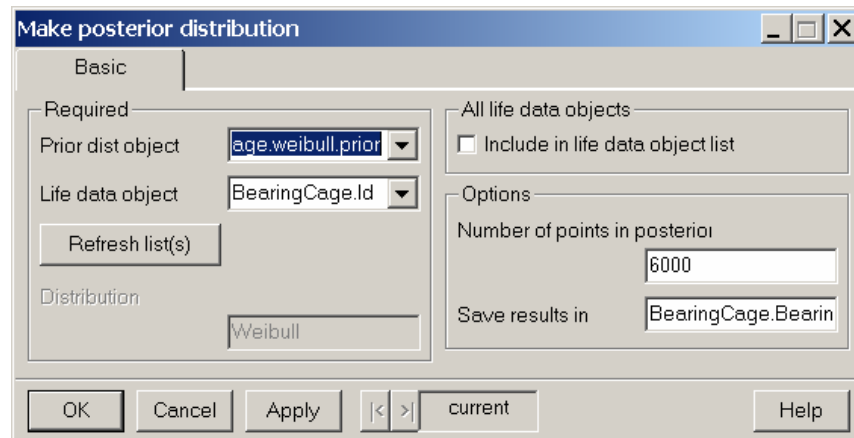
Figure 43 Dialog to create a posterior distribution for the Bearing Cage data.

Clicking on OK or Apply will generate the posterior distribution object (with progress in the generation and a summary being printed).

5.3   Summarizing a posterior distribution

A posterior distribution can be summarized in a number of different ways, depending on the interests of the analyst. Using **SPLIDA ► Single distribution Bayesian analyses ► Summarize prior/posterior distribution** brings up the dialog in Figure 44. From this dialog, the analyst can request plots of the prior distribution sample or the posterior distribution sample. The plots can be for the joint distribution of the

shape parameter and another quantity of interest (the scale parameter, a particular quantile, or a failure probability at a specified time).

Figure 44 Dialog to request a summarization of a posterior distribution.

Figure 44 is a dialog to request a plot of the joint distribution of the prior distribution, the result is shown in Figure 45.
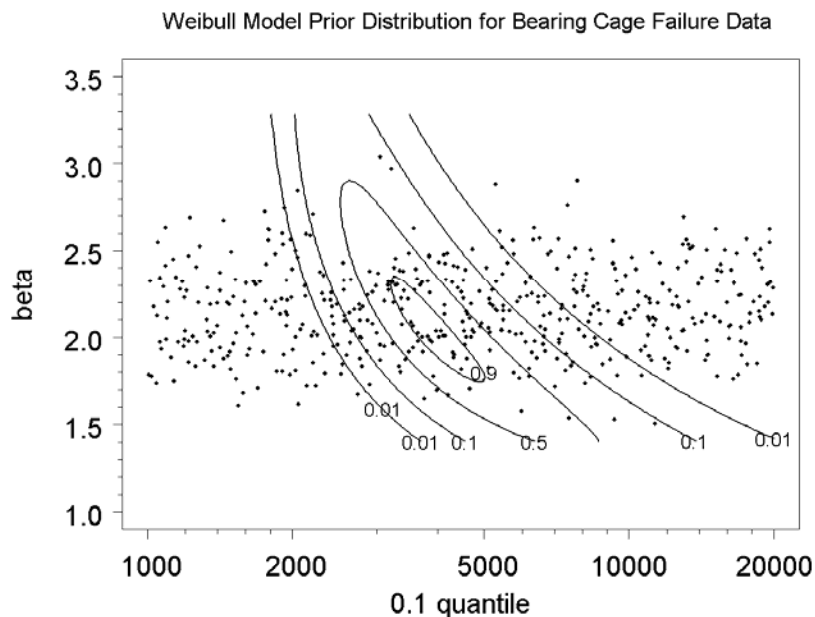
Figure 45 Sample from the Bearing Cage prior distribution (diffuse for 0.1 quantile and somewhat informative for β) and likelihood contours.

Changing "prior" to "posterior" in the dialog in Figure 44 and clicking on `Apply` will result in the plot shown in Figure 46.
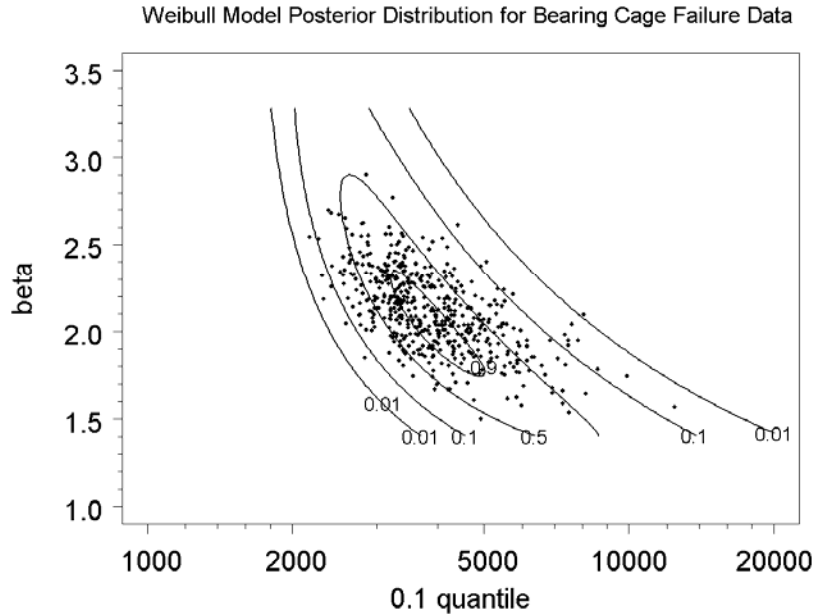
Figure 46 Sample from the Bearing Cage posterior distribution (based on a prior that is diffuse for 0.1 quantile and somewhat informative for ß) and likelihood contours.

It is interesting to compare Figure 45 and Figure 46 to see the effect that the data has on the sample from the prior distribution, to create the sample from the posterior distribution.

To focus on the posterior marginal distribution of a particular quantity of interest, ask for "Marginal only" under "Joint or Marginal." Figure 47 shows the results of asking for the marginal posterior distribution of .1 quantile of the bearing cage life. The vertical lines represent 95% Bayesian credibility limits (obtained by eliminating 2.5% in each tail of the marginal posterior distribution).

Weibull Model Posterior Distribution for Bearing Cage Failure Data
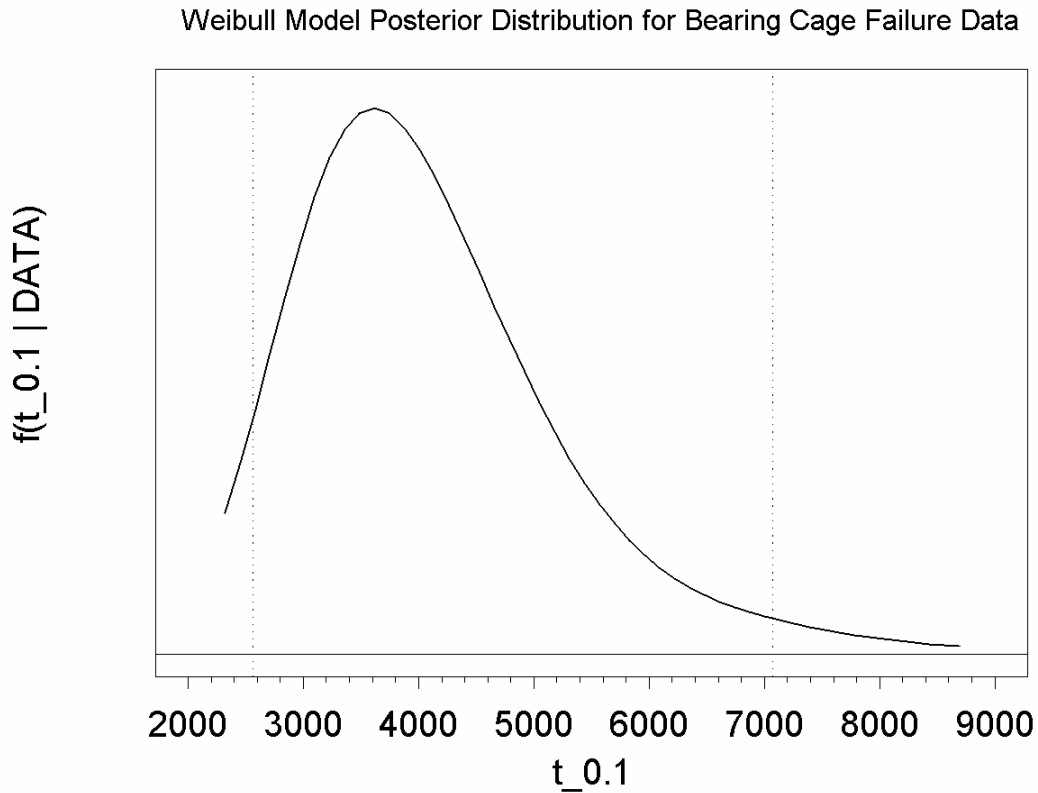
Figure 47 Marginal posterior distribution for the .1 quantile for the Bearing Cage data, based on a prior that is diffuse for .1 quantile and somewhat informative for ß.

6   Planning a Single Distribution Study

This section describes tools for evaluating proposed lifetest plans. The tools presented here are based on theory and methods described in Chapter 10 of Meeker and Escobar (1998).

SPLIDA single distribution life test evaluations are done in three stages. SPLIDA provides three different dialoges to accomplish these tasks:

1. In the first stage (see Section 6.1 and 6.2) the user specifies the testing situation by providing planning values (essentially, best guesses for the underlying model, including information on the life distribution, including the distribution's parameters values) in the form of a plan value object. Given such planning information, it is possible to evaluate any specified test plan or plans.
2. In the second stage (see Section 6.3) one can ask for a plot of approximate required sample size as a function of desired precision. This evaluation, based on large sample approximate variance computations, allows the test planner to see, at a glance, the

effect of sample size for a given test length (as specified by time for Type I censoring and number of test units for Type II censoring).

3. In the third stage of evaluation (see Section 6.4), one can simulate a specified test plan a specified number of times. Plots of the samples from a few of the individual simulated experiments and a summary of ML estimates from all of the simulations (both presented on appropriate probability plots) provides a visualization of the possible results from a life test experiment and the reasons why estimates of certain quantiles might or might not be estimated precisely.

6.1   Specifying test planning information (planning values)

Using **SPLIDA ► Plan Single Distribution Analyis ► Specify test planning information (planning values)** brings up the dialog in Figure 48, allowing the user to specify planning values needed to evaluate and compare different life test plans. The default name for the planning value object is last.xxx.pv where xxx is the chosen distribution. Generally it is a good idea to edit the suffix "last" to be something that describes the actual application (in the example below we use DeviceP). This "planning values object" will be used in the simulation/evaluation stage of planning. To specify the required distribution-parameter plannning values, start by choosing a method to specify (on probability plot) the failure time distribution: a) point and slope or b) two points. Either method can be used to specify a line on a probability plot (with an underlying log-location assumed distribution).

DeviceP is an electronic device tat will be tested at standard accelerated test conditions of 80 Degrees C and 80% RH. We will assume for now that estimate of life at thes conditions is of interest (in the actual applications, the engineers intended to multiple the final answer by an "acceleration factor" known to be approximately correct for the failure mechanism under study).

For DeviceP, the engineers expect approximately 10% of the tested units to fail by 1000 hours and that the failure time distribution will be adequately described by a lognormal distribution with a shape parameter sigma (σ) in the neighborhood of 2. After specifing the needed information, click "Apply" and examine the resulting probability plot reflecting the inputted information, as illustrated in Figure 49. Some tabular information is also provided in Table 5.

Figure 48 Dialog for specifying life test planning information.
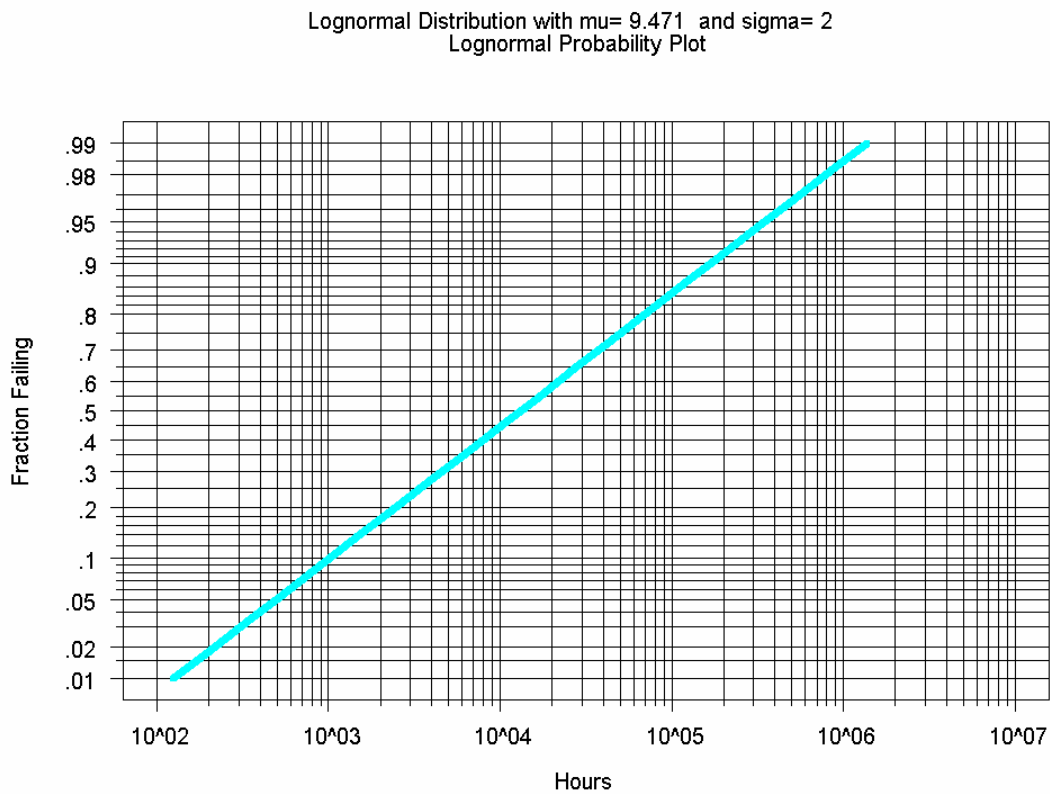
Figure 49 Probability plot depicting life test planning information.

Table 5 Tabular output from the dialog used to specify planning values.

```
Distribution:  Lognormal
Time units are:   Hours


mu =   9.47
sigma =   2



Failure probabilities from the Lognormal distribution
Hours fail.probs
  1e+001   0.000169
  1e+002   0.007490
  1e+003   0.100000
  1e+004   0.448181
  1e+005   0.846381
  1e+006   0.985084
  1e+007   0.999556
```

6.2   Plot test planning information (planning values)

Using **SPLIDA ► Plan Single Distribution Analysis ► Plot test planning information (planning values)** allows the user obtain a plot and table, like those in Figure 49and Table 5, describing a previously created plan value object.

6.3   Plot of approximate required sample size

Using **SPLIDA ► Plan Single Distribution Analyis ► Plot of required sample** will produce the dialog in Figure 50.
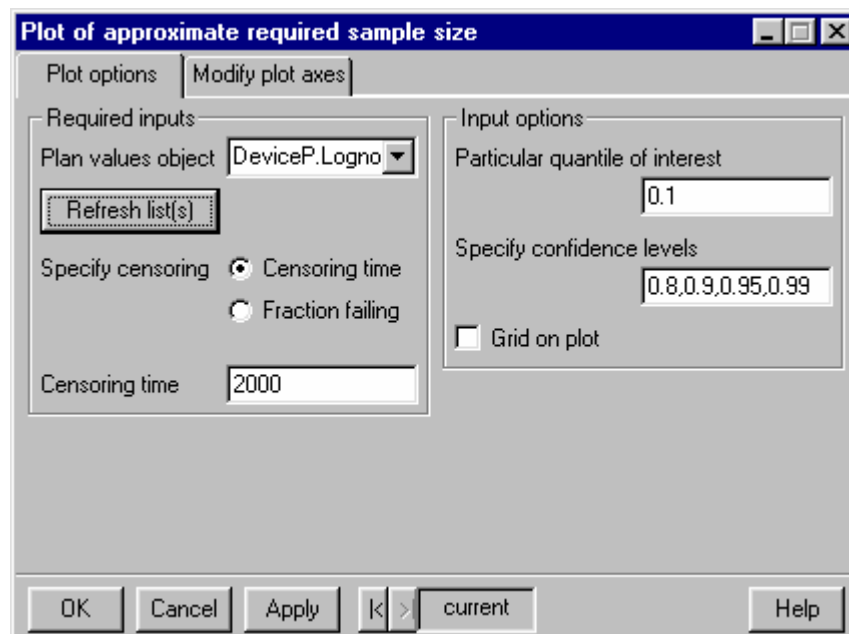
Figure 50 Dialog for requesting a required sample size plot.

Note that the most recently created plan value object (within the current session) is remembered and displayed as the default to be used. Alternatively, a different, previously created, plan values object can be chosen. Then one must choose a censoring type and either a censoring time or number of failures (for Type I and Type II censoring, respectively) and, if desired, a quantile of interest of other than the default of .1. Clicking on "OK" or "Apply" will produce a plot like the one in Figure 51. It is useful to use the rectangle tool from the S-PLUS annotation toolbar (right click on a blank part of the toolbar section of the SPLUS GUI and choose annotation, then click on the rectangle) to help read off values from the plot.

For log-location-scale distributions (Weibull, lognormal, loglogistic) the confidence interval precision is described in terms of a factor R in that the normal-approximation confidence interval for a quantile has the form $[\hat{t}_p / R, \hat{t}_p R]$ where $R > 1$ and the interval is more precise for small values of R (see Chapters 7 and 8 of Meeker and Escobar 1998). For location-scale distributions (smallest extreme value, normal, logistic) precision is given in terms of the half-width of the two-sided confidence interval.

Needed sample size giving approximately a 50% chance of having a confidence interval factor for the 0.1 quantile that is less than R lognormal Distribution with mu= 9.47 and sigma= 2 Test censored at 2000 Hours with 17.5 expected percent failing
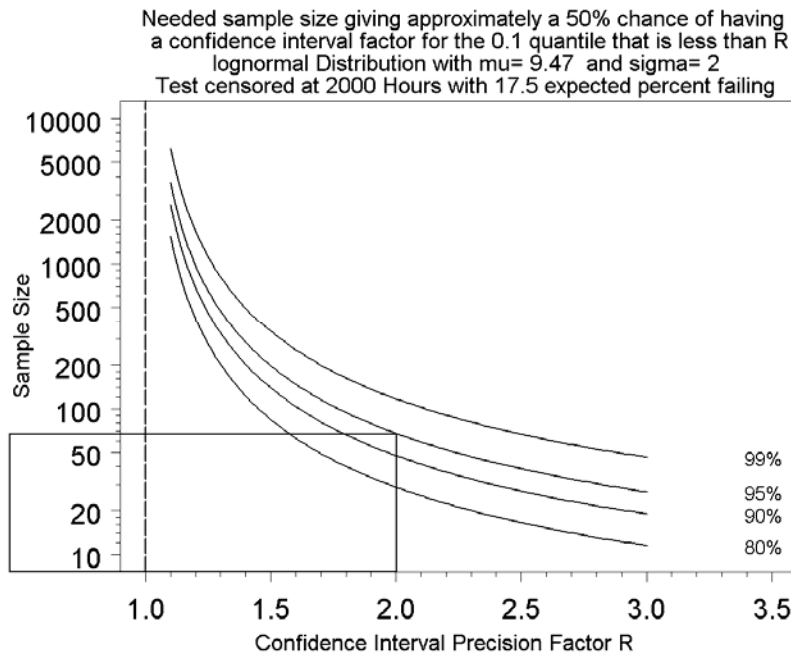
Figure 51 Plot giving sample size needed as a function of the target confidence interval precision factor.

For example, Figure 51 shows that a sample size of approximately 70 is required to have a 95% confidence interval target precision factor of 2. The actual precision factor R, because it depends on the data, is random for any given life test, but should be no more than the target value with probability of approximatley .5. To get a sense of how much R

might vary from the target value, one can use the simulation tool described in Section 6.4.

6.4   Life test simulation

Although the plot of required sample size versus precision factor is useful for determining the approximate sample size needed for a life test, simulating the life test avoids large sample approximations, provides more insight into the results that might be obtained, and gives one a better sense of the trade-off between sample size and test length. Using **SPLIDA ► Plan single distribution analysis ► Simulate a life test** requests a dialog like that shown in Figure 52. This dialog allows one to specify the inputs needed to produce a simulation of a user-specified life test plan.

Figure 52 Dialog to request a life test simulation.

The user specifies the number of units to be tested ("Sample size") and test length ("Censoring time" for time or Type I censoring) or number of units to fail (for failure or Type II censoring) and, perhaps some other optional inputs (e.g., which quantile of the distribution is of primary interest).

One can (and in an unfamiliar situation, should) choose to look at the individual samples from some of the simulated samples by entering a small integer (usually 5 or 6 is sufficient) into the box marked "View detail for how many samples?" If an integer $k > 0$ is given in this cell, then after completion of the simulation, the graphsheet will contain k probability plots showing the planning value line (with a dark thick line) along with the observed failures and the ML estimate line (a thin line to contrast with the "true" planning value line) for each of the k samples. Figure 53 provides an example of such a plot. It is interesting to observe the variability in the failure times and corresponding ML estimate lines from sample to sample in a sequence of these plots.

The final probability plot, illustrated in Figure 54, contains, in summary, the ML estimates from *all* of the simulated samples. When a quantile (number between 0 and 1)

is given in the "Quantile line at" cell, a horizontal line is drawn at that level on the probability plot, allowing a visualization of the sampling variability in the estimates of that quantile, and the average precision factor for the confidence intervals for the corresponding confidence intervals is shown in the plot. The average precision is computed as a geometric average of the individual precision factors for quantiles of log-location distributions and an arithmetic average of the individual precision factors for quantiles of location distributions.
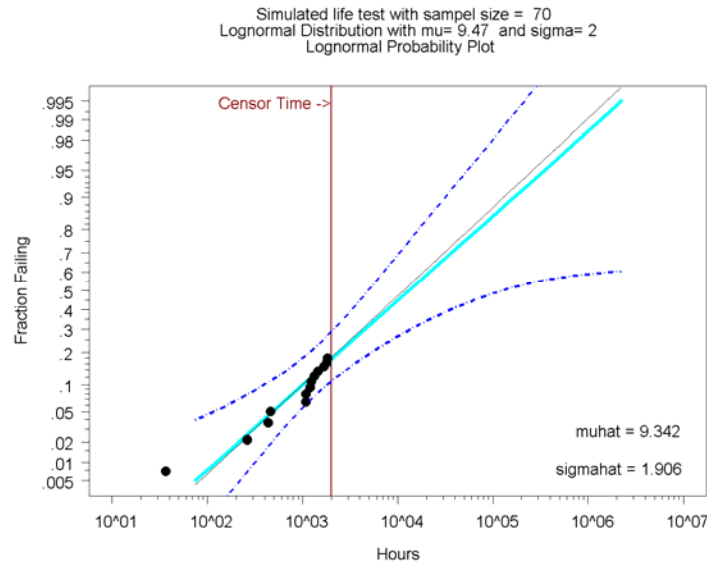
Figure 53 Probability plot and ML estimate for the first simulated life test.

Figure 53 shows the probability plot for the first simulated sample. Generally looking at five or six of these individual simulation results (we only show one here) is sufficient for getting a good idea how the samples in the proposed life test might behave (assuming that the inputted planning information is reasonably accurate). One can see clearly the potential effect that censoring will have on the information from samples and the effect of changing the sample size and censoring time.
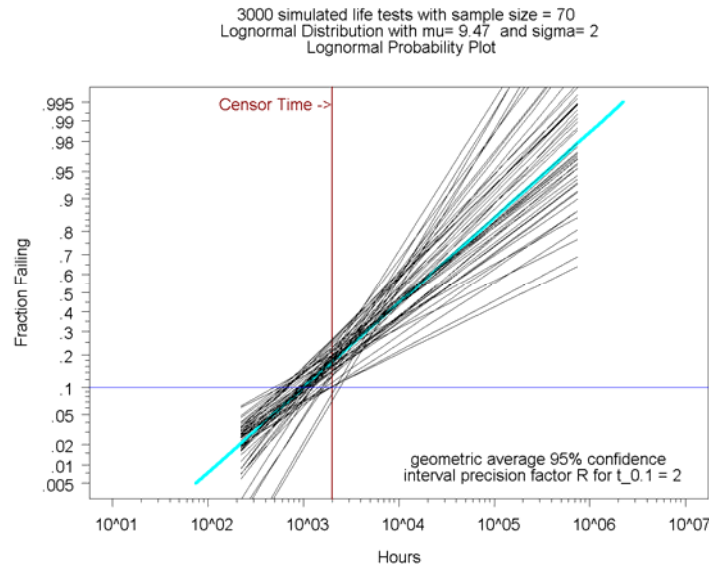
Figure 54 Summary showing ML estimates for all 30 life test simulations.

Figure 54 shows a summary of the ML estimates from 50 of the 3000 simulations (if more than 50 lines are plotted, it becomes difficult to judge the density of the lines in certain parts of the plot). The geometric average of the 3000 individual precision factors printed in this simulation-summary plot provides a useful measure of sampling variability and the corresponding precision that one can expect to see in the results of one's life test.Unless the number of simulations is very large (e.g., 1000's of simulated life tests) the value of this average vary somewhat from simulation to simulation. When the probability of 0 failures is relatively small (e.g. .005 or less) this numerical summary of the simulation will agree very well with the value predicted from the sample-size tool (e.g. Figure 51).

6.5   Probability of successful demonstration

After conducting a life test, a successful demonstration of reliability has occurred if the lower confidence bound on reliability exceeds the given standard $q = 1 - F(t)$ [which is the same as the upper confidence bound on fraction failing being less than a standard $p = F(t)$]. Equivalently, the demonstration is successful if lower confidence bound on the corresponding p quantile of the life distribution is larger than the corresponding standard (e.g., for demonstrating 95% reliability, the standard would be based on the .05 quantile of the life distribution). For a log-location-scale distribution, the probability of a successful demonstration for a Type II censored life test (test run until a specified number of units fail) is a function of the level of reliability to be demonstrated, the number of units to be tested, the number needed to fail, and the underlying true reliability.

SPLIDA has a tool to allow one to compute and make plots of the probability of successful demonstration. The dialog in Figure 55 allows the user to specify the

necessary inputs. In Figure 55, sample sizes n=20,30,40 with corresponding number failing r=10,15,20 (i.e., 50% failures) were chosen. In general, the number failing r must be at least 2 but no greater than n. The probabilities of successful demonstration are computed via simulation. The default number of simulations was increased from the default 300 (which allows quick but rough evaluations) to 3000, which is good enough for most practical purposes (although 10 thousand simulations might be used to make a publication-quality figure). The default reliability to be demonstrated was changed from the default 0.5 to 0.9. Completing the 3000 simulations requires less than a minute of computer time on a modern PC).

Figure 55 Dialog for requesting the computation of probability of successful demonstration curves from q=0.9.

Figure 56 is a plot of the probability of successful demonstration as a function of the actual (unknown) reliability. As expected, it is easier to make the demonstration (i.e., the probability of success is higher) if the actual reliability is much larger than the standard to be demonstrated.
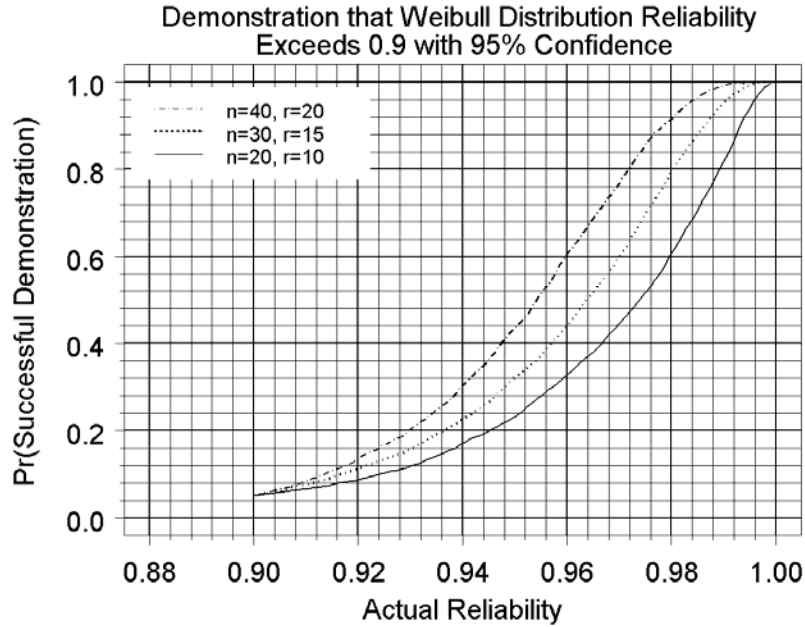
Figure 56 Weibull life test probability of successful demonstration curves for *q*=0.9.

After completing a simulation to compute the probability of successful demonstration, SPLIDA will save the simulation results as an object, making it possible to reuse the results to make evaluations for other given values of reliability as the standard. Figure 57 shows a dialog in which the results from the previous simulation will be used to compute another set of probability of successful demonstration curves with *q=0.7*. From this dialog, it is also possible to choose from among the combinations of sample size *n and the number of* failures *r* used in the previous simulation to limit the number of curves to appear on the plot.

Figure 57 Dialog using existing results (previously computed simulation) to compute probability of successful demonstration curves for *q*=0.7.

The resulting plot appears quickly, because the simulations do not need to be run again and is shown in Figure 58. As expected, the required sample sizes in this plot are smaller because of the less demanding demonstration.
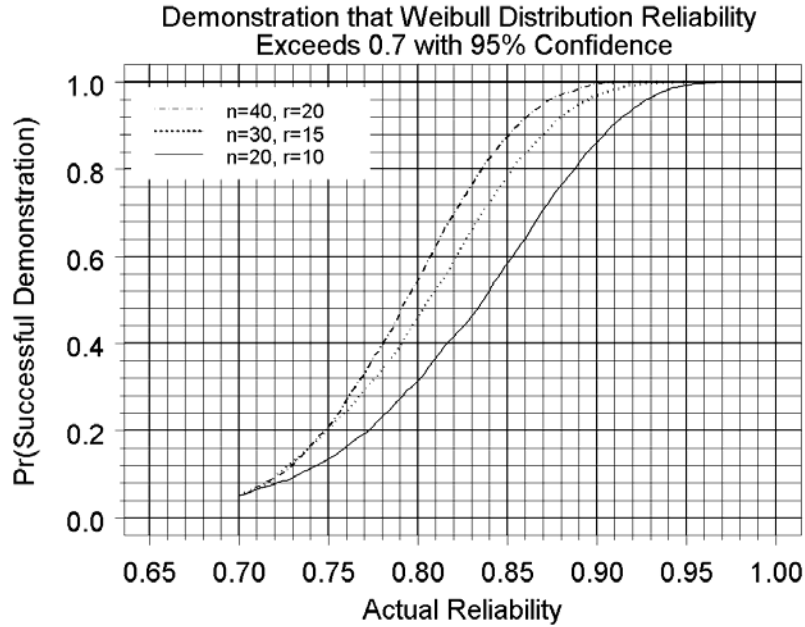
Figure 58 Weibull life test probability of successful demonstration curves for *q*=0.7.

7 Analysis of Data with More Than One Cause of Failure

Both laboratory life tests and field-failure reporting systems (e.g., warranty repair processes) result in data with more than one failure mode. For example, a system may have several components and the failure of any one of these components could cause the system to fail. For many purposes, it is important to use failure mode information in the analysis and interpretation of one's data. This is especially true when the purpose of the analysis is to identify opportunities for improving product reliability.

One commonly used model for multiple failure modes supposes that each unit has a "potential" failure time for each of the possible "modes" of failure and that the distribution of these failure times can be described with a joint distribution. When it can be assumed that the different potential times to failure are approximately statistically independent, this modeling approach provides a simple method of modeling and analysis that allows estimation of the failure time distribution for any particular failure mode or for the entire system with any specified combination of failure modes active. See Chapter 15 of Meeker and Escobar (1998) for technical details and other information about the analysis of such data.

7.1 Summary analysis of individual failure modes

Consider the Device-G data described in Section 15.4 of Meeker and Escobar (1998). The devices that failed during the study failed from either a surge or a wear out failure

mode. The underlying mechanisms causing these two failure modes were different and it would be reasonable to assume that the times to failure for these two different modes would be independent. In order to estimate the failure time distribution of the individual failure modes, under the assumption that the failure modes failure times are independent, use **SPLIDA ► Multiple failure mode life data analysis ► Individual modes.** Then choose the life data object (DeviceG.ld in this case) and distribution (Weibull for this example) in the dialog, as shown in Figure 59.
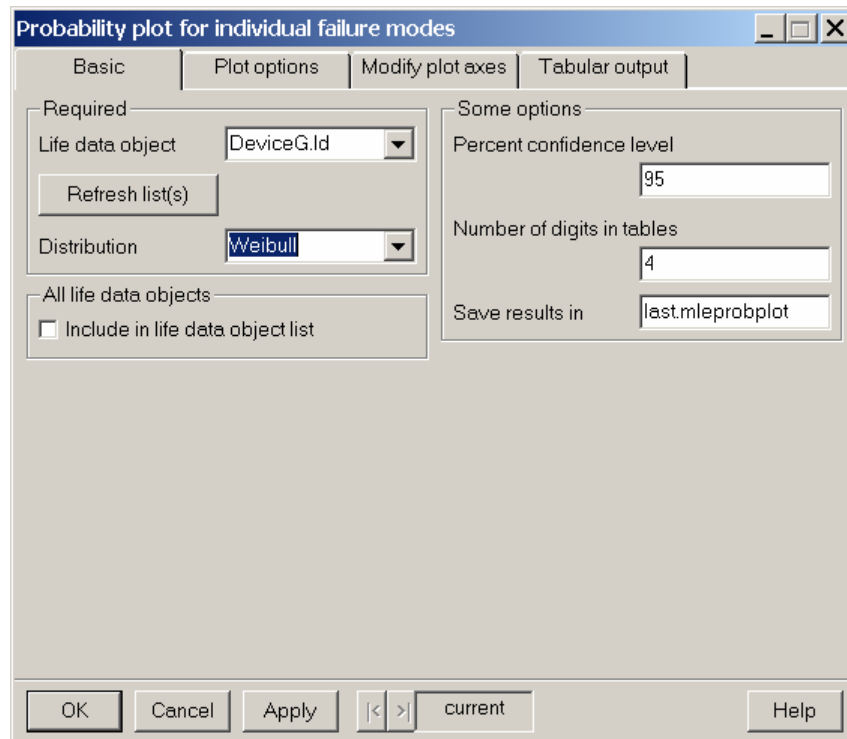
Figure 59 Dialog requesting separate analyses of different failure modes for the Device-G data.

Other options for graphical and tabular output are available in the back pages of the dialog. When done, click "OK" or "Apply," producing the probability plot with estimated distributions for the two individual failure modes in Figure 60. Tabular output giving the Weibull parameter estimates is also provided, as shown in Table 6.

In this application, management was interested in answering a number of different questions with these data. Among the requested outputs was the failure-time distribution for the wear out failure mode alone, which would be the failure time distribution for the device if the surge failure mode could be eliminated. A table of these estimates (requested with the "Tabular Output" page of the dialog) is shown in Table 6.

Individual Device-G Field Data Failure Mode Weibull MLE's
Weibull Probability Plot
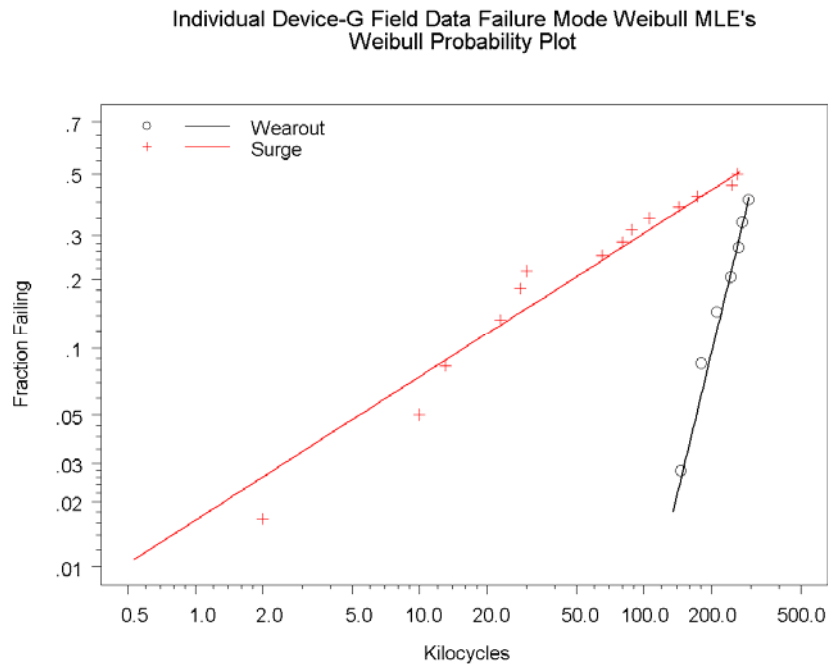
Figure 60 Analysis of the individual failure surge and wear out failure modes for the Device-G field data.

Figure 60 shows immediately that the surge failures tend to occur anytime during life, from early to late. No wearour failures were seen until after 100 thousand cycles. Table 6 provides a summary of separate ML estimations for both the Surge and the Wearout failure modes and some additional details for the Wearout failure mode

Table 6 Summary of ML estimation for Surge and Wearout failure modes and details for the Wearout failure mode

```
Response units: Kilocycles
Weibull Distribution
         Likelihood   eta   se_eta  beta se_beta
1 Wearout     -47.16 340.4  36.14 4.337  1.4506
2   Surge    -101.36 449.5 191.94 0.671  0.1578


Total likelihood= -148.5
Weibull Distribution Failure Probability Estimates
From DeviceG Wearout Failure Mode Group Wearout
with Weibull MLE and Pointwise 95% Confidence Intervals
Kilocycles    Fhat     Std.Err. 95% Lower 95% Upper
140          0.02098  0.02270  0.002453    0.1574
160          0.03714  0.03330  0.006179    0.1931
180          0.06113  0.04510  0.013764    0.2330
200          0.09482  0.05712  0.027638    0.2785
220          0.13983  0.06853  0.050521    0.3318
240          0.19722  0.07932  0.084269    0.3961
260          0.26718  0.09088  0.128000    0.4752
280          0.34864  0.10569  0.176941    0.5713
300          0.43911  0.12529  0.224124    0.6797
```

7.2 Combined modes (series system) analysis

SPLIDA also computes estimates of the series-system failure-time distribution with all failure modes operating. To do this, use **SPLIDA ► Multiple failure mode life data analysis ► Combined modes** and again choose the life data object and distribution and click "OK" or "Apply," producing the probability plot with estimated distribution for the system with both failure modes active and acting independently, shown in Figure 61.

Series System Combined Failure Mode ML Estimates
Device-G Field Data
Weibull Probability Plot
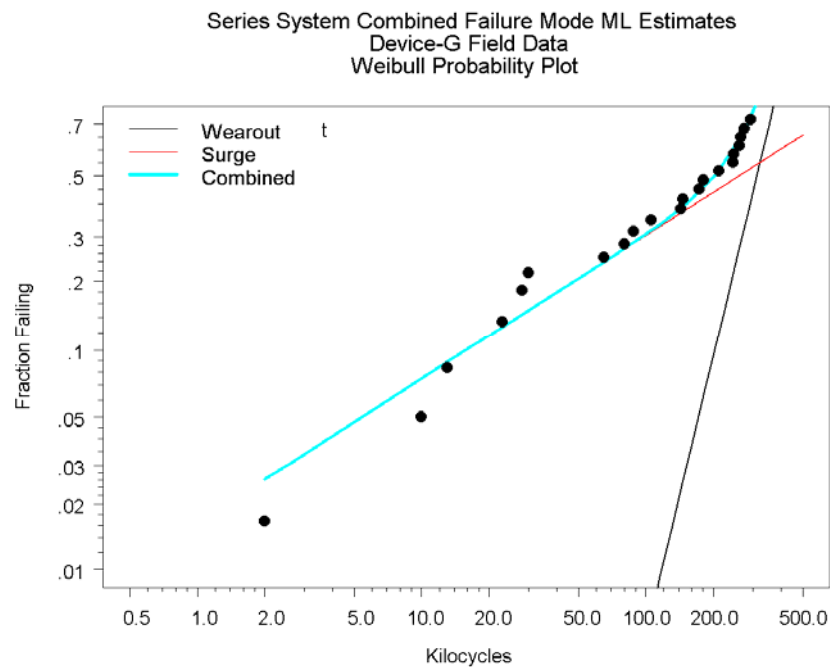
Figure 61 Series system competing risk analysis for the Device-G data.

7.3 Creating life data objects for individual failure modes

Using **SPLIDA ► Multiple failure mode analysis ► Make life data objects for individual failure modes** will bring up the dialog in Figure 62, allowing the user to create separate life data objects for each failure mode in the multiple failure mode life data object.
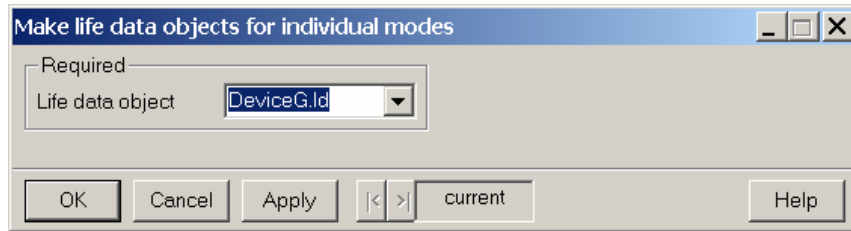
Figure 62 Dialog to ask that individual life data objects be made for individual failure modes.

Names for the life data objects are chosen automatically by appending the failure mode name to the root of the failure mode life data object name (DeviceG.Surge.ld and DeviceG.Wearout.ld in the example). Once these individual life data objects have been created, all of the SPLIDA methods for single-distribution analyses can be applied to the analysis of the individual failure modes, under the (important) assumption that the individual failure modes are acting independently.

8   Comparison of Two or More Distributions Using Life Data Analysis

Experiments are commonly conducted for the purpose of comparing the life distributions of different or competing materials, formulations, designs, manufacturers, and so on. Similar data arise from warranty data and field tracking studies. The analysis methods presented here for such data generalize methods in classical statistical methods commonly known as two-sample comparison and one-way analysis of variance. SPLIDA generalizes these methods, allowing for censoring, non-normal distributions, and comparisons with non-constant shape (slope) in the distributions being compared.

8.1   Comparison with different shape (slope) parameters

Using **SPLIDA ▶ Comparison of distributions life data analysis ▶ Probability plot and ML fit: different shapes (slopes)** brings up a dialog like that in Figure 63 asking for input of a comparison life data object and a distribution, as shown below. A comparison life data object is a life data object that contains at least one categorical explanatory variable (in S-PLUS, an explanatory variable is assumed to be categorical (a "factor" in S-PLUS jargon) if it contains any alphabetic characters). The Life data object list will contain all of the "comparison life data objects" found in the SPLIDA data base or in the user's local data base. As done for single distributions life data objects, comparison life data objects are created by using the **SPLIDA ▶ Make/edit/summary/view data object ▶ Make life data object** part to the SPLIDA menu. If there is more than one categorical explanatory variable in the comparison life data object, then one needs to choose which one to use in the comparison. The following example uses the Snubber data (life data object Snubber.ld) from Nelson (1984, page 529) and reanalyzed in Examples 17.13 to 17.15 in Meeker and Escobar (1998), comparing two different Snubber designs (a snubber is a toaster component). Following these previous analyses, the normal distribution is fit to the data.
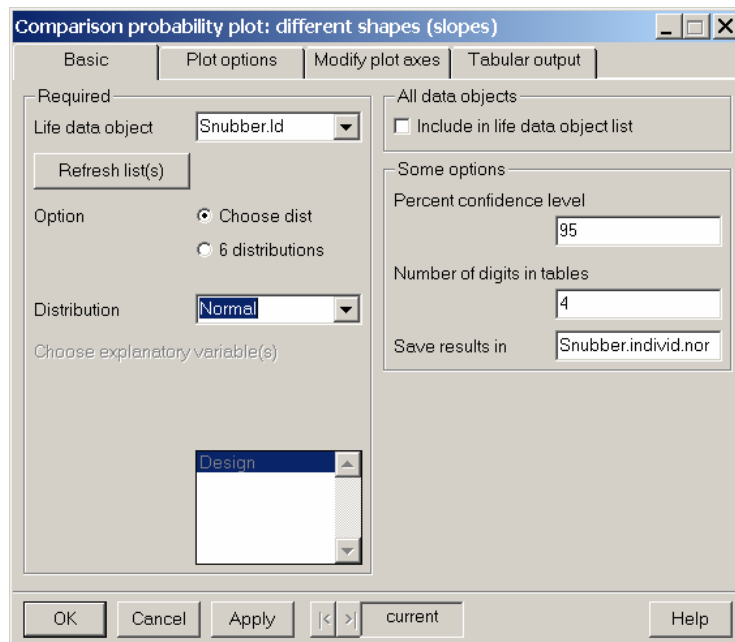
Figure 63 Dialog requesting comparison of the Snubber using separate normal distributions.

Using SPLIDA ► Comparison of distributions life data analysis ► Probability plot and ML fit: different shapes (slopes) brings a dialog like that shown in Figure 63. After the needed and desired optional inputs have been given in the dialog, click on "Apply" to do the analysis.

Figure 64 gives the resulting individual probability plots for each (in this case two) of the samples in the the data set. Table 7 gives a tabular summary of the results. Tests or confidence intervals to compare particular quantiles or other parameters are easy to construct, following the general approach given in Section 17.8 of Meeker and Escobar (1998).

Table 7 Normal distribution ML estimates for the Snubber data using separate normal distributions .

```
Maximum likelihood estimation results:

Response units: Toaster Cycles

Normal Distribution
            Log likelihood      mu  se_mu sigma se_sigma
1 NewDesign         -138.6 1126.6 123.21 546.0     99.5
2 OldDesign         -146.8  908.1  76.19 362.4     63.4

Total log likelihood= -285.4
```
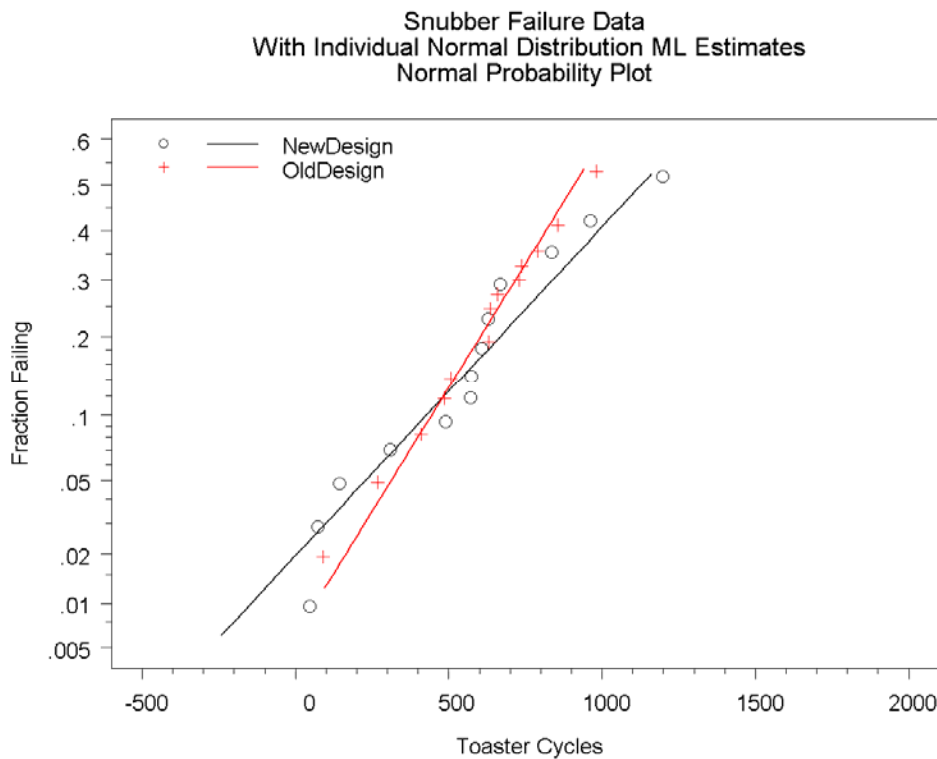

Figure 64 Normal probability plot comparing the new and old snubber designs with separate normal distribution standard deviations.

8.2   Comparison of likelihood contour plots

Using **SPLIDA ► Comparison of distributions life data analysis ► Comparison likelihood contour plot** brings up a dialog like the one in Figure 65, allowing one to specify a comparison life data object and distribution in order to produce corresponding two-parameter contour plots for each of the groups being compared.

Figure 66 shows a comparison contour plot of normal distribution joint confidence regions for μ and σ for the Snubber data.
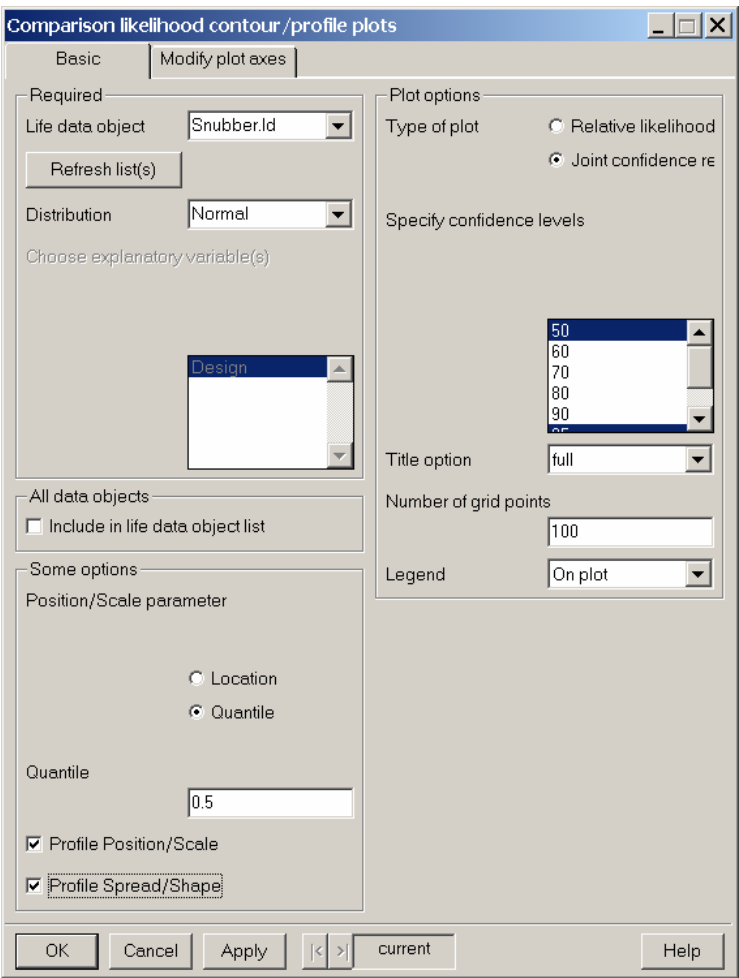
Figure 65 Dialog requesting contour plots to compare the new and old snubber designs assuming separate normal distributions.
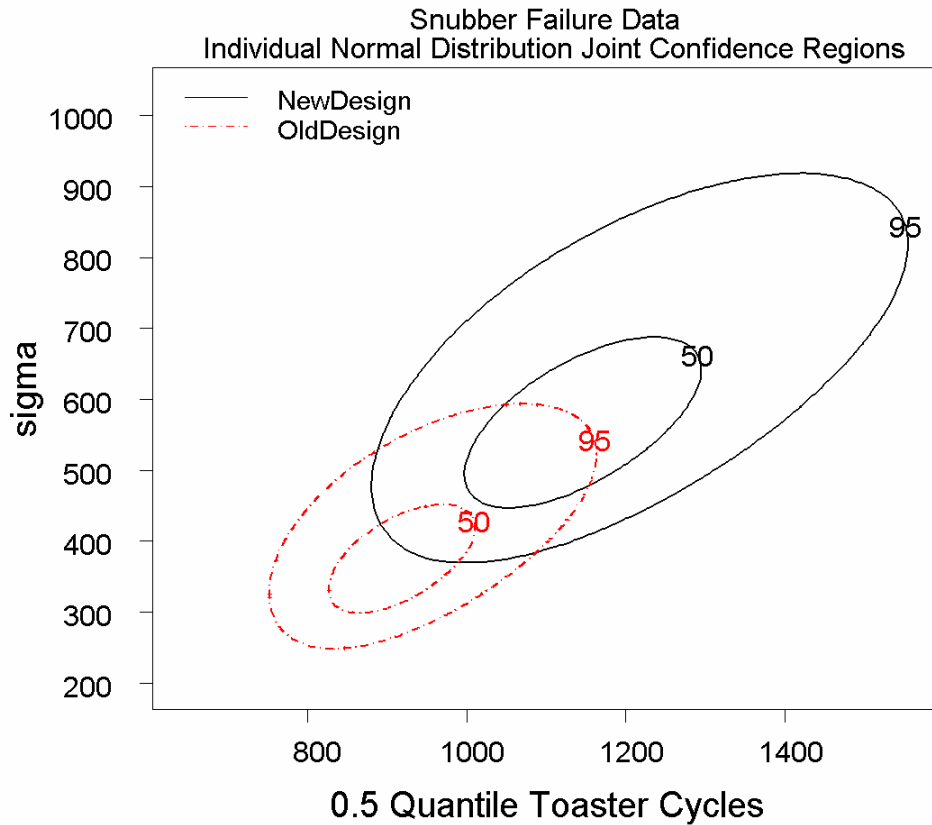
Figure 66 Contour plot showing joint confidence regions for the normal distribution parameters for the two snubber designs.

The SW to NE orientation of the contours in Figure 66 is a result of the right censoring. The degree of overlap provides an alternative visualization of the statistical "closeness" of the parameters of the lifetime distributions for the two different snubber designs.

The dialog in Figure 65 also produced the optional comparison profile likelihood plots (not shown here) comparing the distribution median and the lognormal shape parameters from the New and Old groups.

8.3   Comparison with common shape (slope) parameters

For some applications, it is reasonable to model the different populations or processes being compared as having the same shape (or slope) parameters, differing only with respect to scale (for a log-location-scale distribution) or location (for a location-scale distribution). This tends to simplify the analysis because the equality of any particular distribution quantile would imply the equality of other distribution quantiles (this can be seen by noting that, with this assumption, the probability plots of the individual fraction failing are parallel).

Using **SPLIDA ► Comparison of distributions life data analysis ► Probability plot and ML fit: common shapes (slopes)** brings up a dialog like that shown in Figure 67. This dialog is similar to that used for the comparison with non-common shapes (slopes).
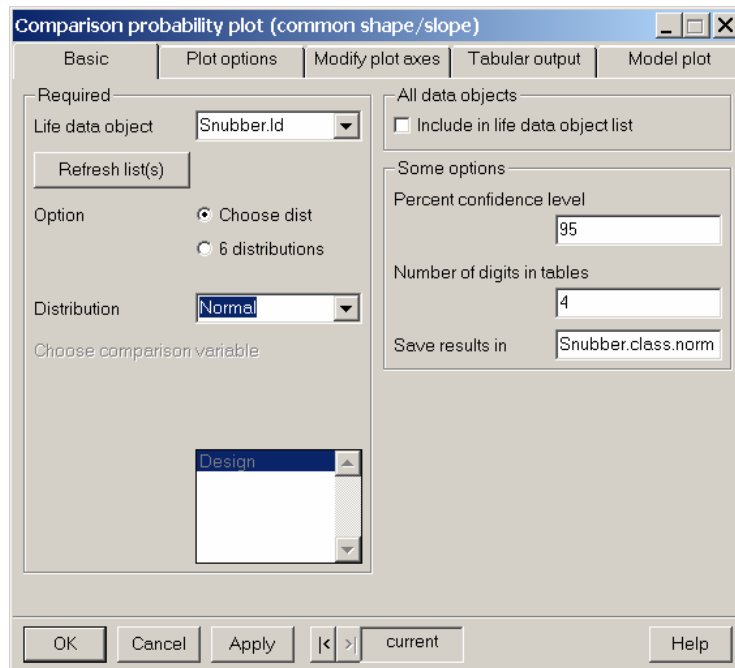
Figure 67 Dialog requesting a probability plot comparing the new and old snubber designs assuming normal distributions with a common standard deviation.

As before, choose a "life data object" (by default SPLIDA puts in the list only life data objects with a comparison or factor explanatory variable) and a "distribution." An estimate of the model parameters is given in the S-PLUS report window. The right-most back page of the dialog provides a dialog that allows one to obtain a plot of the estimated model (a "model plot" is sometimes called a "stress plot" when the explanatory variable is a stress variable). Tables of estimates of distribution quantiles and failure probabilities are also available among the options in the back pages of the dialog. These tabular and graphical outputs for the Snubber data common standard deviation analysis are given in Table 8 and Figure 68.
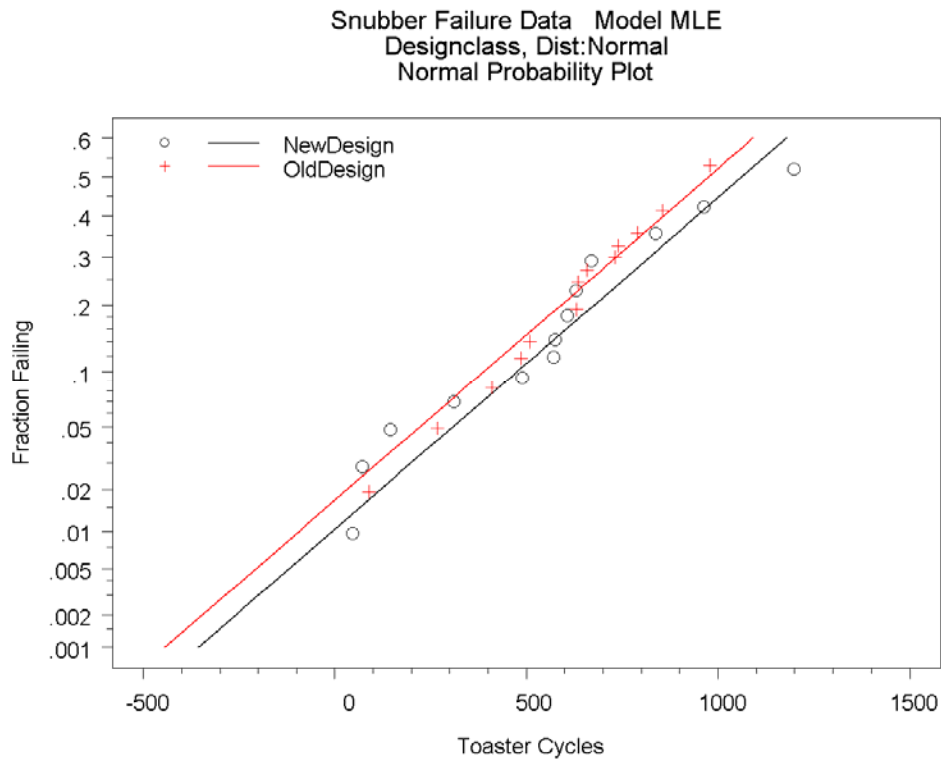
Figure 68 Normal probability plot comparing the data from the new and the old Snubber design assuming normal distributions with a common standard deviation.

The probability plot in Figure 68 shows nonparametric estimates of the life time distributions for both snubber designs along with ML estimates of the assumed normal distributions. The lines representing the ML estimates of the fraction failing for the two designs are parallel because of assumption of common normal distribution standard deviations. For the normal distribution, the slope corresponds to an estimate of the reciprocal of the standard deviation of life (the reciprocal of the scale parameter).

Table 8 Normal distribution ML estimates for the Snubber data with a common normal distribution standard deviation.

```
Snubber Failure Data
Maximum likelihood estimation results:
Response units: Toaster Cycles
Normal Distribution
Relationship
1 : class
Log likelihood at maximum point: -286.7

Parameter                  Approx Conf. Interval
          MLE      Std.Err. 95% lower 95% upper
Intercept 974.63    89.11      800.0    1149.3
DesignNew  86.67   114.21     -137.2     310.5
    sigma 458.56    57.67      358.4     586.7
```

73

The table reproduced in Table 8 gives ML estimates and standard errors for an underlying dummy variable regression model used to distinguish between the two different designs (see Section 17.8 of Meeker and Escobar for a detailed description of this model). The Intercept row corresponds to the "baseline" category (old design in this case) giving the estimated parameter for the corresponding distribution (mean of the normal distribution in this case). The `DesignNew` row in the table gives the ML estimate of the difference in mean life between the baseline and the new design. The 95% approximate confidence interval for the `DesignNew` coefficient is $[-137.5, 310.5]$. This interval is wide enough to contain 0, indicating that the observed difference between the two designs could well have been caused by the random variability in the snubber life distributions. The last row in the output gives the estimate of the normal distribution standard deviation, which, in the model, is the same for both designs.

The "model plot," shown in Figure 69, plots estimates of the normal distribution density functions and distribution quantiles. This figure shows clearly that the observed difference in the designs is dominated by the variability in snubber life.
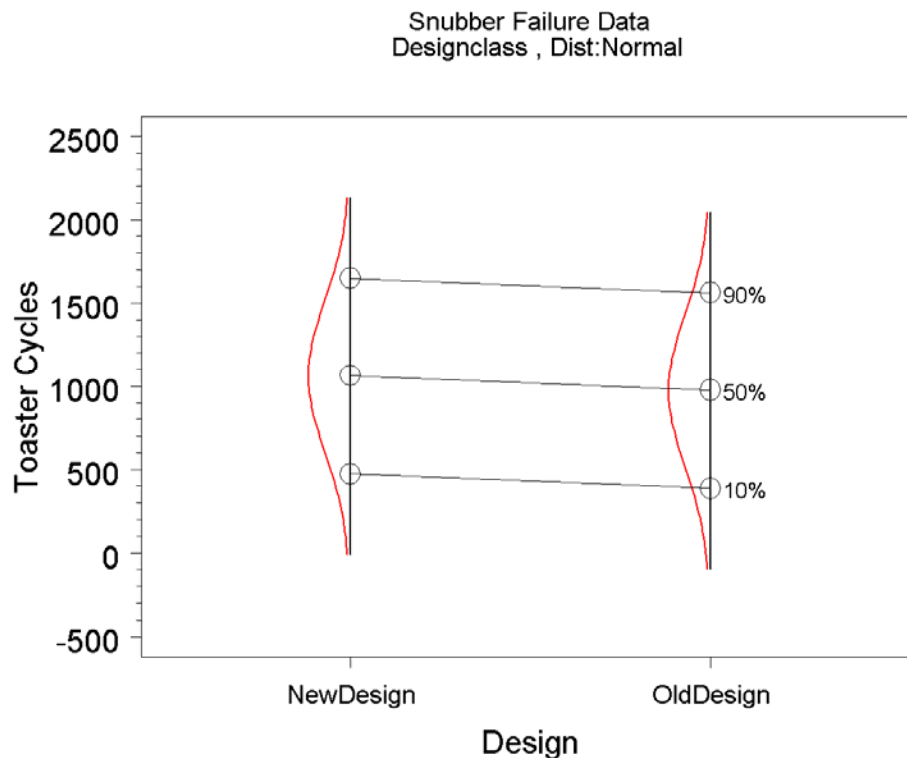
Figure 69 "Model plot" showing the estimated normal distribution densities for the new and the old snubber designs assuming normal distributions with a common standard deviation.

8.4 Make life data objects for individual groups

After comparing different groups, it is often desired to select one or more of the groups for more focused analysis. It is possible, for example to choose a particular group or groups and do single-distribution analyses for them. We will illustrate this by making a separate life data object for the New snubber design. Using **SPLIDA ► Comparison of distributions life data analysis ► Make life data objects for individual groups** brings up the dialog in Figure 70.
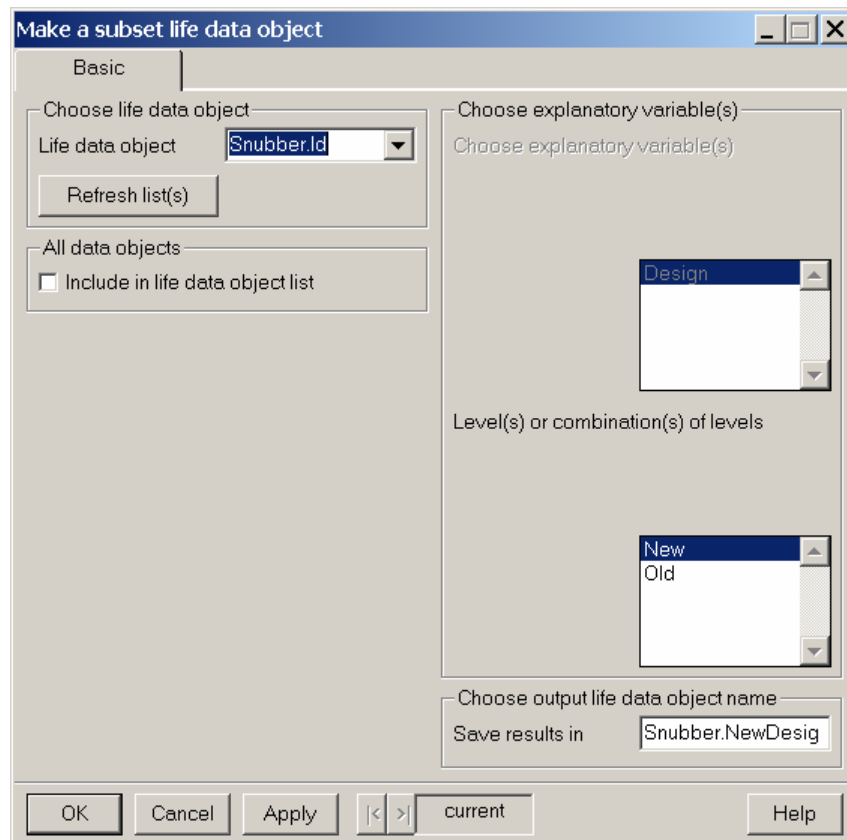
Figure 70 Dialog used to choose a data subset for a new life data object

For the example, we choose the Snubber.ld comparison life data object. Design is the only explanatory variable in this life data objects, so it is automatically highlighted. Then choose "New" under the list of available levels (or in general combinations of levels). A name for the output data object is chosen automatically when the level(s) or combinations of levels are chosen, but it is possible to edit this name. Clicking on "OK" or "Apply" will execute the function to produce the new data object. For the example, the message "Saving subset data object Snubber.NewDesign.ld" is printed and this data object will appear subsequently in the lists of single-distribution life data objects.

8.5 Probability of correct selection

When planning a life test to compare two or more distributions, an important consideration is the sample size needed to make the comparison. For example, when

comparing two different designs (as in the snubber example) if one only tested two units from each design, it is unlikely that the experiment would indicate a statistically significant difference, even if there were a difference of practical importance. Thus we would not have the information needed to choose one design over the other. Correspondingly, if the experiment had used 10,000 units of each design, it would be highly likely that any difference of practical importance (or indeed, small differences that would not be important!) would be detected with high probability. Additionally, when comparing life distributions, one also needs to decide how long to run the test. Generally it will not be possible to wait until all units have failed. An important part of planning the experiment is to determine how long to run the test or, alternatively, how many of the tested units need to fail before a decision should be made.

Typically, in comparative experiments, with all other things being equal, a decision would be made to choose the design that gives the best results in the experiment. Such a decision rule would have the largest probability of making the correct decision. When planning such an experiment it is important to have some idea about the actual probability of making a correct decision. An assessment of this probability can be obtained by using **SPLIDA ► Comparison of distributions ► Probability of correct selection for specified test plans.**

In the dialog in Figure 71, one specifies the assumed distribution, the number of populations to be compared, as well as the sample sizes (separated by commas) and the corresponding number to fail within each sample size. Then, as with other SPLIDA dialoges, click "Apply" and minimize dialog.

Figure 71 Dialog requesting a probability of correct selection comparison of three different proposed comparison life tests.

Depending on how many simulations have been requested, the complete simulation may take a substantial amount of time to run. Periodic messages will be printed in the report window however, to give an indication of the simulation's progress. Separate simulations are required for each sample size/number of failures pair and the computational time is approximately proportional to the number of failures. Figure 72 illustrates the output, giving a Probability of Correct Selection curve for each combination of sample size and number of failures, as a function of the standardized difference d. See Escobar, Pascual, and Meeker (2000) for more information on how to choose d.
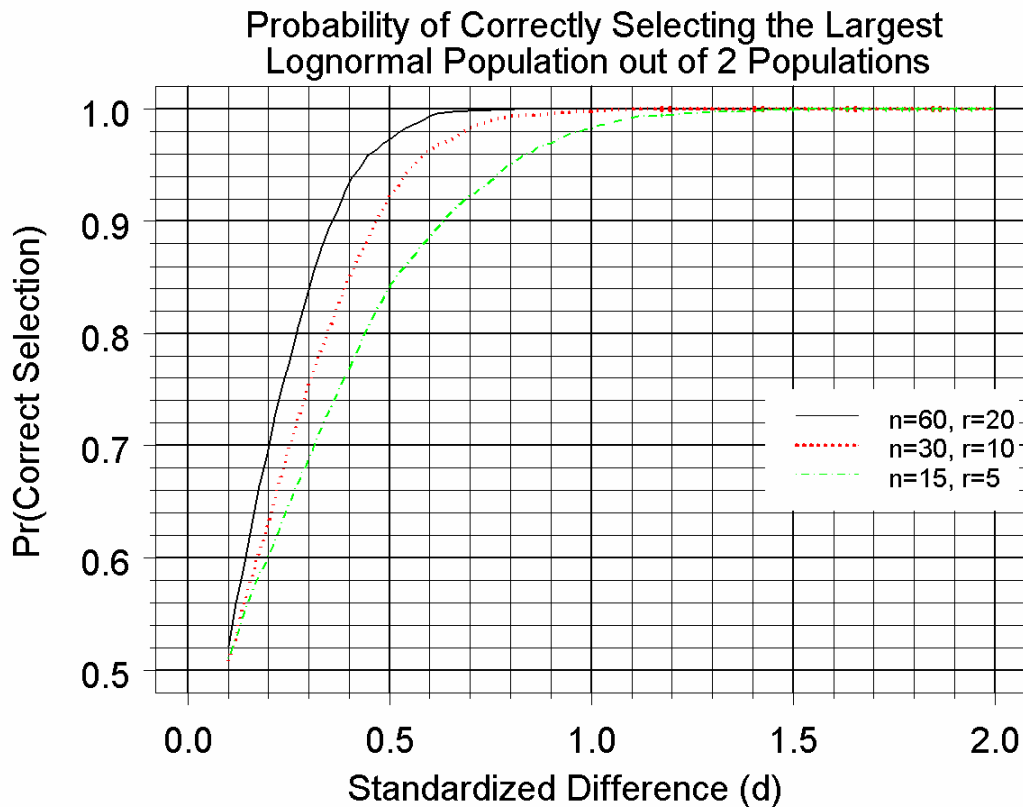
Figure 72 Plot showing the probability of correct selection as a function of the standardized difference between the populations being compared.

The jaggedness in the lines is due to the limited number of simulations that were done in this example, but such simulations are quick and adequate for most purposes. If smoother curves are desired, the simulation sample size can be increased to something like 10000. With a capable computer the time needed for the computations for this example would be on the order of a minute.

9   Simple Regression and Single-Variable Accelerated Life Test (ALT) Data Analysis

This Chapter describes methods for analyzing life data when there is a single explanatory variable. Technical background and further examples illustrating the statistical methods used here are given in Chapter 17 of Meeker and Escobar (1998). A common application of the methods in this chapter arises in accelerated testing where test units are, for example, tested at high temperature with the purpose of obtaining reliability information in a timely manner. Models and methods for accelerated testing are described in detail in Nelson (1990) and Chapters 18 and 19 of Meeker and Escobar (1998). Chapter 10 extends the methods of this chapter, allowing for more than one explanatory variable.

9.1   Scatter plot of censored data

One should always look first at available data through graphical displays. For single-variable accelerated test data, a scatter plot of the failure times will provide useful information. To illustrate the tools in this section, we use the Device-A data from Hooper and Amster (1990), reanalyzed in Section 19.2 of Meeker and Escobar (1998). In order to make a scatter plot of the Device-A data, use **SPLIDA ► Simple regression (ALT) data analysis ► Censored data scatter plot** to bring up the dialog shown in Figure 73. Then choose the one of the life data objects from the list. We use the Device-A life data object in this example (only those life data objects with explanatory variables appear in the list). There is only one explanatory variable in the Device-A data object, so it is chosen automatically. Then choose the axis scales for the response and the explanatory variable. Linear response and linear explanatory variable axes are the defaults, but log and Arrhenius axes are chosen for this example, corresponding to the physically-suggested lognormal-Arrhenius accelerated lifetime model, described and illustrated in Chapters 18 and 19 of Meeker and Escobar (1998).
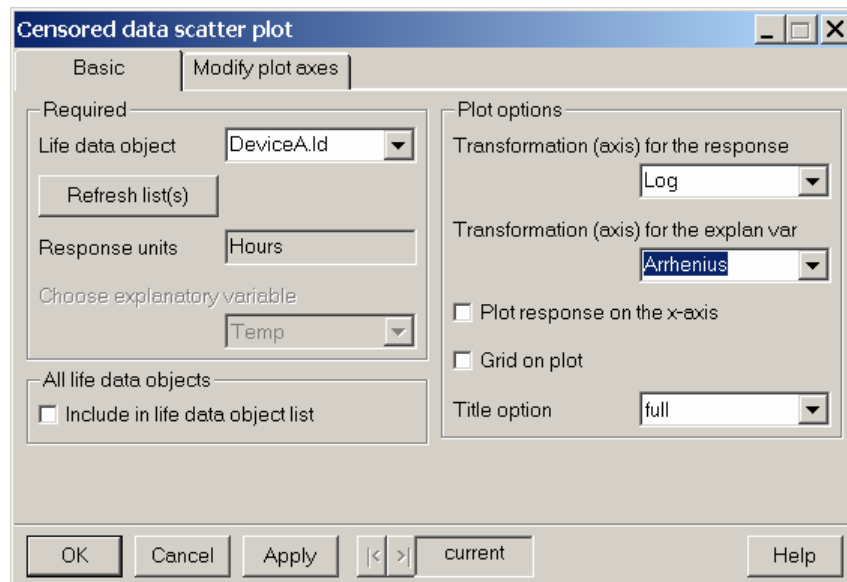
Figure 73 Dialog requesting a censored-data scatter plot of the Device-A data.

Finally, click on "Apply" to produce the Arrhenius plot of the data shown in Figure 74.
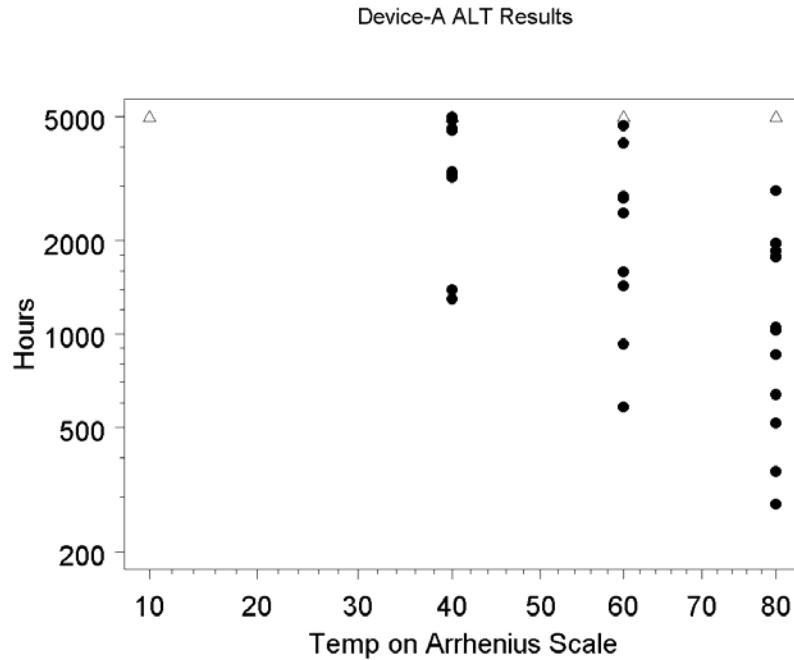
Figure 74 A censored-data scatter plot of the Device-A data.

Notice the nonlinear Arrhenius axis for temperature. The open upward-pointing triangles in the plot indicate right-censored observations For the Device-A data (as in most examples having heavy censoring), the censoring makes it difficult to assess the adequacy of the linear relationship between log time and transformed temperature. Still, however, there is nothing in the plot to suggest that the relationship does not provide a reasonable description of the relationship.

9.2   Group individual probability plots

After loooking at a scatter plot of data from a single-variable accelerated life tests, the next step in the analysis is usually to make an individual probability plot of the data at each level of the accelerating variable (e.g., at each temperature for the Device-A data). Such a plot can be made by using **SPLIDA ► Simple regression (ALT) life data analysis ► Probability plot and ML fit for individual conditions** to request the dialog in Figure 75. Then one chooses the life data object and distribution (Lognormal was used in this example). Clicking "OK" or "Apply" produces ML estimates of the fraction failing at each individual level of temperature.

Figure 75 Dialog requesting a lognormal probability plot for the individual temperature levels in the Device-A accelerated test.
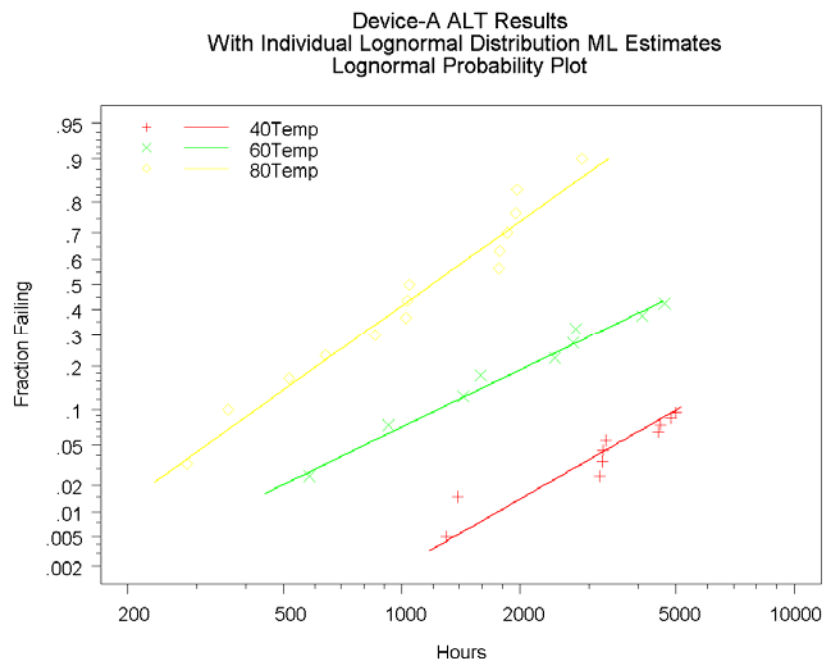
Figure 76 Lognormal probability plot showing the individual nonparametric and parametric estimates of life at the different temperatures in the Device-A accelerated life test.

Figure 76 shows that the lognormal distribution fits well the data at each level of temperature. The slopes of the lines (corresponding to the reciprocal of the lognormal shape parameters) vary somewhat (as expected from variability in the data), but not systematically.Table 9 summarizes the lognormal distribution ML estimation at each individual level of temperature for the Device-A accelerated life test. At 10 degrees C, there were no failures so neither nonparametric nor ML estimates could be computed.

Table 9 Tabular output summarizing lognormal distribution ML estimation at each individual level of temperature for the Device-A accelerated life test.

```
[1] "Skipping 10 because too few failures"

Device A data

Maximum likelihood estimation results:

Response units: Hours

Lognormal Distribution

  Temp Likelihood    mu  se_mu  sigma se_sigma
1   10          NA    NA     NA     NA       NA
2   40     -115.46 9.815 0.4221 1.0083   0.2737
3   60      -89.72 8.644 0.3474 1.1876   0.3167
4   80     -115.58 7.084 0.2087 0.8046   0.1553

Total likelihood = -320.8
```

9.3   Probability plot with common shape (slope) parameter

Many accelerated life test models relate the scale parameter of the life-time distribution (location parameter of the log life distribution) to the accelerating variable but have the shape of the life-time distribution (scale parameter of the log-life-time distribution) constant over all levels of the accelerating variable. Some physical-failure models, such as the Arrhenius relationship, suggest such a statistical model. In order to check the adequacy of such a model, it is useful to fit the candidate distribution to the data with a common shape (slope on a probability plot) assumption. This analysis is similar to that in the comparison-of-distributions example in Section 8.3, with Temperature being used as a class variable. This analysis can be done with **SPLIDA ► Simple regression (ALT) life data analysis ► Prob plot and ML fit for indiv cond: common shapes (slopes).** This dialog, shown in Figure 77, is very similar to that in Figure 75.
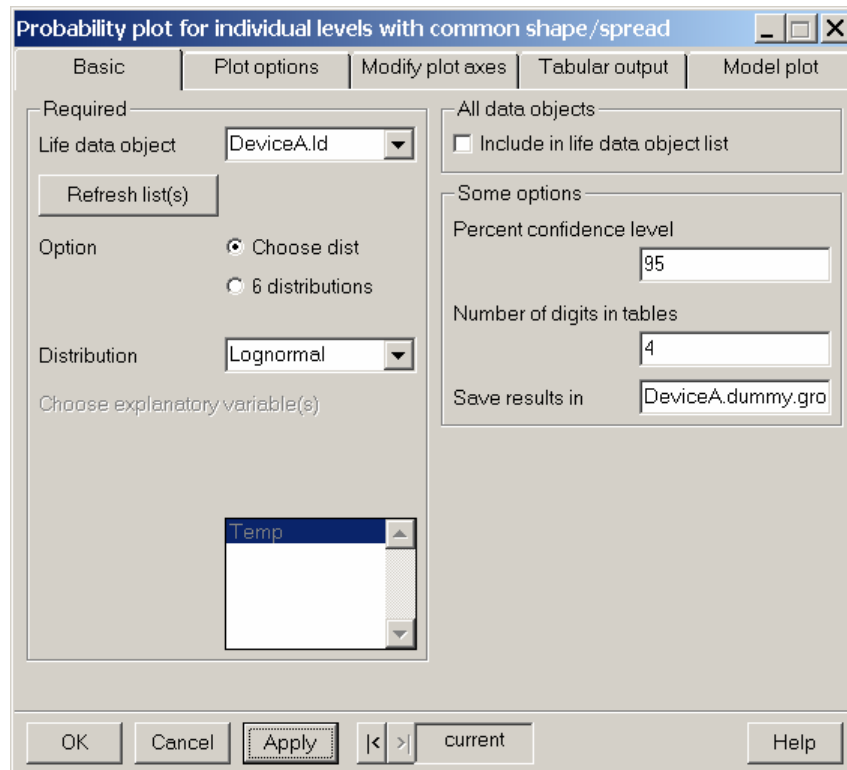
Figure 77 Dialog requesting ML estimates at the different temperatures in the Device-A accelerated life test, with lognormal shape parameters constrained to be the same.

Specify the life data object and the distribution and click "OK" or "Apply" to obtain the multiple probability plot in Figure 78.

Device-A ALT Results subset Estimable Subsets   Model MLE
Tempclass, Dist:Lognormal
Lognormal Probability Plot
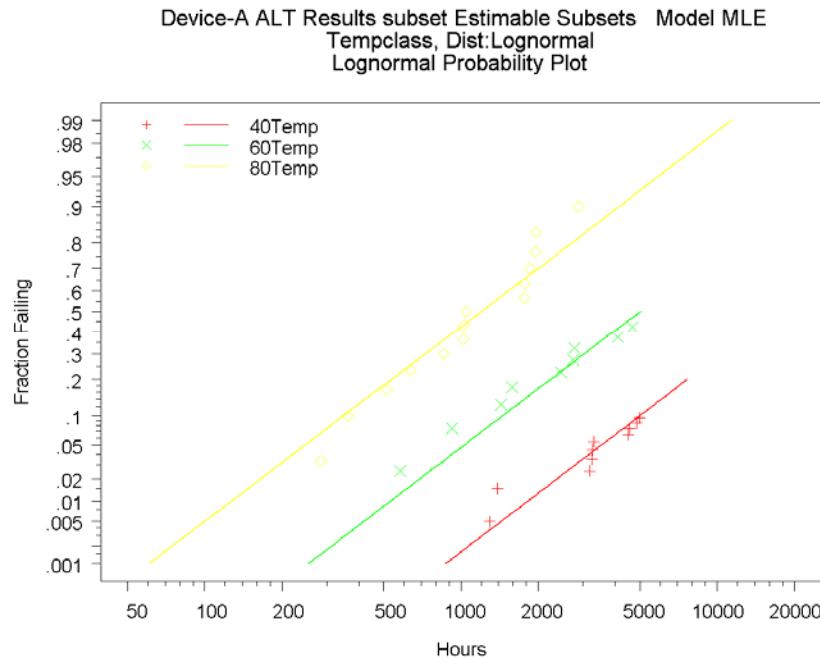
Figure 78 Lognormal probability plot showing ML estimates at the different temperatures in the Device-A accelerated life test, with lognormal shape parameters constrained to be the same.

Note the parallel lines in Figure 78, due to the common σ (standard deviation of log life) assumption in the fitted model.

The tabular output in Table 10 summarizes the lognormal distribution ML estimation for the Device-A accelerated life test. The estimate b_0 is the estimate of μ_{40} the median of log life for the baseline of 40 degrees C (there were no failures at 10 degrees C, so these data are not used in the present analysis). The estimates corresponding to Temp60 and Temp80 give estimates for the differences $\mu_{40} - \mu_{60}$ and $\mu_{40} - \mu_{80}$. As expected, these latter two estimates are negative because of shorter life times at the higher temperatures. The tabular output in Table 10 can be used to compare this model fit with the unconstrained model fit to see if there is strong evidence for a departure from the constant-slope assumption. Figure 78 provides a visual assessment of this assumption.

Table 10 Tabular output summarizing lognormal distribution ML estimation for the Device-A accelerated life test.

Device A data

Maximum likelihood estimation results:

Response units: Hours

Lognormal Distribution

    Relationship

1 : class

```
Log likelihood at maximum point: -321.5
      Parameter          Approx Conf. Interval

            MLE      Std.Err.      95% Lower    95% Upper

      b0   9.7543    0.2465        9.2711      10.2374
Temp60  -1.2330    0.3119       -1.8443      -0.6217
Temp80  -2.6634    0.3484       -3.3463      -1.9806
 sigma   0.9656    0.1320        0.7386       1.2623
```

One can compare the loglikelihoods from fitting the previous two models in order to see if the differences among the shape parameters at the three different temperature levels could be explained by random variability. In a formal maner we compute $-2[(-320.8 - 321.5)] = 1.4,$ which is small relative to 5.99, the .95 quantile of a chi-square distribution with 2 degrees of freedom (the unconstrained model had six parameters while the constrained model had four and thus the difference in dimension is two). This numerical result suggests that there is not strong evidence of differing slopes.

In addition to the table of ML estimates of the distribution parameters, one can also request a table of failure probabilities or quantiles for any of the levels of the explanatory variable in the data by using the options on the Tabular Output of the dialog in Figure 77.

9.4   Fitting simple regression and accelerated life test models

Accelerated life tests are often conducted for the purpose of using data at higher-than-usual levels of some explanatory variable to make predictions of life at lower typical or use levels of that variable. A model is used to describe the effect that the explanatory variable will have on life. For example, when temperatutre is used to accelerate a failure mechanism related to a chemical reaction, the Arrhenius relationship is often suggested as a model to describe the effect that temperature will have on life. The data can be used to estimate the parameters of this model, as described in Section 19.2 of Meeker and Escobar (1998). Use **SPLIDA ► Simple regression (ALT) life data ► Probability plot and ML fit of a regression (acceleration) model** to bring up the dialog in Figure 79.
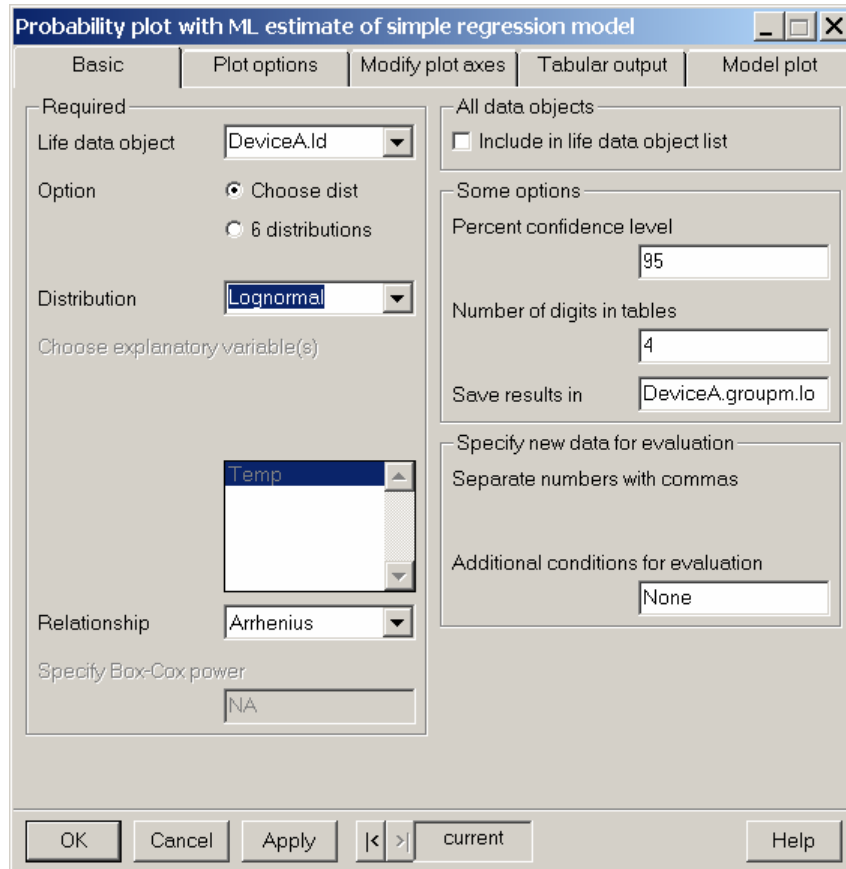
Figure 79 Dialog used to request a probability plot showing the ML fit of the lognormal/Arrhenius acceleration model to the Device-A data.

For many single-variable accelerated life tests, the model to be used is initially suggested by previous experience or physical theory. For the Device-A example, the well-known Arrhenius relationship will be used to describe the relationship between life and temperature, as shown on the dialog. One can also specify additional levels of temperature at which to compute and plot estimates of life and associated confidence intervals. As with other dialoges, various options for customizing the plots and for requesting tabular output are available in the back pages of the dialog.

Options on the "Model plot" page of the dialog, shown in Figure 80, allows one to request and customize a plot of the life versus stress along with the other analyses (hours to failure as a function of temperature on log/Arrhenius scales for the Device-A example).
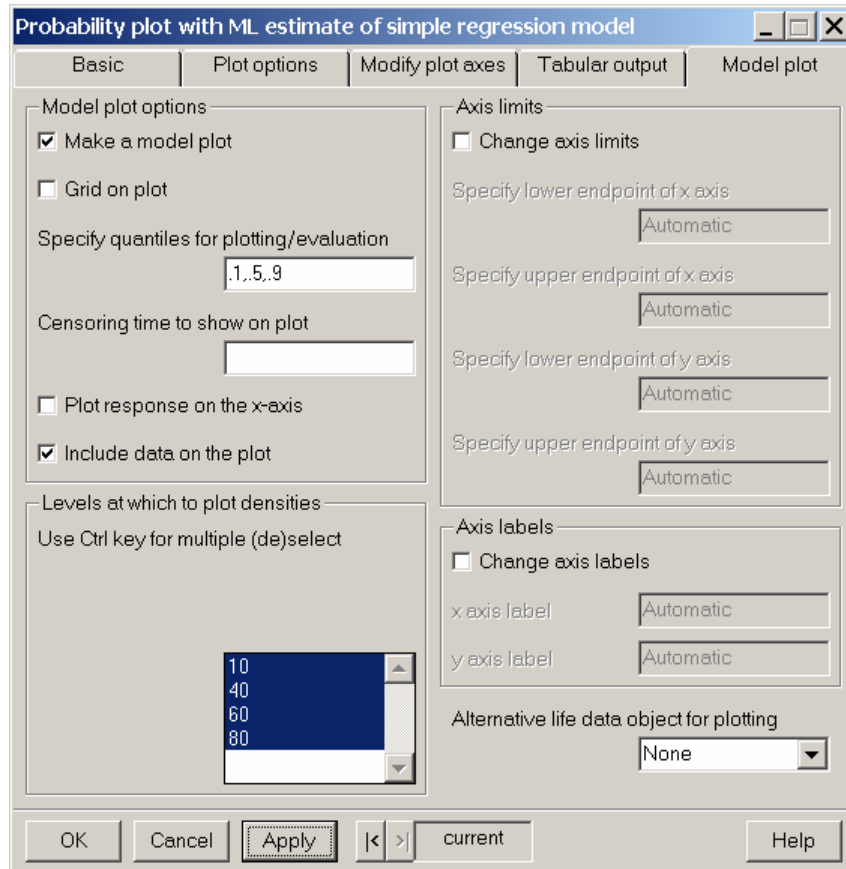
Figure 80 Dialog showing the options on the "model plot" dialog page.

To add confidence limits to the plot, visit the Plot options page, ask for "pointwise" confidence bands, and highlight the level or levels for which the bands should be plotted (generally the use conditions, 10 in this case).

When all of the requested options have been selected, click on "Apply." This will produce the multiple probability plot in Figure 81, showing the lognormal/Arrhenius model estimates of fraction failing as a function of time at the diferent levels of temperature in the original (including 10 Degrees C, where there were no failures) and the Model plot in Figure 82.
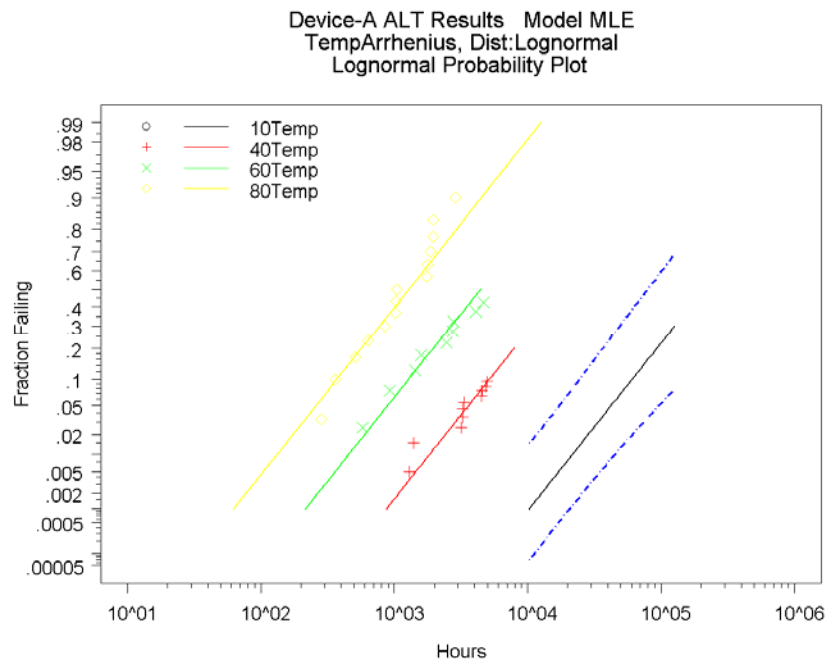
Figure 81 Lognormal probability plot showing the lognormal/Arrhenius model fit to the Device-A data with an extrapolation to the use conditions of 10 degrees C, including approximate pointwise 95% confidence intervals.

Table 11 gives a summary of the ML regression output, similar to what one would obtain from a standard least squares regression output.

Table 11 Maximum likelihood estimation results for the lognormal/Arrhenius model fit to the Device-A data

Device-A ALT Results

Maximum likelihood estimation results:

Response units: Hours

Lognormal Distribution

    Relationship

1 : Arrhenius

```
Log likelihood at maximum point: -321.7
     Parameter             Approx Conf. Interval
```

    MLE Std.Err. 95% Lower 95% Upper

```
Intercept -13.4686  2.88719  -19.1274   -7.8098
     Temp   0.6279  0.08284    0.4655    0.7902
    sigma   0.9778  0.13265    0.7495    1.2756
```

87
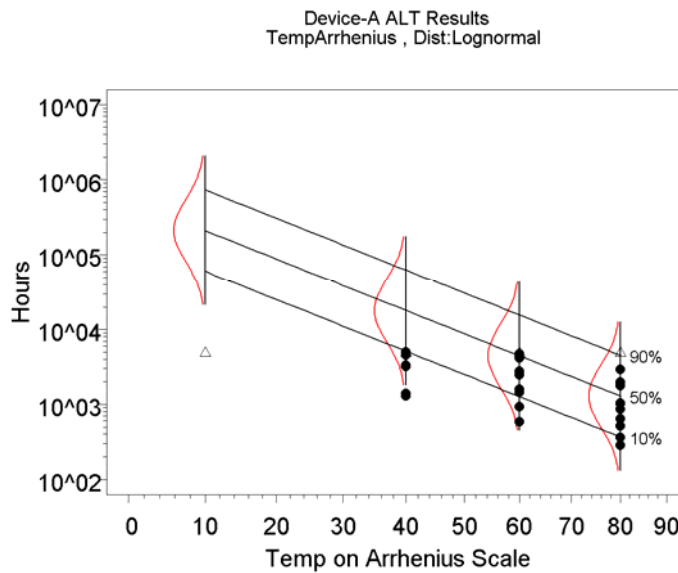
Figure 82 Model plot on Arrhenius paper for the Device-A accelerated life test data and accelerated failure-time densities as a function of temperature.

There is nothing in either of these plots to indicate any serious departure form the fitted model. Of course it should be emphasized that just because the data fits the model well within the range of the data this does not imply that one can safely extrapolate out side of the range of the data (10 degrees C for the Device-A example). The justification for the extrapolation comes from the Arrhenius rate-reaction model from physical chemistry. If this model does not adequately describe the effect of temperature on the underlying degradation mechanism, then seriously incorrect extrapolations could result.

9.5   Make life subset data objects for individual groups

In many applications, under certain circumstances, it will be desired to select one or more of the groups of regression data set for more focused analysis. It is possible, for example to choose a particular level or levels of an accelerating variable and do single-distribution analyses for them. Alternativley, one may want to delete one or more subsets from a data set either as a form of sensitivity analysis or because the subset data are incorrect. We will illustrate this by making a subset life data object for the Mylarpoly.ld life data object. As described in Section 19.3.1 of Meeker and Escobar (1998), the specimens tested at 361.4 kV/mm failed from a failure mode different than those at other levels of voltage stress because the voltage stress of 361.4 kV/mm was too high. It is thus appropriate to drop these data from the analysis. Using **SPLIDA ► Simple regression (ALT) life data ► Make life data objects for individual groups** brings up the dialog like that shown in Figure 83.
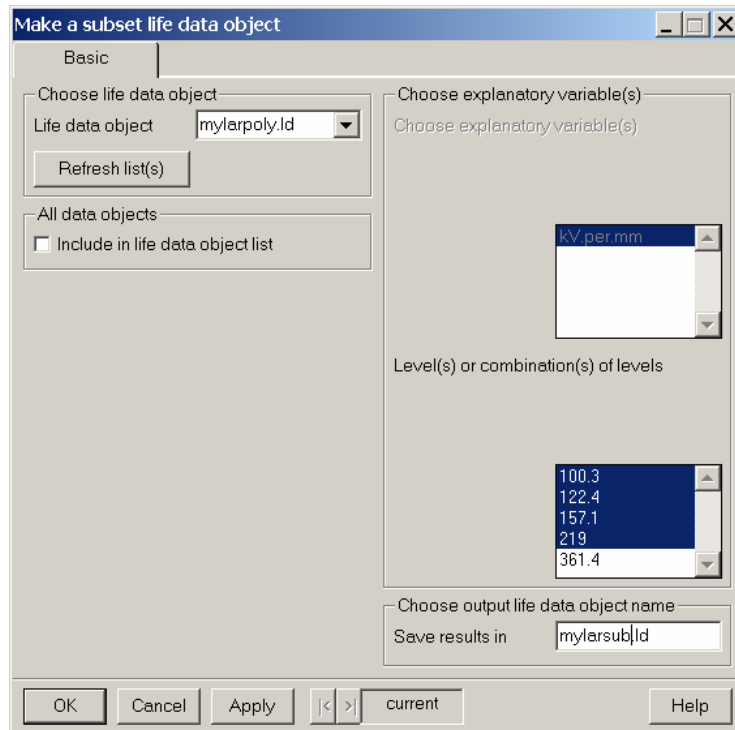
Figure 83 Dialog requesting the creation of a subset data object for the mylarploy data, omitting the 361.4 kV data.

For the example, we choose the mylarpoly.ld comparison life data object. The voltage stress variable kV.per.mm is the only explanatory variable in this life data object, so it is automatically highlighted. Then choose the desired levels (all except 361.4 in this case) under the list of available levels (or in general combinations of levels). A name for the output data object is chosen automatically when the level(s) or combinations of levels are chosen, but it is possible to edit this name. In this example, because so many levels were chosen, the automatic name is long, so it was edited to read mylarpoly.subset.ld. After clicking on "OK" or "Apply" the new life data object is created, in the commands window, we get the message "Saving subset data object mylarsub.ld," and this life data object will appear in subsequent lists of accelerated life test or regression life data objects.

10 Multiple Regression (ALT) Life Data Analysis

The methods illustrated in Section 9 are easily extended to allow one to fit failure time regression models with two or more explanatory variables. The methods presented here provide useful extensions to the multiple regression methods covered in a standard course in the subject of multiple regression analysis, allowing for non-normal distributions and censoring. To illustrate the methods for failure-time regression, we will use the New Spring life data (data object NewSpring.ld) from Meeker, Escobar, and Zayac (2003). The data came from a factorial experiment to evaluate the life of a new processing

method for manufacturing a spring as well as to assess the effect of a processing temperature and spring displacement (stroke) on spring life. Spring displacement was also to be an accelerating variable (using high values to speed up the test). The tabular summary of these data in Table 12 was obtained by using **SPLIDA ►
Make/summary/view/edit data object ► Summary/view data object.**

```
Number of rows in data matrix= 80
Response units:  kCycles
Response minimum:  89
Response maximum:  5000
Number of cases in data set= 108
Number of exact failures in data set= 73
Number of right censored observations in data set= 35
No truncation information

Summary of numeric columns in X matrix:
       min   max    mean       sd      cv
Stroke  50    70   62.38    7.671  0.123
  Temp 500  1000  775.00  250.316  0.323

List of unique X values (or combinations)
   Stroke Temp Method min-Resp max-Resp #Exact #R-cen Total
 1     50   500    New     5000     5000      0      9      9
 2     60   500    New     1016     5000      3      6      9
 3     70   500    New      752     5000      7      2      9
 4     50   500    Old      997     5000      4      5      9
 5     60   500    Old      551     4006      9      0      9
 6     70   500    Old      211     2029      9      0      9
 7     50  1000    New     1715     5000      2      7      9
 8     60  1000    New     1595     5000      6      3      9
 9     70  1000    New      808     4563      9      0      9
10     50  1000    Old      489     5000      6      3      9
11     60  1000    Old      371     2630      9      0      9
12     70  1000    Old       89     1301      9      0      9
```

Table 12 Summary of the NewSpring life data object

10.1 Censored data pairs plot

As described earlier, data analysis should begin with exploration using graphical tools. The dialog in Figure 84, obtained from **SPLIDA ► Multiple regression (ALT) life data ► Censored data pairs plot,** allows the user to plot all pairs of variables in one "matrix plot" (also known as a "pairs plot" in S-PLUS).
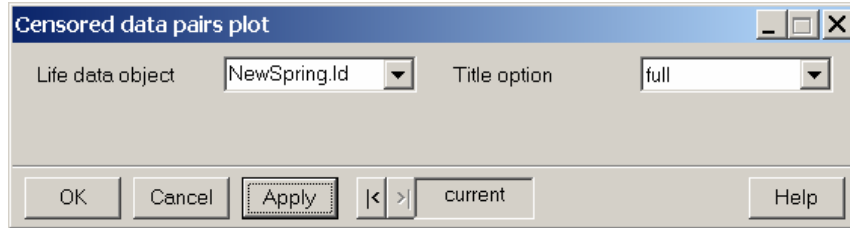
Figure 84 Dialog to request a censored data pairs plot.

Figure 85 shows a pairs plot for the NewSpring data. As shown in the simple regression example with the Device-A data, the censoring makes it more difficult to interpret a scatter plot. The pair-wise plots between Method, Temp, and Stroke show the factorial arrangement of the experiment.
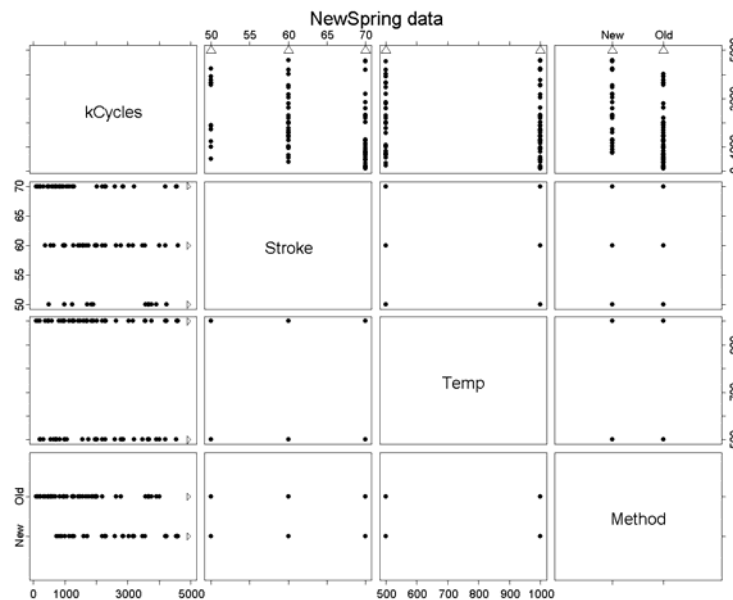
Figure 85 Censored data pairs plot showing the NewSpring life versus Stroke, Temperature, and Method.

10.2 Censored data scatter plot

The censored data scatter plot dialog obtained with **SPLIDA ► Multiple regression (ALT) life data ► Censored data scatter plot** is exactly the same as that shown in Figure 73, obtained with **SPLIDA ► Simple regression (ALT) data analysis ► Censored data scatter plot**. As before the dialog allows one to request a single scatter plot of the response versus one explanatory variable, with an option to transform either or both of the data axes.

10.3 Probability plot and ML fit for individual conditions

The dialog obtained from **SPLIDA ► Multiple regression (ALT) life data ► Probability plot and ML fit for individual conditions,** shown in Figure 86 allows one

to request separate analyses for all combinations of experimental variable levels in one's data set, with all of the results plotted on one probability plot.
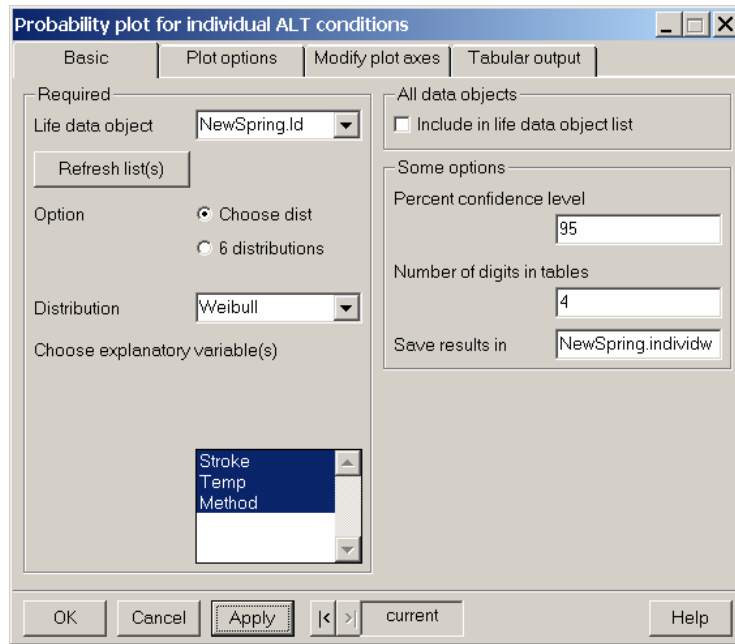
Figure 86 Dialog requesting a probability plot showing individual nonparametric estimates for each experimental factor level combination on lognormal paper.

After the dialog appears, choose the life data object (only life data objects with explanatory variables appear in the list), a distribution, and all or some of the explanatory variables. Then click "Apply" or "OK," producing a probability plot like that in Figure 87.
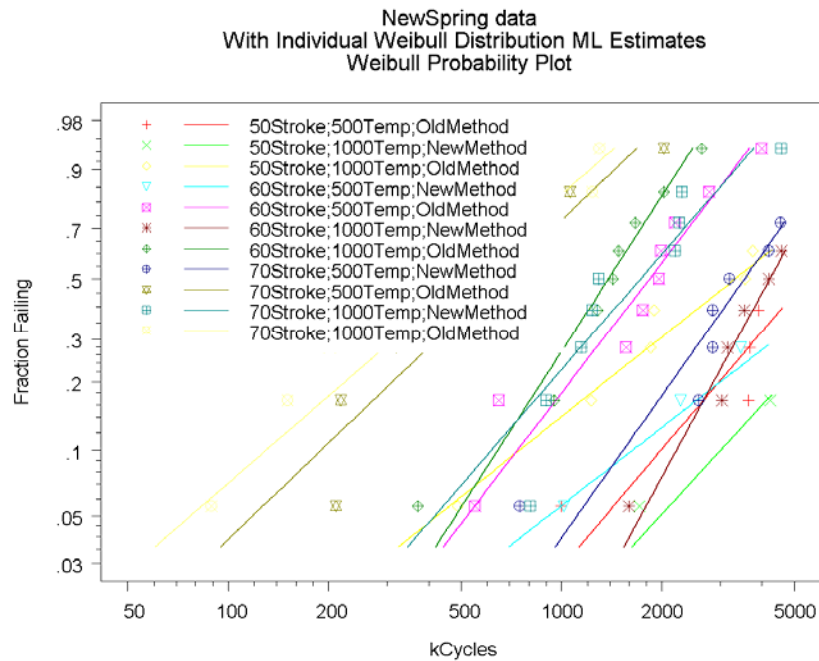
Figure 87 Probability plot showing individual nonparametric estimates for each experimental factor level combination of the NewSpring data on Weibull paper.

By using the Plot Options page of the dialog in Figure 86, it is possible to suppress the printing of the legend or have it print on a separate page. There is a considerable amount of variation among the slopes of the ML estimate lines in Figure 87. This is due to the small number of failures at some combinations of voltage and temperature. Corresponding numerical results are given in Table 13. NAs are shown for the test conditions for which there were not enough failures to compute ML estimates.

```
Maximum likelihood estimation results:

Response units: kCycles

Weibull Distribution
                                   Log likelihood    eta se_eta  beta se_beta
 1  50Stroke;500Temp;NewMethod              NA      NA     NA     NA      NA
 2  50Stroke;500Temp;OldMethod           -39.85  6639.6 2122.7 1.868  0.8845
 3 50Stroke;1000Temp;NewMethod           -21.59 11104.8 7802.5 1.723  1.1772
 4 50Stroke;1000Temp;OldMethod           -56.45  4430.4 1446.9 1.265  0.4536
 5  60Stroke;500Temp;NewMethod           -31.17 10101.2 6480.1 1.236  0.6723
 6  60Stroke;500Temp;OldMethod           -74.39  2193.1  372.5 2.067  0.5406
 7 60Stroke;1000Temp;NewMethod           -55.10  4781.6  677.9 2.920  1.0591
 8 60Stroke;1000Temp;OldMethod           -70.45  1606.2  228.4 2.466  0.6460
 9  70Stroke;500Temp;NewMethod           -63.58  4169.5  700.5 2.250  0.7468
10  70Stroke;500Temp;OldMethod           -67.43   835.9  194.1 1.521  0.3799
11 70Stroke;1000Temp;NewMethod           -74.49  2108.8  408.3 1.831  0.4420
12 70Stroke;1000Temp;OldMethod           -66.08   665.7  169.7 1.378  0.3730

Total log likelihood= -620.6
```

Table 13 Individual ML estimates for the different NewSpring life test conditions

10.4 Probability plot and ML fit for individual conditions: common shapes (slopes)

The methods described in this section are similar to those described in Sections 8.3 and 9.3 except that more than one explanatory variable can be specified for the analysis. Using **SPLIDA ► Multiple regression (ALT) life data ► Prob plot and ML fit for indiv cond: common shapes (slopes)** brings up a dialog like that in Figure 88. This dialog is very much like that obtained with **SPLIDA ► Simple regression (ALT) life data analysis ► Prob plot and ML fit for indiv cond: common shapes (slopes),** except that one is allowed to choose more than one explanatory variable (including categorical variables, known as "factors" in S-PLUS). Using maximum likelihood, a model is fitted that allows the scale parameter of log-location-scale distributions (location parameters of location-scale distributions) at the different combinations of experimental conditions to vary, but constrains the shape (slope) parameter of the log-location distributions (scale parameters of location-scale distributions) to be the same in the model.
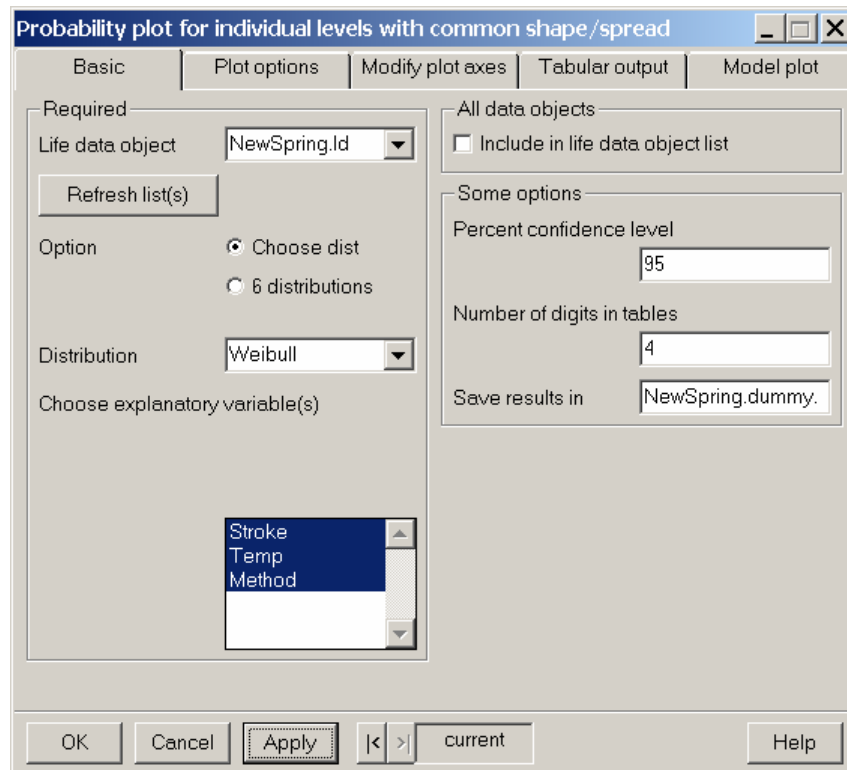
Figure 88 Dialog to request, for the NewSpring ALT data, a probability plot and maximum likelihood estimation for individual combinations of the NewSpring experimental conditions, but with a common Weibull shape parameter.

As with previous similar dialoges, choose the life data object, a distribution, the explanatory variables to be used, and then click "Apply" or OK, giving the probability plot shown in Figure 89.
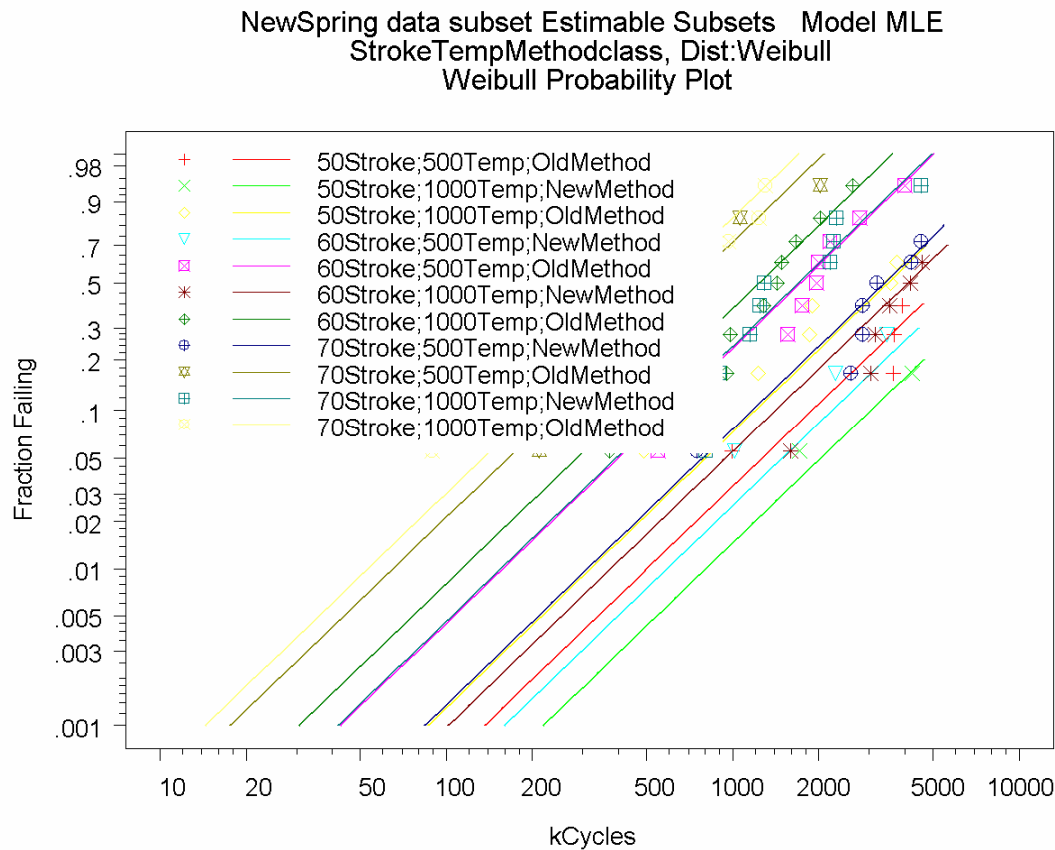
Figure 89 NewSpring ALT data probability plot and maximum likelihood estimation for individual combinations of the NewSpring experimental conditions, but with a common lognormal shape parameter.

10.5 Probability plot and ML fit of a regression (acceleration) model

This section describes the fitting a multiple regression model. The dialog for doing this, while similar to the dialog for fitting a one-variable relationship, differs significantly. First, the model specification is more complex because more than one variable/relationship combination has to be specified. Relatedly, it is also more complicated to specify additional conditions ("new data" in S-PLUS jargon) at which to do evaluations of distribution properties like failure probabilities and quantiles. Additionally, there is no option on the dialog for a "model plot." Instead, after a model is fit, the user can make a "conditional model plot," as described in Section 10.6.

Using **SPLIDA ► Multiple regression (ALT) life data ► Probability plot and ML fit of a regression (acceleration) model** brings up a dialog like the one in Figure 90. In this box, one chooses the life data object, a distribution (or the "6 distribution" option) and which explanatory variables to use. Then one moves to the model page of the dialog to specify the model (i.e., life-explanatory variable relationships). To illustrate these features, we continue with the NewSpring example.

After the NewSpring.ld life data object has been chosen, one chooses the distribution and explanatory variables. The Plot Option and Tabular Output pages of the dialog will show (when output options are actually selected) lists of explanatory variable combinations from which one can choose one or more combinations for particular focus. For example, as with simple regression analyses, one can ask for confidence intervals for ML estimates of the cdf (fraction failing as a function of time) on probability plots and for tables of distribution quantiles or failure probabilities at specified explanatory variable combinations. It is also possible to input user-specified combinations of levels of the explanatory variables ("new data" in S-PLUS jargon) other than those implied by the levels in the input data. This can be done either by the "Direct method" of entering numbers in the "Additional levels for evaluation" box or by specifying an Splus data set containing the information with appropriate column headings corresponding to the variable names in the lifedata object. The first method is illustrated in the dialog shown in Figure 90 where the string of characters 30;600;New,20;600;New is used to specify two new conditions for evaluation. Note that semicolons are used to separate the levels within a condition and that a comma is used to separate the two conditions. When entering these numbers it is sometimes convenient to use the S-PLUS right-click Zoom option. The "Save results in:" box gets filled and becomes editable after a the life-explanatory variable relationships are chosen on the `Model` page.
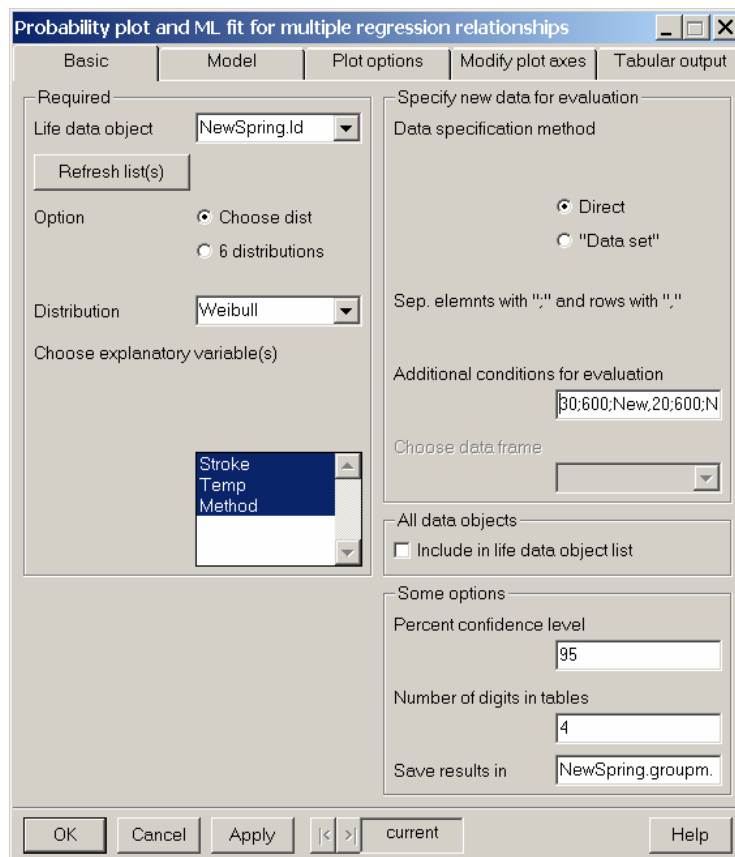
Figure 90 Basic page of the multiple regression dialog requesting a probability plot and a fitted regression model for the NewSpring data.

On the `Model` page of the dialog (shown in Figure 91), the user first chooses the relationships for each of the variables (except that categorical variables-like Method---factors in S-PLUS jargon---are automatically chosen to be class variables). This choosing process is initiated by clicking on the "relationships" button. For the example below, the standard log (equivalent to inverse power rule) was chosen for Stroke, a linear (i.e., no transformation) for Temperature variables, respectively. By default, the fitted model will include only main effects for each of the variables chosen to be in the model (note the default equation). It is, however, possible to specify a more complicated model involving interactions and polynomial terms, if desired. This is done by specifying a model equation, using standard S-PLUS modeling formula. The easiest way to specify such a model is by using the Create Formula button, specify main effects and other terms, as desired. Then, after dismissing the equation-specification dialog, edit the resulting equation (e.g. to remove unwanted interactions). The right-click zoom option for editing the equation is useful.

As with previous dialogs for requesting ML estimation, a Tabular Output page allows the user to request tables of distribution quantiles or failure probabilities for specified levels or combinations of levels of the explanatory variables.

Figure 91 Model-specification page of the multiple regression dialog requesting a probability plot showing a fitted regression model for the NewSpring data.

The Tabular Output page of the dialog in Figure 90, shown in Figure 92, lists the available levels of the explanatory variables (this list is full only if the explanatory variables have been chosen on the basic page and if one of the boxes for Quantiles Estimates or CDF estimates are checked). The user-inputted values appear at the bottom of the list. In this case, only the two user-inputted values have been chosen for generating the output tables.
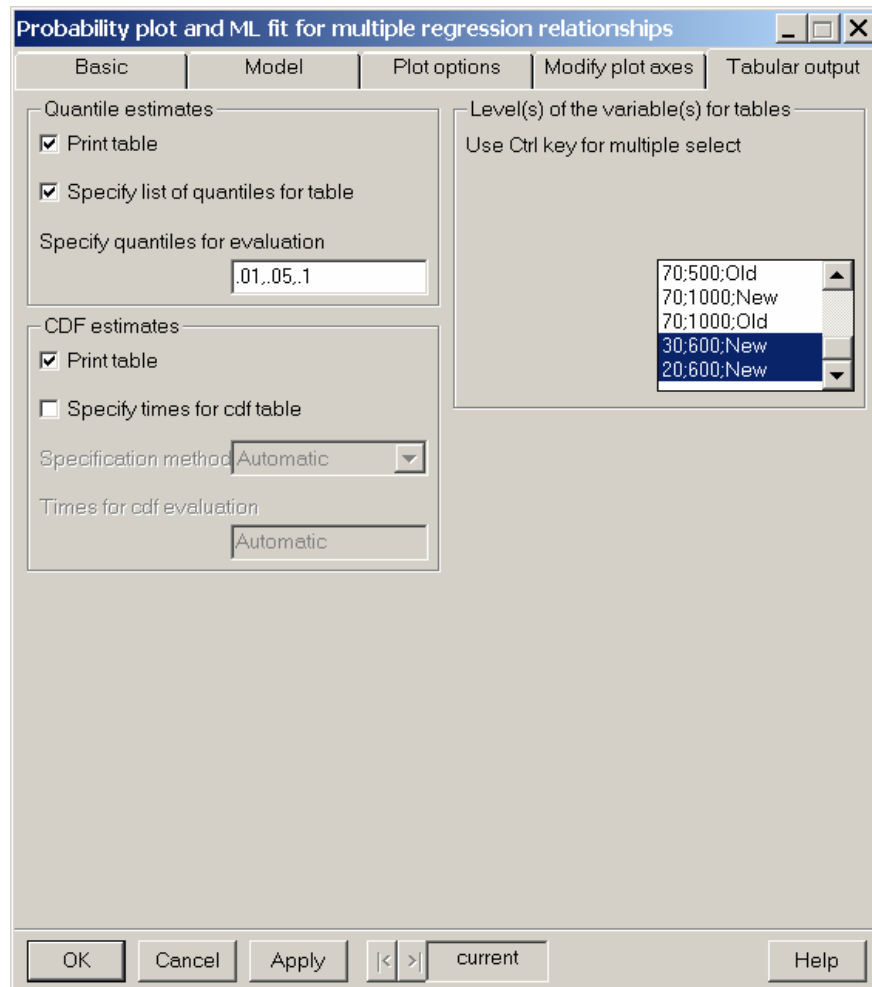
Figure 92 Tabular Output page of the multiple regression dialog requesting tables of quantile and failure probability estimates at particular levels of the experimental variables for the NewSpring data.

Clicking on "OK" or "Apply" produces the probability plot in Figure 93 and the S-PLUS tabular output shown in Table 14 and Table 15. Table 14 gives the usual regression output. Table 15 shows ML estimates for the (Stroke; Temp; Method) conditions of (30; 600; New). The estimates for the conditions (20; 600; New) are not displayed here to save some space..

Figure 93 shows the probability plot for this example with lines also drawn for the user-specified combinations of 30 Volts and 5 Degrees C and 30 Volts and 10 Degrees C. The legend is covering some of the plotted information, so one might want to exercise the Plot Option to suppress or move the legend to a new page. Note that when the model is fit, the results are saved in a "results object" namedNewSpring.groupm.weibull.StrkLog.TempLinr.MthdClss.out. Note that the name of the results object contains the life data object prefix, distribution name, and explanatory variable/relationship combinations, relieving the user from the chore of specifying such names and making it easier to distinguish among different results objects

```
NewSpring data
Maximum likelihood estimation results:
Response units: kCycles
Weibull Distribution
Variable: Relationship (g)
1 Stroke: Log
2 Temp: Linear
3 Method: Class

S-PLUS dummy variable coding method used for Class
variable(s): contr.treatment,contr.poly
Model formula:
Location ~ g(Stroke) + Temp + Method
Log likelihood at maximum point: -625.8

  Parameter                              Approx Conf. Interval
                    MLE    Std.Err.   95% Lower    95% Upper
 (Intercept)  32.026955 2.4843573   27.157704   36.8962056
   g(Stroke)  -5.509575 0.5872085   -6.660482   -4.3586674
        Temp  -0.000883 0.0002709   -0.001414   -0.0003521
      Method  -1.272388 0.1475136   -1.561510   -0.9832671
       sigma   0.569491 0.0539010    0.473067    0.6855692
weibull.beta   1.755953 0.1661969    1.458642    2.1138649
```

when requesting subsequent analyses (e.g., residual plots, conditional model plots, and sensitivity analyses).

Table 14 ML estimates for the Weibull distribution acceleration model fit to the NewSpring accelerated life test data
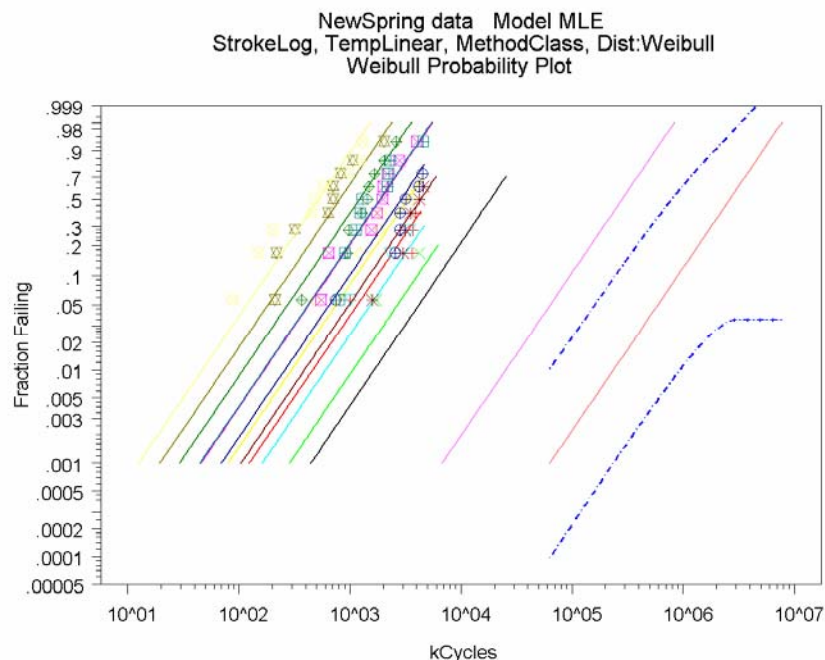
Figure 93 Probability plot showing a fitted Weibull regression model to the NewSpring data.

```
Failure Probability Estimates

Using NewSpring data at  Stroke30;Temp600;MethodNew
Parametric ML CDF Estimates
Pointwise Approximate 95% Confidence Intervals

 Weibull Distribution

  kCycles         Fhat     Std.Err.    95% Lower    95% Upper
       50 1.7949e-007 2.5544e-007 1.1031e-008 2.9204e-006
      100 6.0622e-007 8.0559e-007 4.4822e-008 8.1990e-006
      200 2.0475e-006 2.5350e-006 1.8087e-007 2.3178e-005
      500 1.0233e-005 1.1511e-005 1.1282e-006 9.2798e-005
     1000 3.4560e-005 3.6150e-005 4.4482e-006 2.6846e-004
     2000 1.1672e-004 1.1372e-004 1.7290e-005 7.8753e-004
     5000 5.8320e-004 5.2102e-004 1.0119e-004 3.3535e-003

Quantile Estimates

Using NewSpring data at  Stroke30;Temp600;MethodNew
Parametric ML Quantile Estimates
Pointwise Approximate 95% Confidence Intervals

 Weibull Distribution

     p Quanhat Std.Err. 95% Lower 95% Upper
  0.01   25293    11573     10316     62011
  0.05   63992    28770     26512    154459
  0.10   96419    43532     39797    233600
```

Table 15 Maximum likelihood estimates of Failure probabilities and distribution quantiles.

10.6   Conditional model plot

When a fitted regeression model has more than one explanatory variable, it is still possible to make a "model plot" showing time as a function of stress (or other explanatory variable). In order to do this, however, it is necessary to specify fixed values of the other explanatory variables. Using **SPLIDA ► Multiple regression (ALT) life data ► Conditional model plot** brings up the dialog in Figure 94. When the dialog appears, it will remember the most recent results object (if the analysis was done in the current session). In this case the remembered name is NewSpring.groupm.weibull.StrkLog.TempLinr.MthdClss.out. Suppose now that design engineers want to estimate the life distribution of the spring manufactured with the new method with a processing temperature of 600 degrees F for different levels of stroke ranging between 10 and 80 mils. After specifying this information and clicking on "Apply" or "OK," produces a plot like that in Figure 96.
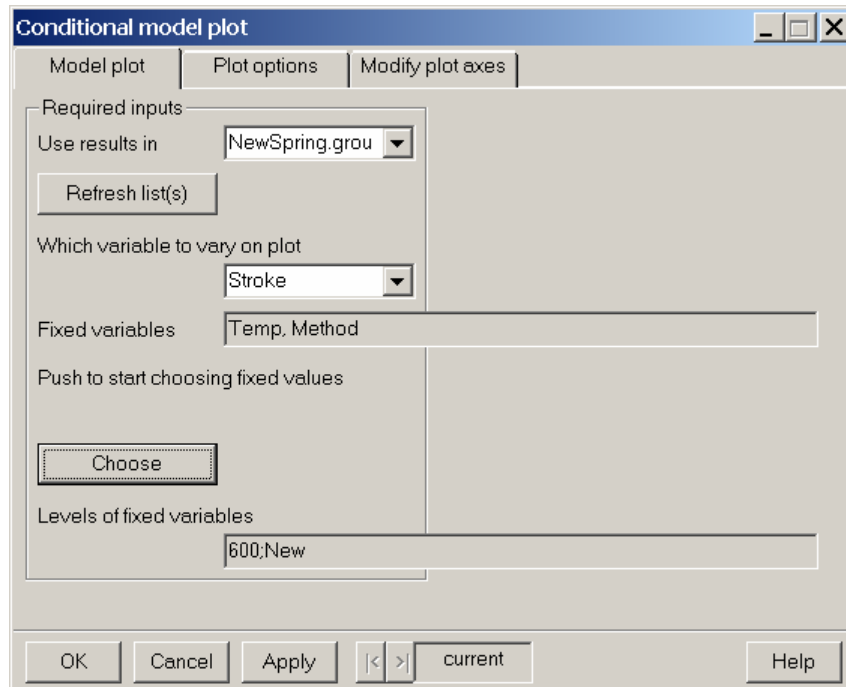
Figure 94 Dialog for a conditional model plot.
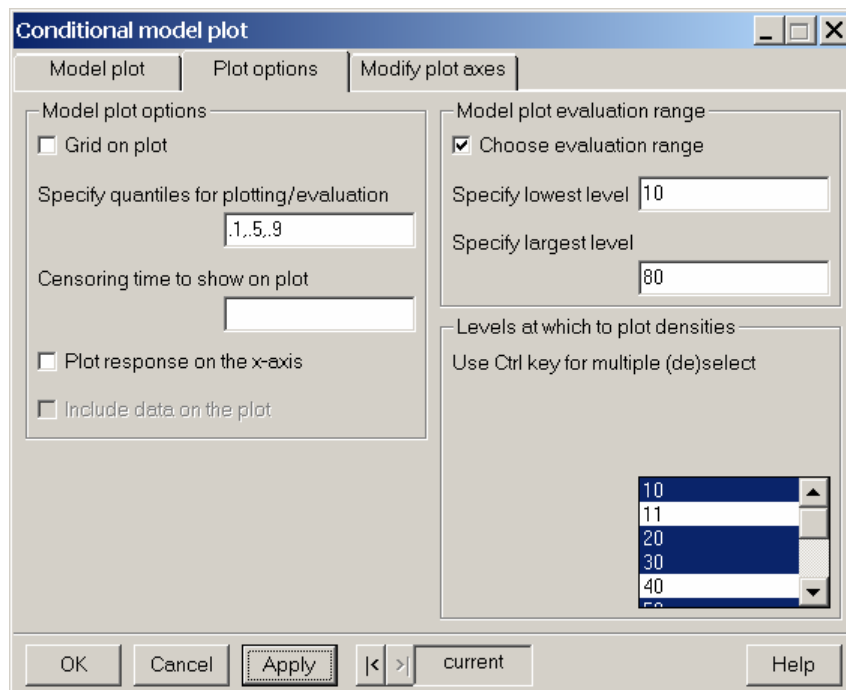
Figure 95 Plot options page for a conditional model plot.
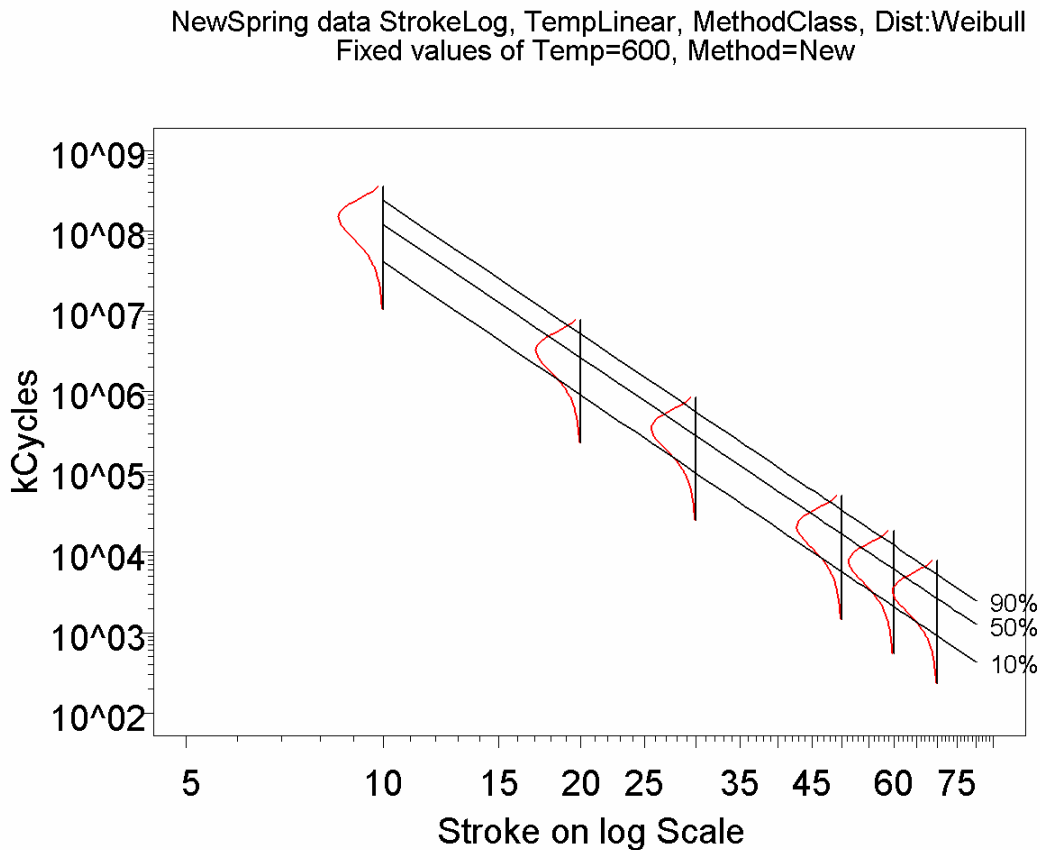
Figure 96 Conditional model plot showing estimated NewSpring life versus stroke for the new processing method and temperature fixed at 600 degrees F.

10.7 Sensitivity analysis plot

Generally when fitting several alternative models to data, among the models that fit the data well, there is usually relatively little difference in the results from the fitted models, as long as one is working within the range of one's data. SPLIDA has a powerful tool that allows one to compare directly alternative analyses with different underlying model assumptions. Using **SPLIDA ► Multiple regression (ALT) life data analysis ► Sensitivity analysis** plot brings up the dialog in Figure 97. Similar to the "Conditional model plot dialog", the name of the last results structure used in the current session is remembered, but any of the other existing results structures could be selected instead. The primary model purturbation used in the sensitivity analysis is the relationship assumption relating a continuous explanatory variable to life. This is done by fitting separate models using several different choices for the parameter in the Box-Cox family of transformations on the chosen numerical explanatory variable. The default values are from –1 to 2, in steps of .5, but this was changed to run from –4 to 3 in steps 0f .25. The output is a plot of the ML estimate of a selected quantile (or quantiles) as a function of the Box-Cox parameter (1 is the same as a linear transformation, .5 is the

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same as a square root relationship, 0 corresponds to a log relationship, and –1 corresponds to a reciprocal relationship).
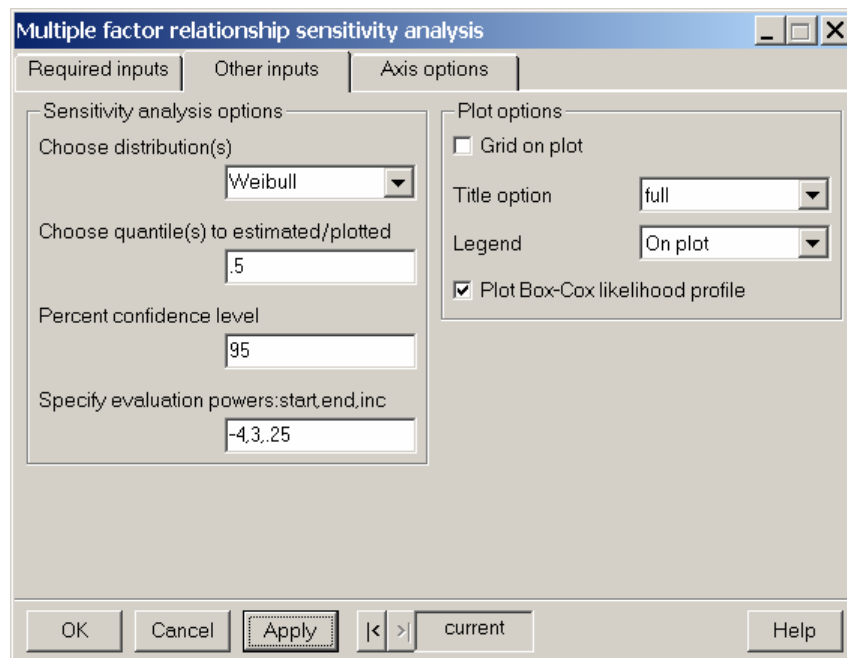
Figure 97 Dialog requesting sensitivity analysis for the assumed relationship between spring life manufactured with the new method at 600 degrees F and operated with a stroke of 30 mils.

If only one quantile and one distribution are used in the comparison (the default), pointwise confidence intervals for the quantile are also plotted, as shown in Figure 98
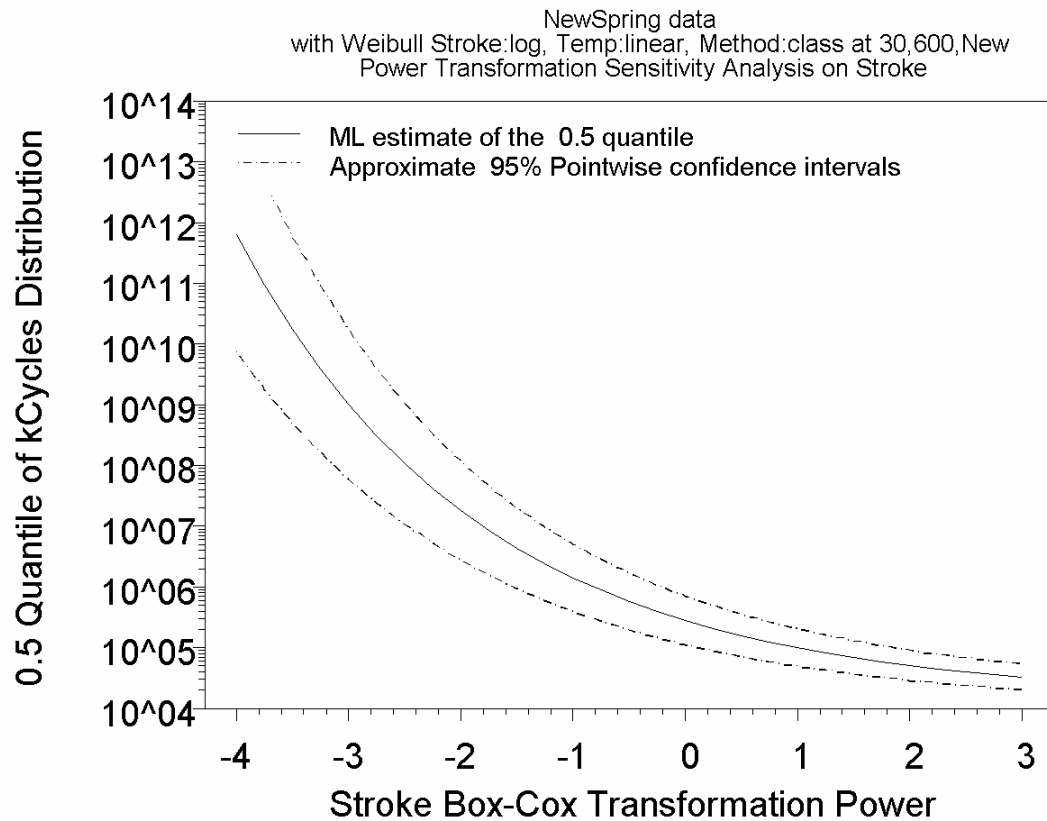
Figure 98 Sensitivity analysis plot showing the effect on spring life of the assumed relationship between life and stroke for a spring manufactured using the new method at 600 degrees F and operated with a stroke of 30 mils.

Also provided is a plot, like that shown in Figure 99, of the profile likelihood function for the Box-Cox parameter. This plot allows an assessment of the statistical power to discriminate among the different relationship parameters. Note that if there are only two levels of the transformed explanatory variable, the profile likelihood will be constant because there is no information in the data to discriminate among different transformations on that explanatory variable.

NewSpring data
Approximate Profile Likelihood and 95% Confidence Interval
for Stroke Box-Cox Transformation Power from the Weibull Distribution
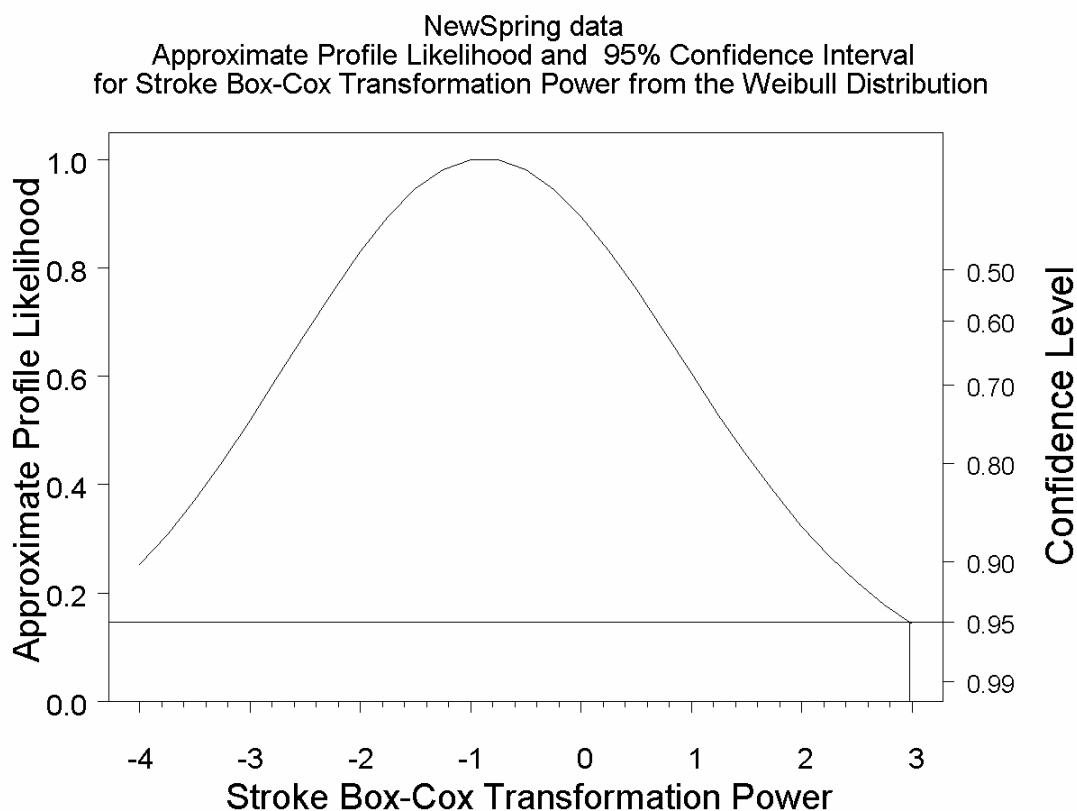
Figure 99 Profile plot showing the relative likelihood (probability) of the NewSpring data as a function of the stroke relationship Box-Cox parameter.

In this case, the profile likelihood suggests that a transformation with the Box-Cox parameter between –4 and 3 are consistent with the data.. Based on previous, transformations used typically used for similar products have been 0 (corresponding to a log transformation). To be conservative, however, values between 1 and 3 might be used in reporting a plausible range of results.

10.8 Make life data objects for individual groups

It may, under certain circunstances be desired to select one or more of the groups of regression data for more focused analysis. It is possible, for example to choose a particular combination of levels of explanatory variables and do separate analyses for them. We will illustrate this by making a separate life data object for the new design within the NewSpring life data object. Using **SPLIDA ► Simple regression (ALT) life data ► Make subset life data object** brings up the dialog in Figure 100.
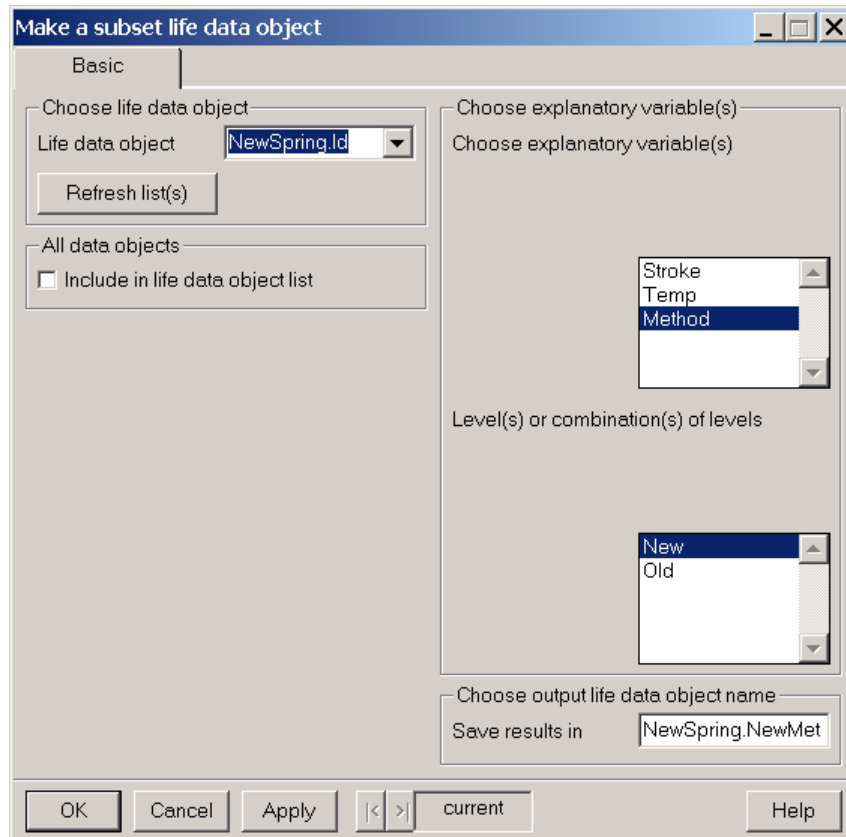
Figure 100 Dialog requesting the creation of a life data object using only the New Design data from the NewSpring lifedata object.

After clicking on "OK" or "Apply" the new life data object is created, we get the message "Saving subset data object `NewSpring.NewMethod.ld`," and this life data object will appear in subsequent lists of single distribution life data object.

11 Regression Residual Plots

In any kind of statistical modeling, it is important to look for departures from models being fitted to data. This is especially true for regression models in which explanatory variables like temperature and humidity are used to describe the characteristics of a response variable like life time. Because it is possible to specify most model assumptions in terms of characteristics of model residuals (deviations between the response and some definition of expected or typical response), a useful method of diagnostic checking is to examine a model's residuals, usually with various graphical displays. Examples illustrating the use of and some technical details concerning the definition of residuals for censored data are given in Section 17.6 of Meeker and Escobar (1998). We will illustrate the use of the SPLIDA's residual plotting capabilities with the mylar-polyurethane insulating structure accelerated life test data, analyzed in Section 19.3.1 of Meeker and

Escobar (1998). The ML analysis results object is
(`NewSpring.groupm.weibull.StrkLog.TempLinr.MthdClss.out`).

Using **SPLIDA ► Multiple regression (ALT) analysis ► Residual analysis**
brings up the dialog in Figure 101. By default, all of the possible residual plots are
selected, but it is easy to select one or a few of the possible plots. The `Modify plot`
`axes` page becomes active and can be used to control the plot axes if there is just one
plot chosen.
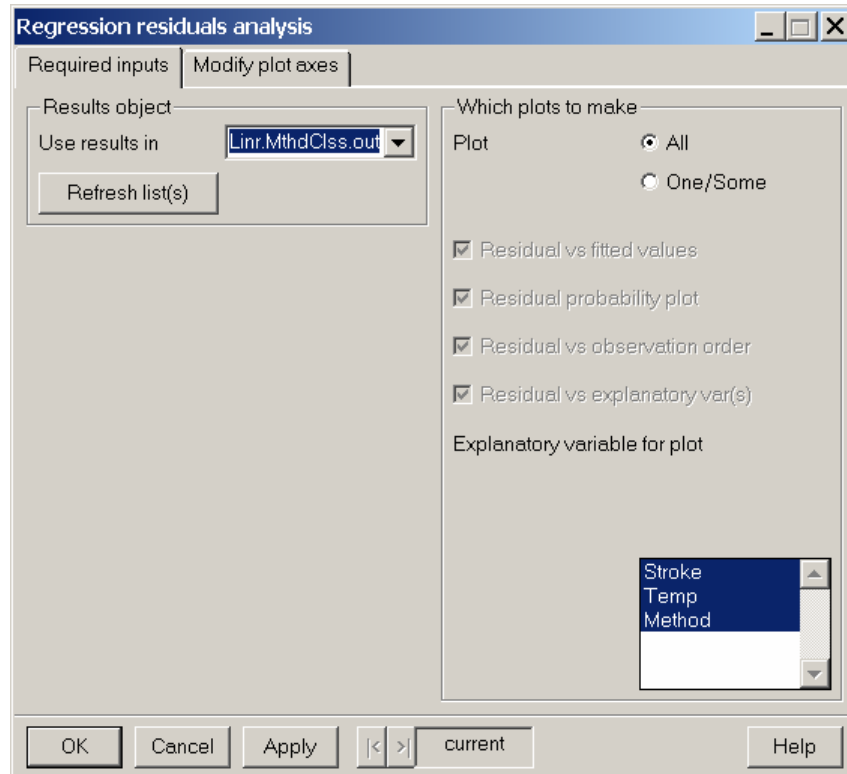
Figure 101 Dialog requesting residuals plots for the NewSpring regression model

11.1 Residuals versus fitted values

It is common practice to plot residuals versus fitted values. Such a plot is useful for
detecting departures from the structural model as well as model departures like
nonconstant spread. Figure 102 shows a plot of the residuals versus the fitted values for
NewSpring regression model. Notice that many of the residuals are right censored and

that this limits the amount of information in the plot.
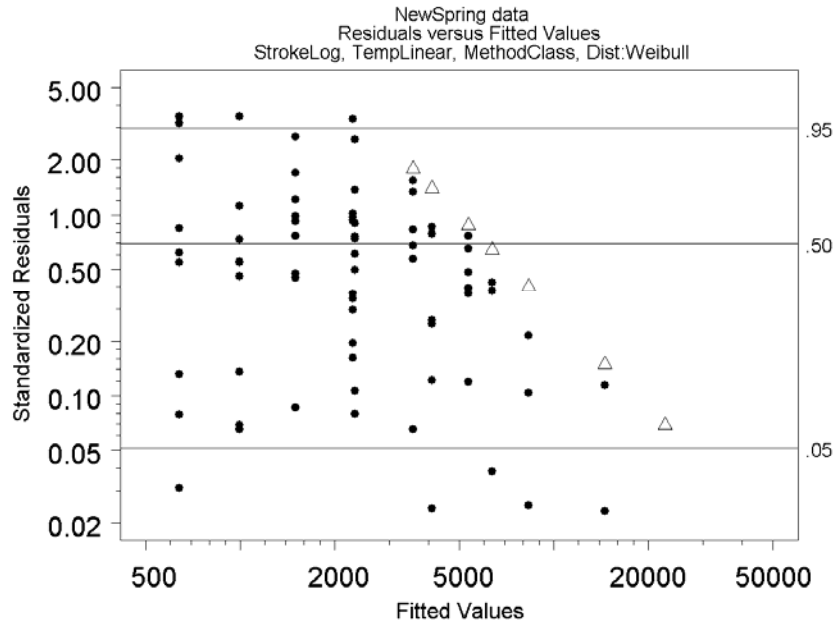
Figure 102 Plot of the regression residuals for the laminate panel data versus fitted values.

11.2 *Residuals versus possible explanatory variables*

The residual from a regression model should not have a strong relationship with any other external variable. If such a relationship is discovered, it suggests that the variable should have been included into the model. To illustrate this we return to the NewSpring data used in Section 10.5, but using a model in which life is related to Stroke and Method only, omitting the temperature variable (this will be referred to as the reduced model).

The results object for the reduced model is chosen (actually it is remembered as the default choice if the model was fit just before bringing up the dialog). Then DegreesC is chosen as the variable against which to plot. Clicking on "Apply" or "OK" gives the plot shown in Figure 103.
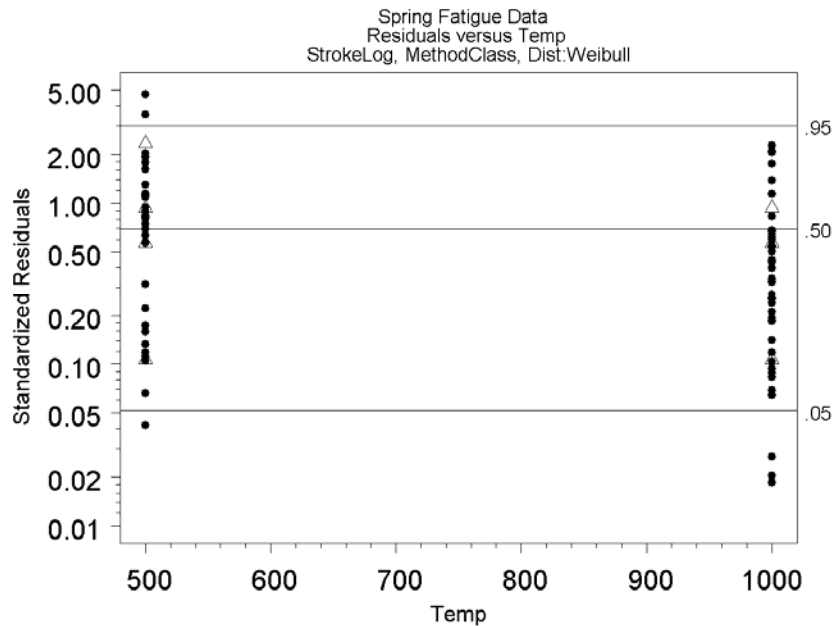
Figure 103 Residuals for the Weibull/inverse power relationship model for the NewSpring data plotted against the omitted variable Temperature.

The plot suggests some relationship between the residuals and Temperature, although the relationship does not seem to be strong.

11.3 Residuals versus observation number

In some studies, data are obtained over time and the order of the cases in the resulting data set may have some importance. For example, if units are tested sequentially, one at a time, on a machine to do fatigue cycling, it is possible that some underlying "lurking factor" has an effect on the response. For example, there may be gradual drift of machine settings or an effect due to changes in ambient temperature or other environmental conditions. An analyst can check for such underlying time-dependent changes in a process by plotting residuals versus time order of the observations. Of course, in order for such a plot to be meaningful, it is necessary that the case ordering in the data set correspond to the actual time ordering that units were tested. If (as is often the case) available data are ordered according to observed failure time (which might be the natural ordering for commonly-used simultaneous testing) or according to explanatory variable levels, the potentially important time-sequence information is lost and plots made versus time order would probably have no meaning.

11.4 Residual probability plot

As described in Section 17.6 of Meeker and Escobar (1998), residuals from a regression model, even with censoring, can be examined with the use of standard probability

plotting methods. Residual probability plots are useful for assessing the reasonableness of particular distribution choice for a regression model. The multiple probability plots used in Sections 9 and 10 are useful for making this assessment at individual conditions. After fitting an overall model, however, plotting the residuals provides more information on the adequacy of the overall distribution.

Figure 104 is a residual probability plot for the laminate panel data.
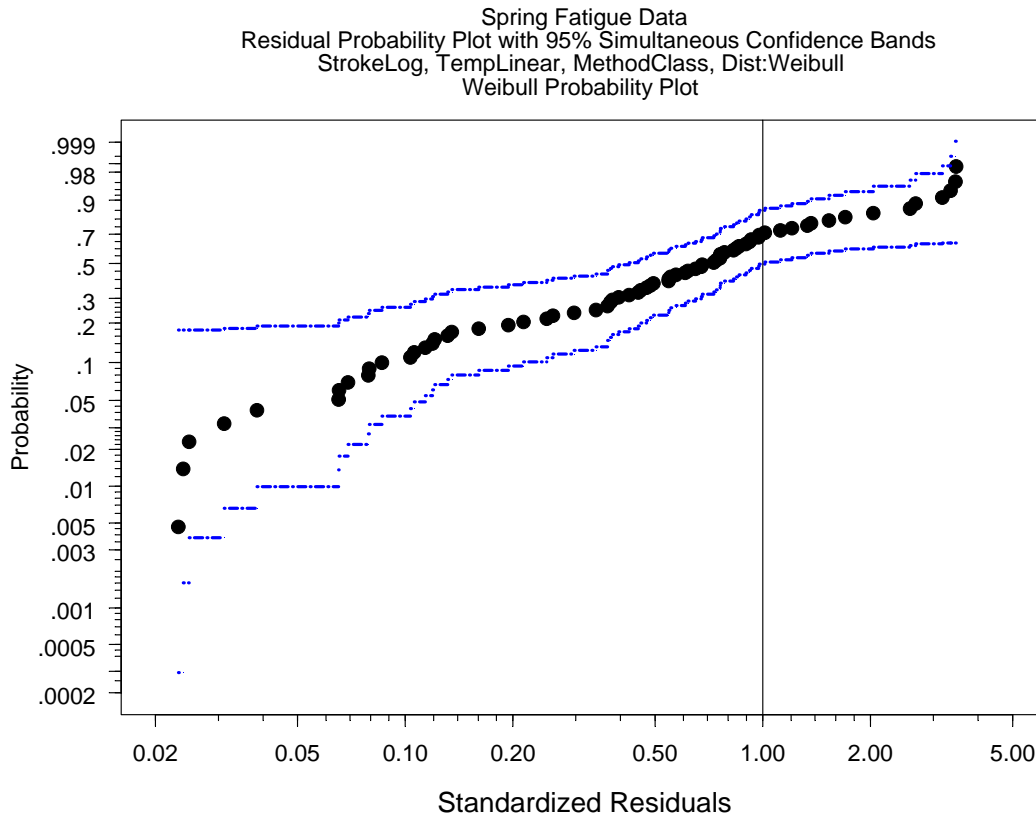
Figure 104 Probability plot of the regression residuals for the laminate panel data.

12 Planning an Acccelerated Life Test (ALT)

Accelerated life tests (ALTs) are complicated and expensive experiments. Careful planning of such experiments is important. SPLIDA provides tools for generating, modifying, and evaluating accelerated life test plans. To use the evaluation tools, on must first specify model information (or planning information) and the proposed test plan(s).

12.1 Specification of ALT plan values

The first step in the development and evaluation of ALT plans is specification of a model and planning values for the model parameters. Generally such informatioin is obtained from previous experience with similar materials or products or engineering

analysis and judgement. Using **SPLIDA ► Plan an accelerated life test ► Specify ALT model information (planning values)** brings up a dialog like the one shown in Figure 105.

Figure 105 Dialog for specifying ALT-planning values.

As an example, we will specify the planning values corresponding to the adhesive bond used in Example 20.1 on page 535 of Meeker and Escobar (1998). On the Basic page of the dialog, change "last.ALTpv" to the more descriptive AdhesiveBond.ALTpv, choose a distribution (Weibull in this case), and edit the time units cell to the more descriptive "Days," as shown in Figure 105. Note that the distribution name was automatically inserted into the "Save results in" cell (*the alt planning values object name is too large to see completely without using the right-click-Zoom option in the cell*). Although it is not required to enter the use conditions, it is convenient to do so (because the use conditions will not have to be specified later, although there is opportunity to modify the use conditions at any time). For the adhesive bond example, the use condition is 50 degrees C.

Then click the Relationships button to launch pop-ups for each accelerating variable. You must choose the relationship (transformation) for each accelerating variable. For the AdhesiveBond example, there is only one explanatory variable; choose Arrhenius. Then, if you visit the Relationship(s) page, you can see the chosen relationship(s).

Now, visit the Parameters page. In order to specify the ALT model parameters, on this page you must specify:

- The failure probability or quantile at a specific set of operating conditions

- The regression (or accelerating variable) model coefficients, and

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- The distribution shape (spread) parameter.

For the adhesive bond example, the responsible engineers in the past had used a Weibull distribution to model the life of adhesives and the Weibull shape parameter was typically in the neighborhood of 1.667. In addition, for purposes of planning, the .001 quantile of the life distribution at 50 degrees C is specified as 183 days (or equivalently, the failure probability at 50 degrees and 183 days is specified to be .001). Additionally, the slope parameter in the life-temperature relationship is the effective activation energy (.7265 in eV). The completed dialog is shown in Figure 106. Click on "Apply" to create the ALT planning value object.

Figure 106 Completed parameter page of the accelerated life test planning values dialog.

12.2 Specification of an ALT test plan

In order to specify an ALT plan, use **SPLIDA ► Plan an accelerated life test ► Specify ALT test plan** to bring up a dialog like that shown in Figure 107**.** There are two different methods for specifying an ALT test plan. For the "Direct method" the plan characteristics for a test plan are entered directly into the dialog. For test plans with more than one explanatory variable, a factorial experiment is produced (and if a fraction of this factorial is desired, this is accomplished by modifying the test plan, as described in Section 12.4. In the "Frame" method, a test plan is first entered into an S-PLUS data set specified in the dialog.

Figure 107 illustrates the direct method. Specify the number of accelerating variable levels; and change the default accelerating variable name(s). At this point, click the Choose levels button. There will be a popup window for each of the accelerating variables. In each popup window, you will need to specify the levels of the experimental variable. In this case the levels for DegreesC are: 78, 98, 120. Next, specify the number of units to be allocated to each subexperiment (only a single number is allowed here; if

you want to have a different number allocated to different subexperiments, this is accomplished by editing the test plan, as shown in Section 12.4). Then, enter the type of censoring, and the censoring time (again, if you want to have different censoring times at different subexperiments, this will be accomplished by editing the plan later). You may also want to specify a particular prefix for the test plan object name (AdhesiveBond1 in the example). The default suffix last.ALTplan is provided automatically. Then provide the actual plan information.

Figure 107 Completed dialog specifying an ALT plan.

After the plan is specified, click on "OK" or "Apply" (in case that you want to make more than one related plan). To specify a second or third test plan, edit the dialog and the suffix of the plan object name (e.g., use AhhesiveBond2 so that the previously specified plan(s) will not be overwritten) and again press "OK" or "Apply".

12.3 Generating special ALT test plans

When planning an ALT with a single accelerating variable, SPLIDA can be used to generate some special plans such as the following:

- Traditional plans that use equally spaced levels of the accelerating variable and equal allocation of test units to each level of the accelerating variable.

- Optimum test plans that minimize the large sample variance of a particular quantile of interest. Such plans may not be practical (e.g., because they test units at only two levels of the explanatory variables) but they provide a baseline for the best that one can do and insight into what is needed to develop a plan that is statistically efficient.

- Constrained optimum (or compromise) test plans that have three distinct levels of the accelerating variable with something like 15% or 20% of the test units constrained to be tested at the middle level of the accelerating variable.

- Equal expected number failing test plans which are constrained to have three distinct levels of the explanatory variable with the expected number failing the same at each of the levels.

Figure 108 shows the dialog to request the optimum test plan for the Adhesive Bond example, this plan and its evaluation are given in Table 16.

More information about plans like these can be found in Chapter 20 of Meeker and Escobar (1998).

Figure 108 Dialog to generate an optimum ALT plan

```
 Optimum test plan based on AdhesiveBond.Weibull.altpv

Test plan summary:
  DegreesC   n ctime   zeta       p efail
1   94.652 212    183 -1.63 0.17838 37.82
3  120.000  88    183  0.84 0.90054 79.25


Evaluation at use conditions 50 DegreesC
 Quantile   Days    Ase R-Factor
      0.1 2990.1 1133.3   2.1019
```

Table 16 Output from the generation of the AdhesiveBond optimum ALT plan

12.4 Modify an ALT test plan

Suppose that we want to modify the plan that we specified in Section 12.2 so that the allocations are 155, 60, and 84 specimens tested at 78, 98, and 120 degrees C, respectively. Use **SPLIDA ► Plan an accelerated life test ► Modify ALT plan** to bring up the dialog in Figure 109. Choose the AdhesiveBond1.altplan test plan object, and then click on the Edit plan button to bring up the plan's edit worksheet.
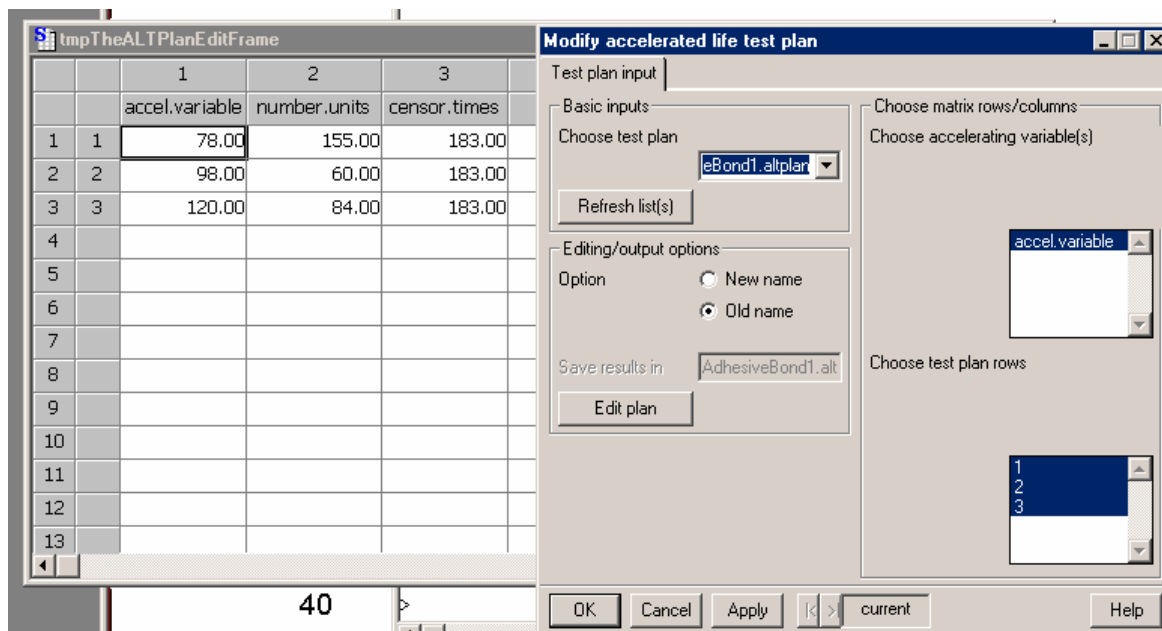
Figure 109 Dialog and edit worksheet for modifying the AhesiveBond1.ALTplan ALT test plan object

Edit the number.units column and click on Apply. This will create the new test plan with the same name. If a new test plan with a new name had been desired, one can choose the New name Option and then edit the Save results in box.

12.5 Sample size for an ALT test plan

Using **SPLIDA ► Plan an accelerated life test ► Sample size for an ALT test plan**
will bring up the dialog in Figure 110. After choosing the plan values and the test plan
objects, enter the use conditions, and click on Apply. This will generate the plot in Figure
111. This plot is used in the same way as the sample size plot for a single distribution,
described in Section 6.3. For example, the rectangle tool in the S-PLUS annotate toolbar
can be used to draw in a rectangle to show that the predicted precision factor for an
approximate 95% confidence interval for the .1 quantile for this ALT plan is R=2.3.
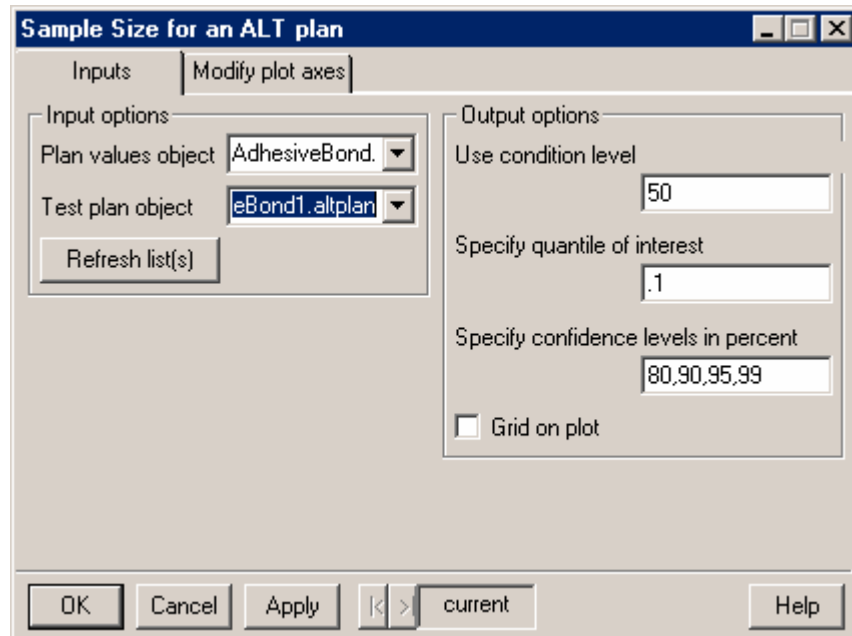
Figure 110 Dialog for the SPLIDA sample size tool
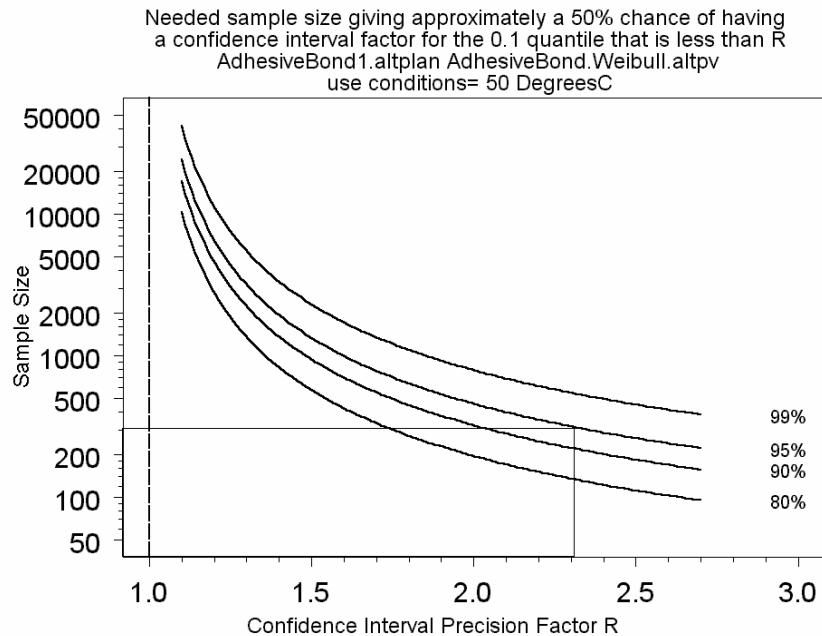
Figure 111 Plot of required sample size as a function of the precision factor R for the AdhesiveBond accelerated life test example.

12.6 Summarization and approximate evaluation of a specified ALT test plan

Using **SPLIDA ► Plan an accelerated life test ► Summarize, evaluate, and plot an ALT test plan** will bring up the dialog shown in Figure 112. In this box, one specifies the ALT plan values object (AdhesiveBond.Weibull.altpv in this case), the ALT test plan object (AdhesiveBond1.altplan) to be evaluated, and the use conditions (50 Degrees C for the adhesive bond). By default, the output produced includes a tabular summary and a plot summary of the specified ALT test plan. Figure 113 shows an Arrhenius plot for the adhesive bond test plan specified in Figure 107, using the planning values that were specified in Figure 106.

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Figure 112 Dialog asking for a summary evaluation and plot of a proposed accelerated life test plan for the adhesive bond example.

The output in Table 17 provides a printed summary of the planning values and the test plan, followed by a listing of the large-sample approximate standard deviation and confidence interval precision factor for confidence intervals corresponding to each of the quantiles listed in the "Plot, summarize/evaluate ALT Plan" dialog shown in Figure 112.

Table 17 ALT planning information and test plan specification for the adhesive bond example.

```
Accelerated test planning values
Distribution:  Weibull
Relationship:  Arrhenius
Time units:  Days

For a censoring time of 183 Days

the failure probability at 50 Degrees C  is:  0.001

Intercept is:  -16.7
slope =  0.726

weibull.beta =  1.67
sigma =  0.6

Accelerated Test Plan

  level  number  censor.time     xi     pi     prfail  Efail
1    78     155          183  0.4478  0.518  0.0315772   4.89
2    98      60          183  0.7264  0.201  0.242189   14.5
3   120      84          183  1       0.281  0.900543   75.6

Use condition is 50 Degrees C
Total number of test units= 299
The large sample approximate standard deviation
of the 0.1 log quantile at 50 Degrees C  = 0.4378
```

```
corresponding to a 95% confidence precision factor or R= 2.358
The large sample approximate standard deviation
of the 0.5 log quantile at 50 Degrees C  = 0.4911
corresponding to a 95% confidence precision factor or R= 2.619
The large sample approximate standard deviation
of the 0.9 log quantile at 50 Degrees C  = 0.532
corresponding to a 95% confidence precision factor or R= 2.837
```

Note that the 95% confidence precision factor for the .1 quantile is 2.358, which agrees closely with the factor read from Figure 111.
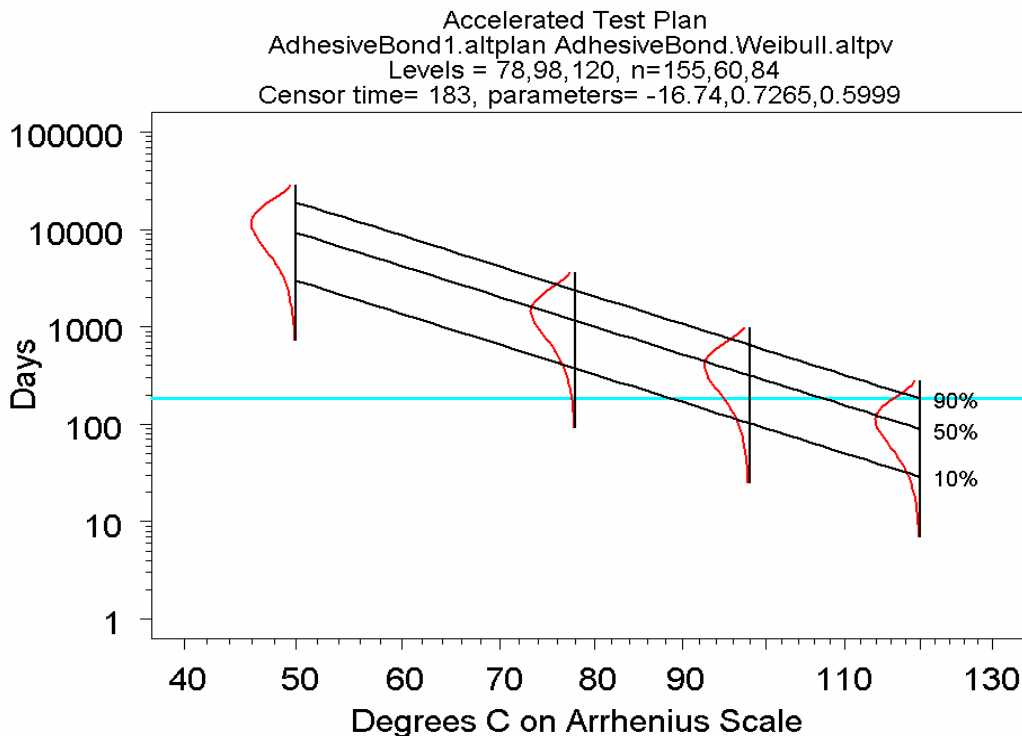
Figure 113 Arrhenius plot showing a proposed test plan for the Adhesive Bond example.

12.7 Simulation of an ALT test plan

Although the approximate evaluation given above is useful, doing a simulation of a proposed ALT test plan avoids the use of large-sample approximations (which may not be adequate when some of the test conditions have expected numbers of failures that are small) and will provide insight into the limitations of a proposed test plan. Using **SPLIDA ► Plan an accelerated life test ► Simulate an ALT test plan** brings up the dialog in Figure 114, allowing the user to request a simulation of a proposed ALT test plan.

Figure 114 Dialog requesting a simulation to evaluate a proposed accelerated life test plan for the adhesive bond example.

Then specify the previously created ALT plan values (e.g., AdhesiveBond.Weibull.altpv), the ALT test plan object (e.g., `AdhesiveBond1.Weibull.ALTplan`). If they were specified in the planning values object, the Use condition level will use that value (those values) as the default entry, but the box is editable. There is also an option to see some details of the analyses of the individual simulated samples. Looking at four or five such samples in a new situation can be useful.

After completing the dialog, click on "Apply." The simulation will then begin to run, giving an indication of progress in the output window (completing the simulation will take from a few seconds to a few minutes depending on the number of simulations requested and processor speed (500 simulations is sufficient for most purposes). When completed, five new multiple probability plots, summarizing the first five simulated data objects, will have been created. For the adhesive bond example, one of these is shown in Figure 115. Note the deviation from a straight line is the lower tails of the distributions at 98 and 120 degrees C. For an experienced analyst, this behavior is expected, as the observations in the lower tail of a normal distribution have the most variability (the same is true of the largest observations from a normal distribution, but these observations have been censored).
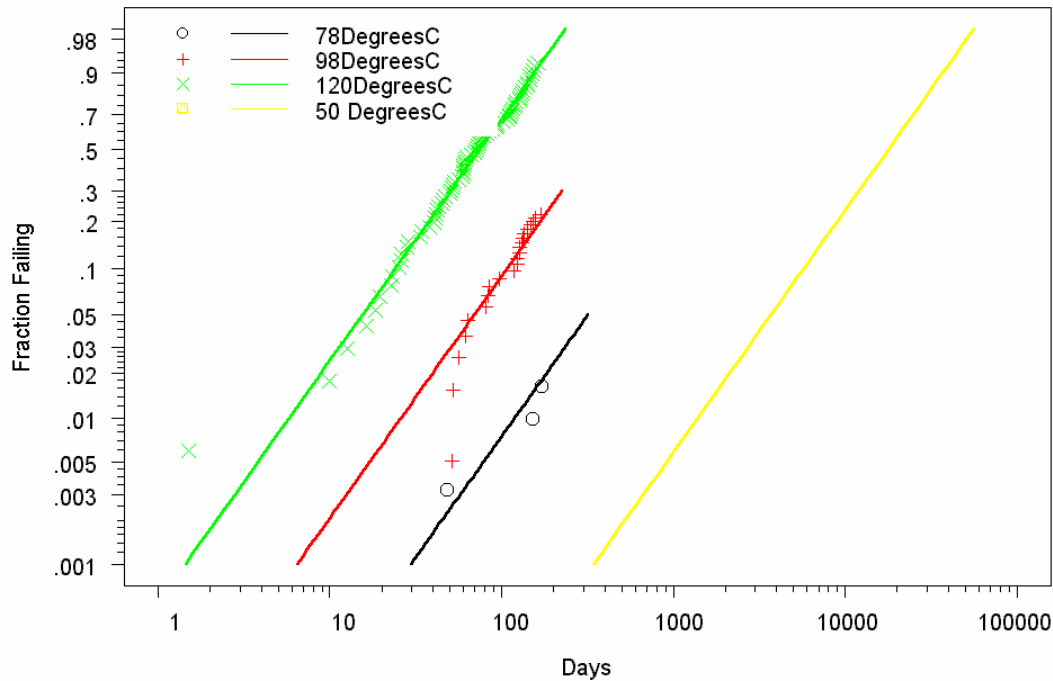
Figure 115 Multiple probability plot for one of the simulated adhesive bond ALT experiments

In addition, a summary of the results is depicted in a quantile-life versus accelerating variable plot for each of the accelerating variables (an Arrhenius plot for the example), as shown in Figure 116. The results of the simulation are saved in a file (by default, the name of this file is obtained by pasting together the names of the planning values object and the test plan object (AdhesiveBond1.altplan.AdhesiveBond.Weibull.altpv.alt.sim.out for the example). By using this file, one can produce other summaries of the simulation results, as described in Section 12.8.

Note also that SPLIDA saves the last simulated ALT life data object as last.sim.ALT.ld. One can use all of the SPLIDA ALT data analysis tools to analyze these data to gain experience in analyzing such data and to get a sense of the information that can be expected from the experiment.

Accelerated life test simulation based on
AdhesiveBond1.altplan AdhesiveBond.Weibull.altpv
Failure time 0.1 quantile vs DegreesC
x:Arrhenius , Dist:Weibull
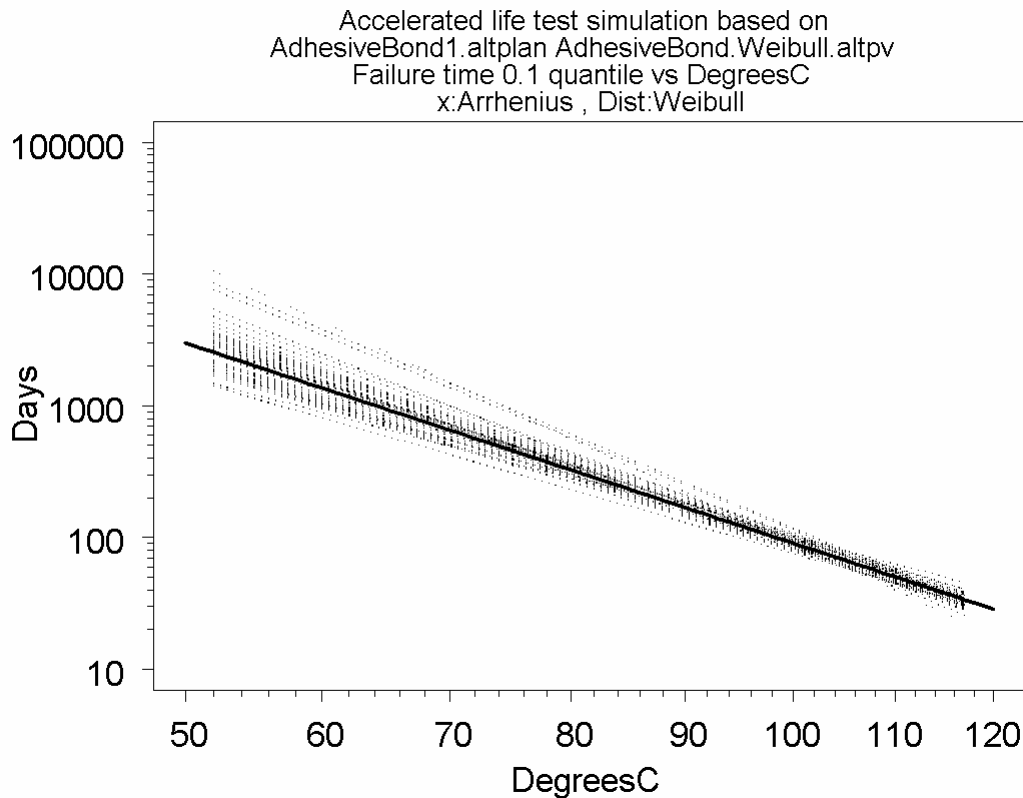
Figure 116 Arrhenius plot showing a summary of the simulation of the proposed accelerated life test for the Adhesive Bond.

12.8 Evaluation of ALT test plan simulation results

Although SPLIDA automatically presents the summary of the simulated results shown in Figure 116, an analyst may be interested in other summaries of the simulation results. To produce such summaries, use **SPLIDA ► Plan an accelerated life test ► Summarize an ALT simulation** to bring up the dialog in Figure 117. This dialog allows the user to request some or all of three different kinds of simulation summaries. The first page, shown in Figure 117, requires specification of the ALT simulation results object (SPLIDA will remember the most recent results object if it was created in the current session). This page also allows the user to choose one or both of two overall summary plots:

- A plot of a specified quantile versus the accelerating variable
- A plot of the fraction failing as a function of time at a specified level(s) of the accelerating variable(s)

For the time versus the accelerating variable plot, the user can choose the quantile to plot. For the fraction failing as a function of time plot, the user can choose the level of the accelerating variable, as shown in Figure 117.
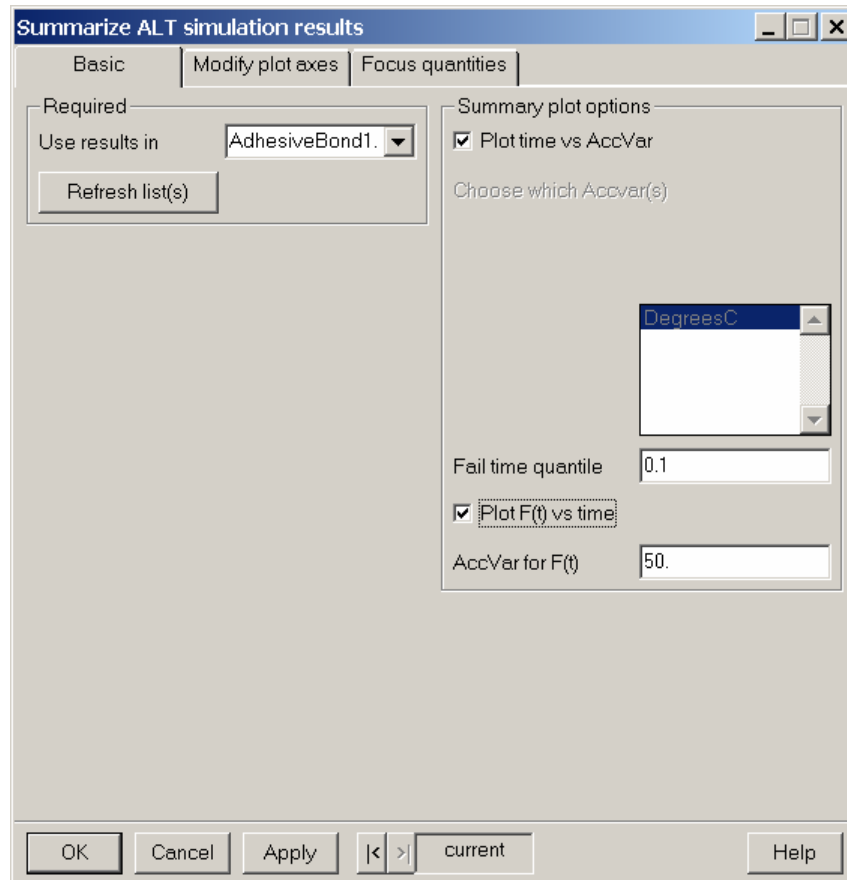
Figure 117 Dialog used to request summarizations of an ALT simulation

In addition to these summary plots, one can also choose to make summary plots of two specified "focus quantities." A focus quantity is a quantity of interest: a parameter, a specified quantile, or a failure probability at a given level (or levels) of the accelerating variable(s). Specification of the focus quantities is from the `Focus quantities` page of the `Summarize ALT simulation results` Dialog shown in Figure 118. Two different focus quantities should be chosen. Then one can choose to look at the joint distribution of the simulated estimates, the marginal distribution, or both at the same time.

Figure 118 Dialog to specify details of summarization of ALT focus quantities

After completing the dialog, click on "Apply" to generate the summary graphics. First, another plot like that in Figure 116 is generated (although a different quantile of a different level of temperature could have been used). Figure 119 is a plot of the ML estimates of the fraction failing at 50 Degrees C, as a function of time. Figure 120 shows the joint distribution of the ML estimates of β_1 (the effective activation energy) and $F(1830)$, the fraction failing at 1830 days (about 5 years), at the nominal use conditions of 50 Degrees C. This type of plot provides important insights and explanations for the results from ALT experiments. For example, the negative association between of β_1 and $F(1830)$ suggests that larger estimates of β_1 will lead to smaller estimates of failure probabilities.
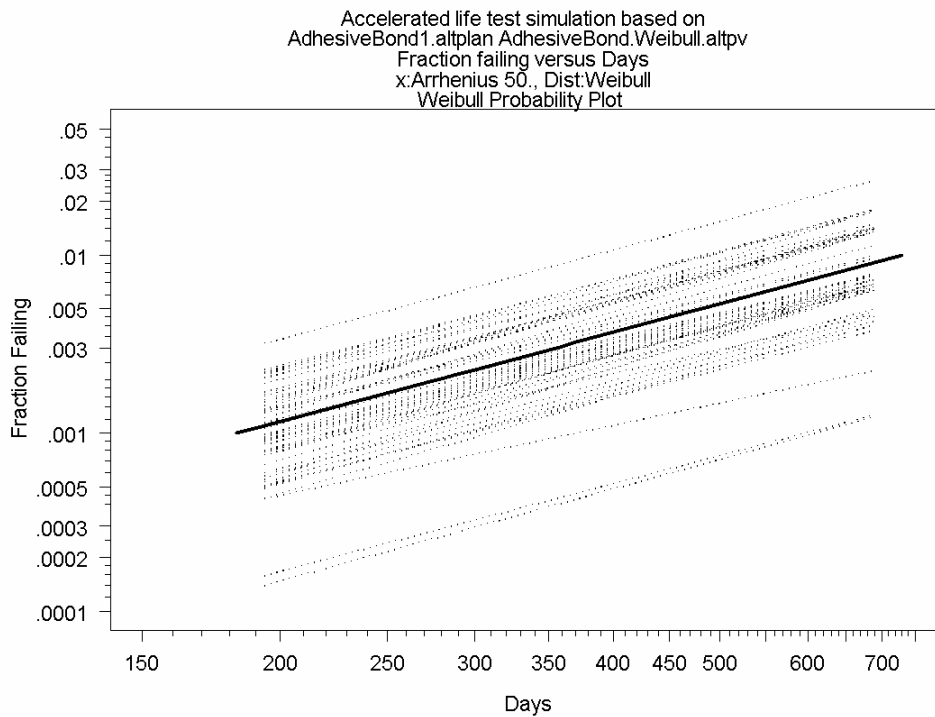
Figure 119 ALT simulation summary plot of ML estimates of fraction failing as a function of time
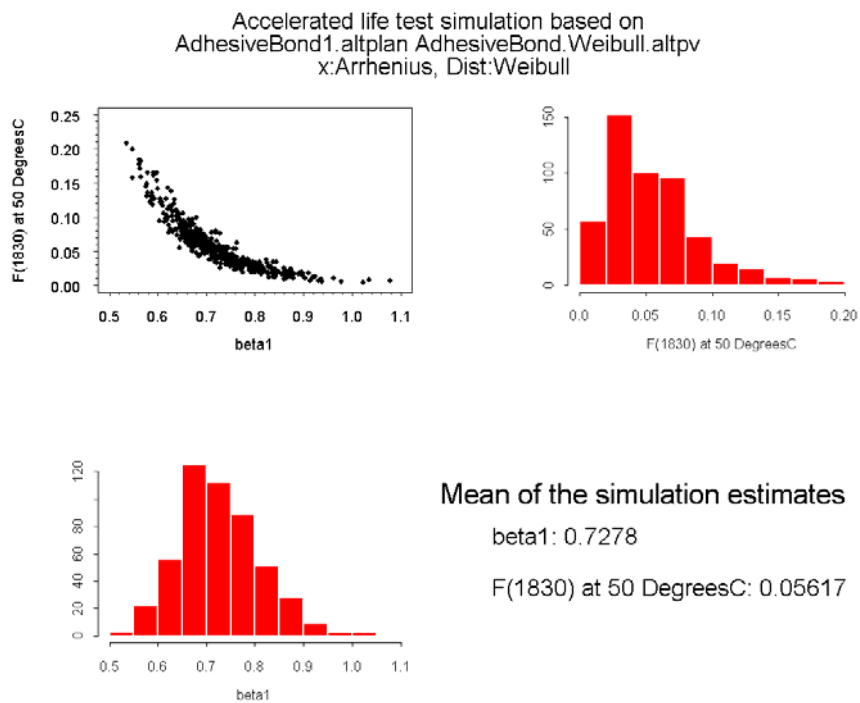
Figure 120 ALT simulation summary plots of ML estimates F(1830) and β_1 (effective activation energy)

SPLIDA chooses default values for the ranges of the axes of these plots. It is possible for the user to control these if the dialog is requesting only one of the three kinds of plots. In order to do this, the dialog must be set so that only one of the three different kinds of lots is being requested. In this case, the controls on the `Modify plot axes` page will become active.

12.9 Compute acceleration factors

SPLIDA provides a tool for computing acceleration factors for the simple acceleration models. Use **SPLIDA ► Plan an accelerated life test ► Compute acceleration factor** to bring up the dialog illustrated in Figure 121. Specify the range of accelerating variable levels of interest (the smallest being the use cnditions and the highest being the highest testing level), the acceleration model. Also, provide the acceleration model regression coefficient (effective activation energy for the Arrhenius model and power for the inverse power relationship). Several values can be provided at one time, as shown in Figure 121. Then click on Apply to get a plot of the acceleration factor across the specified range of the accelerating variable, as shown in Figure 122.

Figure 121 Dialog to compute acceleration factors and make a plot of acceleration factor versus temperature.
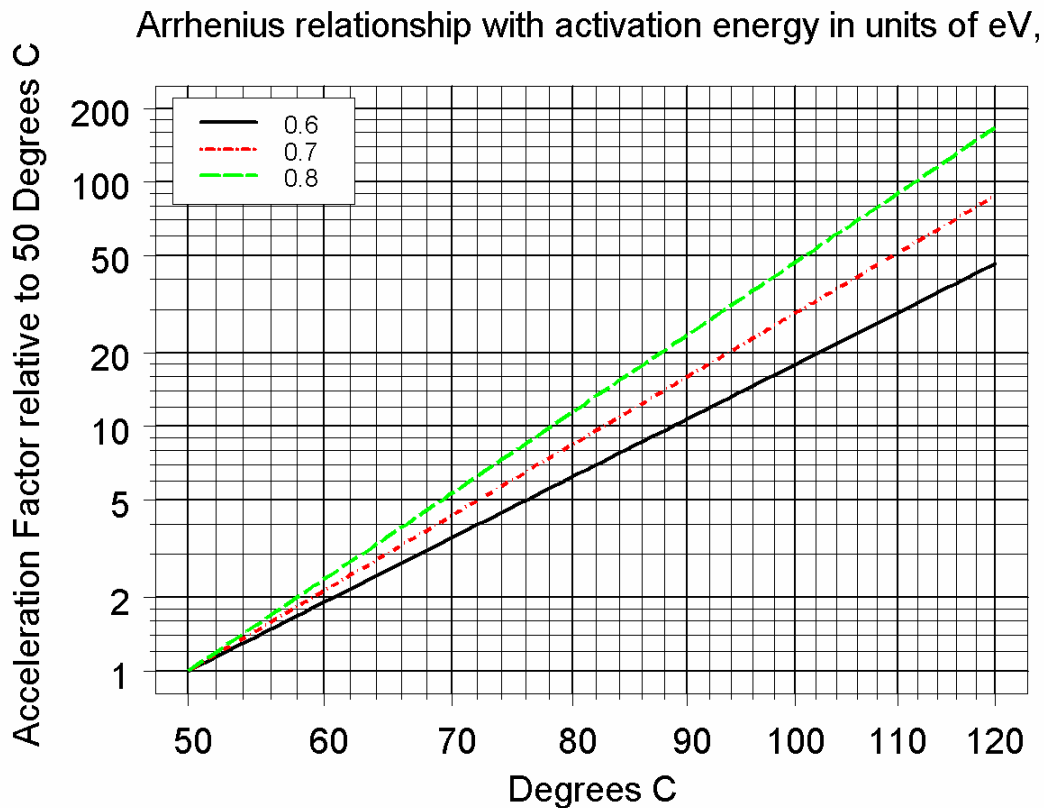
Figure 122 Plot of acceleration factors with different assumed activations energies (in units of eV).

13 Recurrence (point process) data analysis

Recurrence data (also known as point process data) arise when events are tracked on a collection (or sample) of units over time. Such data arise in many different applications. For example, in field tracking, a group of units is monitored over time and a record is kept of repairs for individual units, perhaps consisting of data, type of repair, and the cost of the repair. Interest, for example, may center on the recurrence rate or on the average cumulative cost of system maintenance as a function of time.

13.1 Structure of recurrence data

In its detailed form (used in this chapter), recurrence data consists of information on the times and nature of events on a collection of units under observation. An event can be a recurrence ("Fail" or "Repair" or some other name of the recurrence), the beginning of an observation period ("Start") or the end of an observation period ("End").

Usually, units are under continuous observation for some specified period of time. During that time, there may be one or more "recurrences" or "events." In other applications, recurrences for a unit may be observed only during "windows of opportunity" (outside of these windows events that occur are not observed). Each event may have an associated "cost" or other value (if not, the default value is 1.0, indicating a counting process).

To illustrate the structure of the data needed by SPLIDA, we will present three simple "toy" examples, but point to corresponding real examples available among the example SPLIDA data sets.

Example 13.1 Three vehicles observed continuously over time with events for all vehicles.

Vehicle	Weeks	Event	Cost
V1	0	Start	0
V1	39	Fail	234
V1	44	Fail	441
V1	50	Fail	120
V1	80	Fail	233
V1	81	Fail	671
V1	92	End	0
V2	0	Start	0
V2	31	Fail	211
V2	73	Fail	432
V2	98	Fail	654
V2	102	Fail	112
V2	112	Fail	256
V2	118	Fail	115
V2	119	End	0
V3	0	Start	0
V3	225	Fail	187
V3	340	End	0

Example 13.1 illustrates the simplest and most common structure where units begin observation at time 0 and are monitored continuously (more or less) until observation terminates for some reason (e.g., the unit is taken out of service, fails catastrophically, or it is still in service at the time of data analysis). The Start lines are optional if there is only one per unit and the time of start is 0. The End line is optional if the end of the observation period is the time of the last recurrence. Examples of data set like this in the SPLIDA collection include MachineH.txt, Cylinder.txt, and ValveSeat.txt.

Example 13.2 Same three vehicles from Example 13.1, but observed in windows of opportunity with observed events for two of the three vehicles.

Vehicle	Weeks	Event	Cost
V1	0	Start	0
V1	39	Fail	234
V1	40	End	0
V1	75	Start	0
V1	80	Fail	233
V1	81	Fail	671
V1	90	End	0
V2	0	Start	0
V2	31	Fail	211
V2	40	End	0
V2	100	Start	0
V2	102	Fail	112
V2	112	Fail	256
V2	115	End	0
V3	0	Start	0
V3	55	End	0
V3	120	Start	0
V3	205	End	0
V3	300	Start	0
V3	340	End	0

In Example 13.2, there are (required) Start/End pairs for each unit and these define the observation windows of opportunity. Examples of data set like this in the SPLIDA collection include AMSAAWindow1.xls, and AMSAAWindow2.xls.

Example 13.3 Fifty vehicles observed continuously over time with events for only four vehicles.

Vehicle	Weeks	Event	Cost
V1	0	Start	0
V1	55	Fail	234
V1	150	End	0
V2	0	Start	0
V2	57	Fail	211
V2	78	Fail	432
V2	150	End	0
V3	0	Start	0
V3	112	Fail	187
V3	198	End	0
V4	0	Start	0
V4	225	Fail	187
V4	340	End	0
VG1	0	Mstart	14

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VG1	230	Mend	14
VG1	0	Mstart	32
VG1	200	Mend	32

Example 13.3 contains Mstart/Mend pairs for groups of units that had no recurrences. Such data arise frequently in warranty data in which some units are returned, but large numbers of units are not returned. Using Mstart/Mend pairs for groups of units relieves us of having to specify large numbers of Start/End pairs. An example of a data set like this in the SPLIDA collection is R4490.txt.

13.2   Creating a recurrence data object

As with life data, to do a SPLIDA analysis of recurrence data, one first has to construct a recurrence data object. Because the structure of recurrence data is different from that of life data, there is a different dialog to making a recurrence data object. **SPLIDA ► Make/edit/summary/view data object ► Make a recurrence (point process) data object** to bring up the dialog in Figure 123.

Figure 123 Dialog used to make a recurrence data object.

Clicking on the Choose data set arrow in the dialog illustrated in Figure 123 will show all of the data sets in the SPLIDA database and in the user's working database. Only some of these will be appropriate for making a repeated measures data object. SPLIDA has no way to distinguish among different generic data sets (as it attempts to do with data

objects when it decides which to put into a list available for use), so all data sets appear in the list.

After choosing a data set corresponding to a recurrence data set, one must choose the time, unit identifier, and the status/event columns from the frame (all are required for recurrence data). The dialog in Figure 123 uses the earth-moving machine maintenance data (MachineH is the frame name) from Chapter 16 of Meeker and Escobar (1998). For the earth-moving machine data there is also a cost variable giving the cost of each reported maintenance action. Because cost was of primary interest in this application, this variable is also specified in the dialog. When the needed inputs have been specified, click on "OK" or "Apply" to create the data object. In addition to creating the recurrence data object, a short summary of the data in the object is provided (as shown in Table 18) and, if desired a printout of the data can be requested by using the Output Options page of the dialog.

```
Summary of   Earth-Moving Machine Repair Labor Hours data
Number of rows in data matrix= 573
Number of units in the recurrence data object:  23
Number of event times = 550
Total cost/weight of events in the recurrence data set= 1958.7
Response units:  Hours
Response minimum:  52
Response maximum:  9249.5
```

Table 18 Summary of: Earth-Moving Machine Maintenance Data in MachineH.rdu

13.3 Recurrence data event plot

As with the analysis of other types of data, one should start with simple graphical methods that provide insight into the nature of the data. Using **SPLIDA ► Recurrence data analysis ► Recurrence event plot** will produce the dialog in Figure 124. In this dialog one specifies the name of a recurrence data. There are options to print the events for a subset of the data. This option is useful when there are a large number of units in a data set.

Figure 124 Dialog requesting an event plot for the earth-moving machine (Machine H) maintenance data.

Then, clicking "Apply" produces the event plot in Figure 125, allowing one to visualize the events (maintenance actions in this example) as they unfolded in time (measured in hours of operation for this example).
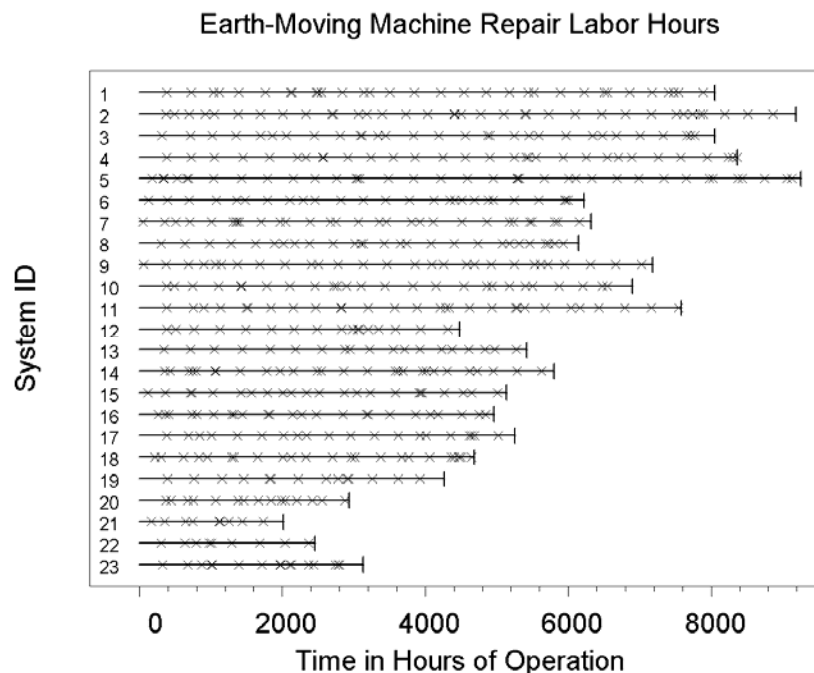
Figure 125 Event plot for the earth-moving machine (Machine H) maintenance data.

13.4 Recurrence data mean cumulative function

Using **SPLIDA ► Recurrence data analysis ► Mean cumulative function plot** will generate a dialog from like that shown in Figure 126, from which one can request a plot of the mean cumulative function (MCF) and corresponding pointwise confidence intervals for a recurrence data set. There is an option to request a table giving the same information. Clicking on Apply will give a plot like that in **Figure 127**. .
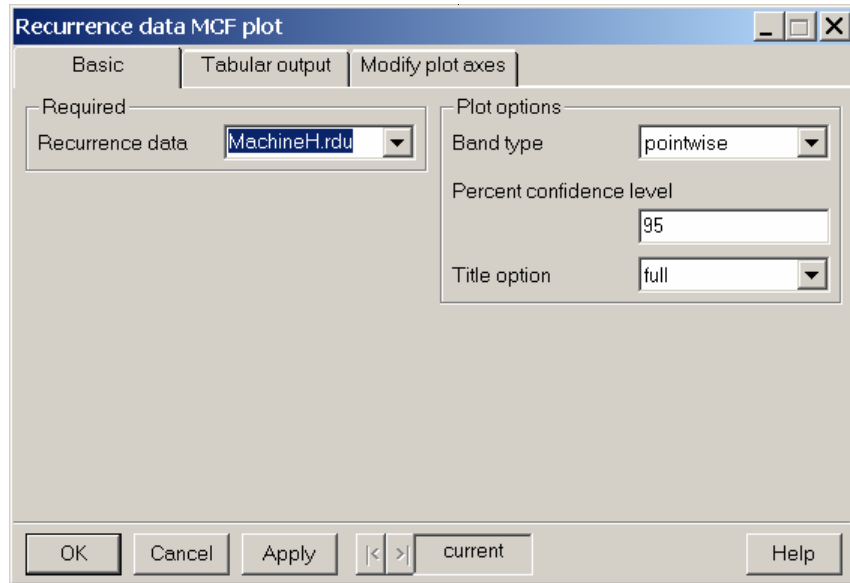
Figure 126 Dialog requesting a mean cumulative function (MCF) plot for the earth-moving machine (Machine H) maintenance data.

Figure 127shows the MCF for the earth-moving machine data. As described in Example 16.6 of Meeker and Escobar (1998), some of the irregular behavior in this function in the early part of life can be traced to the regularly scheduled maintenance of the machines in the fleet.
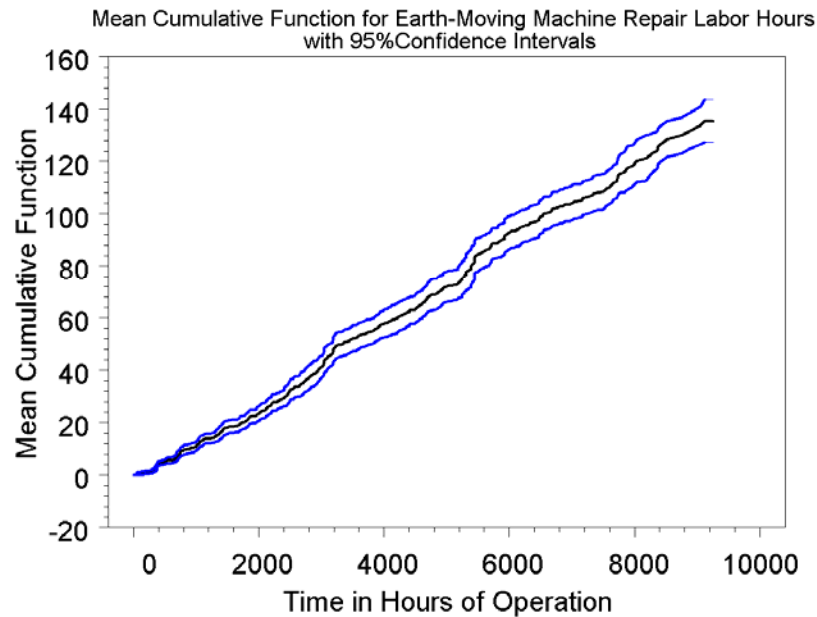
Figure 127 Mean cumulative function (MCF) plot for the earth-moving machine (Machine H) maintenance data.

In some applications, there is a need to compare recurrence data from two fleets, manufacturers, methods, etc. The dialog obtained from **SPLIDA ► Recurrence data analysis ► Compare two mean cumulative functions plot**, shown in Figure 128, allows one to request a plot making such a comparison. As an example, we use data from Doganaksoy and Nelson (1991) on the lifetime of breaking grids used in locomotives. This example was also used in Section 16.3 of Meeker and Escobar (1998). The plot in Figure 129 shows an estimate of the difference between the MCF functions for two different production batches of breaking grids along with corresponding pointwise confidence intervals. Because the zero line falls outside of the pointwise confidence interval, the plot suggests that there is a statistically important difference between the two batches.

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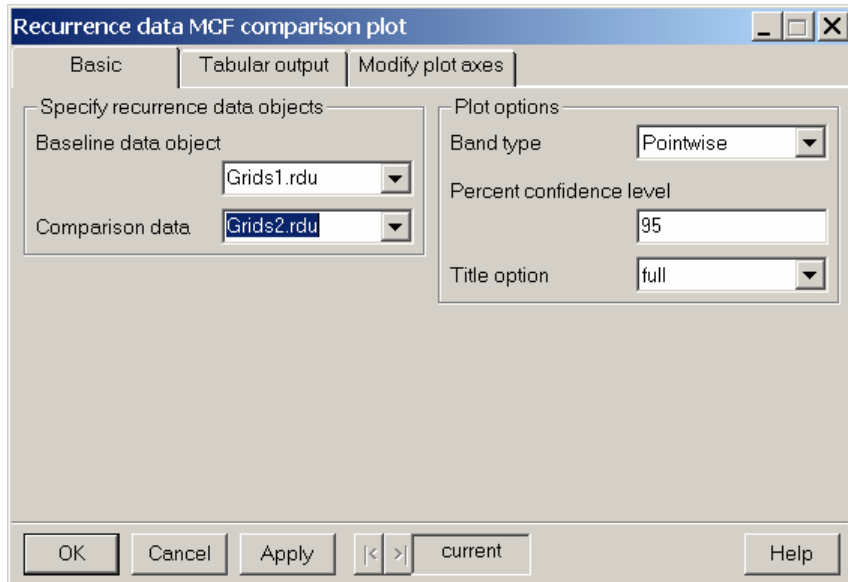
Figure 128 Dialog requesting a plot comparing two mean cumulative function (MCF) plots.
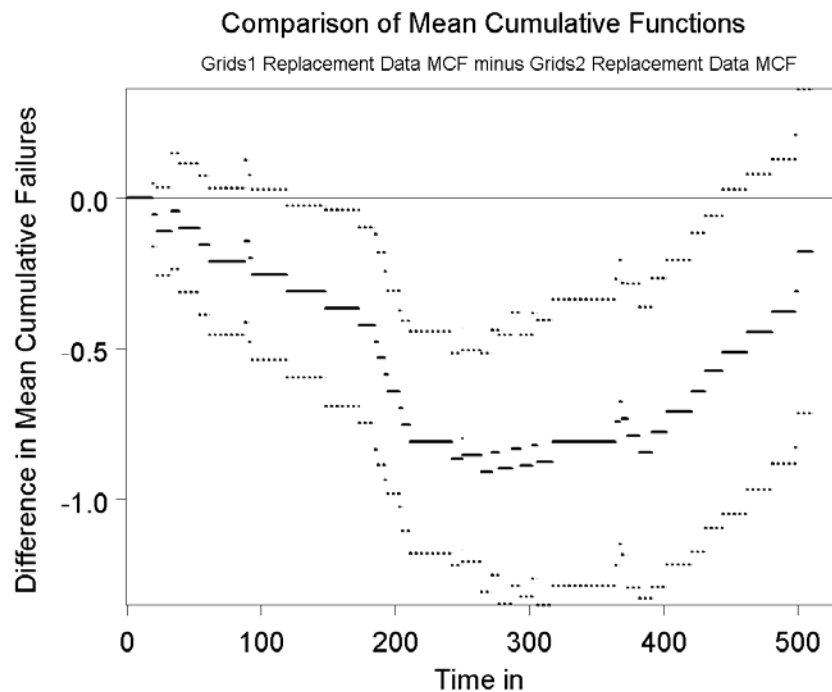
Figure 129 Plot comparing two mean cumulative function (MCF) plots.

13.5 Convert renewal recurrence data to life data (not yet available)

Using **SPLIDA ► Recurrence data analysis ► Convert renewal recurrence data to life data** will allow the user to convert certain kinds of recurrence data into life data. The underlying assumption that is needed for the output of the conversion to provide data that will not be misleading is that the randomness in times

between events (perhaps conditioned on some explanatory variables) in the recurrence data process can be modeled adequately as a sequence of identically distributed independent observations.

14 Repeated Measures Degradation Data Analysis

SPLIDA provides convenient tools for doing the "simple degradation analysis" described in Chapters 13 and 21 of Meeker and Escobar (1998) and elsewhere (as indicated in their end-of-chapter bibliographic notes). In this simple method for degradation analysis, separate regression analyses are run on each individual degradation path to predict failure times (which we call pseudo failure times) for those units that have not failed. The idea is to squeeze a little more information out of one's data. This approach to degradation analysis (indeed any method of degradation analysis that will require any extrapolation) needs to have a solid basis for the underlying degradation path model. Ideally, this will be a model based on well-developed physical-chemical theory (e.g., some simple wear processes are known to be approximately linear after a break-in period). When such a physical model is not available, extrapolation may not be justified.

We do not recommend the use of model fitting and experimentation with different variable transformation in order to find, in a simple way, the relationship of "best fit" when there will ultimately be more than just a little extrapolation in time. In any case, when extrapolation is required, analyses should be accompanied by careful, systematic sensitivity analysis among a collection of models that are plausible. This sensitivity analysis is critical to obtaining an understanding of the magnitude of model uncertainty (which when extrapolating can dominate the statistical uncertainty quantified in our confidence intervals).

14.1 Make a degradation (repeated measures) data object

As with life data and recurrence data, one first has to construct a degradation data object. Because the structure of different data is different from that of life data, there is a different dialog to making a recurrence data object. Using **SPLIDA ▶ Make/summary/view data object ▶ Make a degradation (repeated measures) data object** will bring up a dialog like that shown in Figure 130. Clicking on the Choose data set arrow will show all of the data sets in the SPLIDA database and in your working database. After choosing an appropriate repeated measures data set, one must choose the response, time, and unit identifier (all are required for degradation data), and explanatory variables, if any. There are a number of other optional inputs to change the defaults. By default, the name of the response is inserted into the name of the resulting repeated measures data object. This is especially useful in applications in which there are multiple degradation responses measured on each unit. The example in Figure 130 uses the GaAs Laser (GaAsLaser is the data set name) degradation data from Chapter 13 of Meeker and Escobar (1998).

When the needed inputs have been specified, click on "OK" or "Apply" to create the repeated measures data object. In addition to creating the degradation data object, a short summary of the data in the object is provided and, if desired a printout of the data can be requested by using the "Output Options" back page of the dialog.

Figure 130 Dialog used to make a repeated measures degradation data object.

14.2 Degradation data plot

Using **SPLIDA ▶ Degradation data analysis ▶ Degradation data plot** brings up a dialog, like that shown in Figure 131, which can be used to request a plot of degradation paths. If there are explanatory variables, there is an option to group paths by explanatory variable levels, instead of plotting all paths on a single graph. It is possible to request special axes transforms (e.g., linear, log, or square root axes) to plot either time or the response. The Modify Plot Options page allows one to control the ranges of the plot axes.
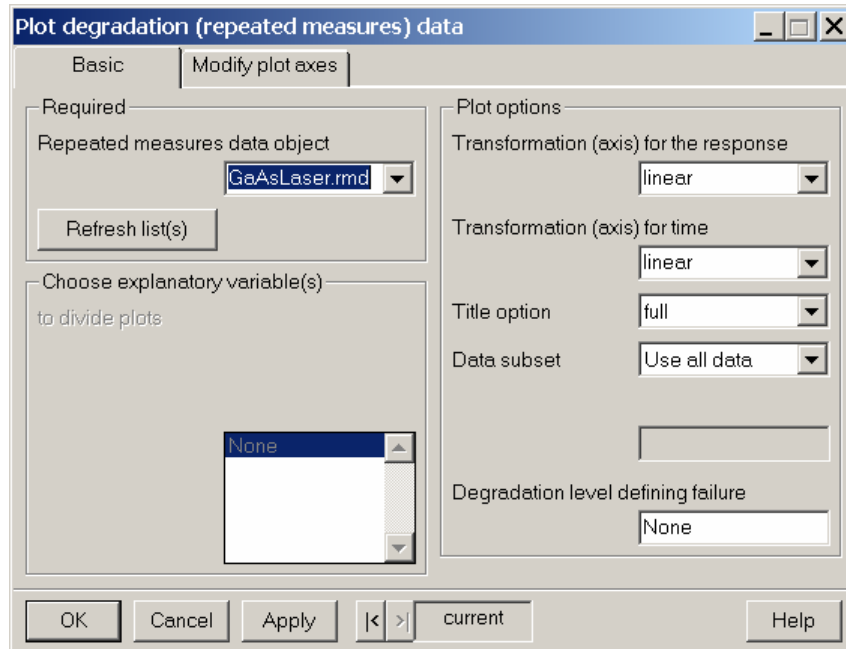
Figure 131 Dialog to request a plot of degradation data for the GaAs Laser data.

Figure 132 shows a simple degradation plot for the GaAs laser data.
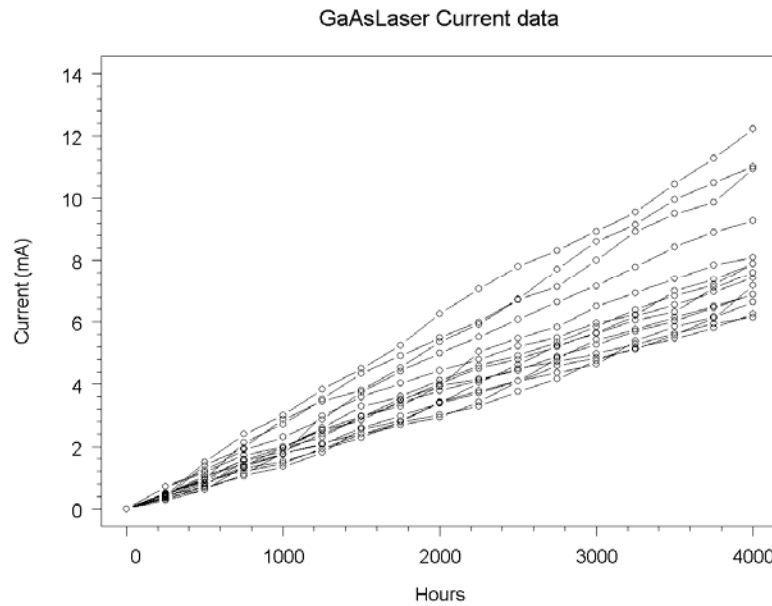
Figure 132 Plot of degradation data for the GaAs Laser data.

14.3 Degradation data trellis plot

Using **SPLIDA ► Degradation data analysis ► Degradation data trellis plot** brings up a dialog like that shown in Figure 133. This dialog is used to request a

trellis plot of degradation paths. If there are explanatory variables, there is an option to group paths by explanatory variable levels, although the individual plots, as illustrated in Figure 134, are recommended because they can be importantly revealing.

Figure 133 Dialog to request a trellis plot of the GaAs laser degradation paths.

Figure 134 Trellis plot of the GaAs laser degradation paths.

14.4 Convert/extrapolate data to pseudo failure-time data

The so-called "simple degradation analysis" method mentioned at the beginning of this section fits a specified model to each of the sample paths in an effort to "squeeze" a little more information out of such data by exptrapolating the paths (or some of the paths) to predict the time of failure. Then the "pseudo failure times" are analyzed using the standard methods for failure-time data. The conversion of degradation path data to failure-time data can be done easily with the **SPLIDA ► Degradation data analysis ► Convert/ex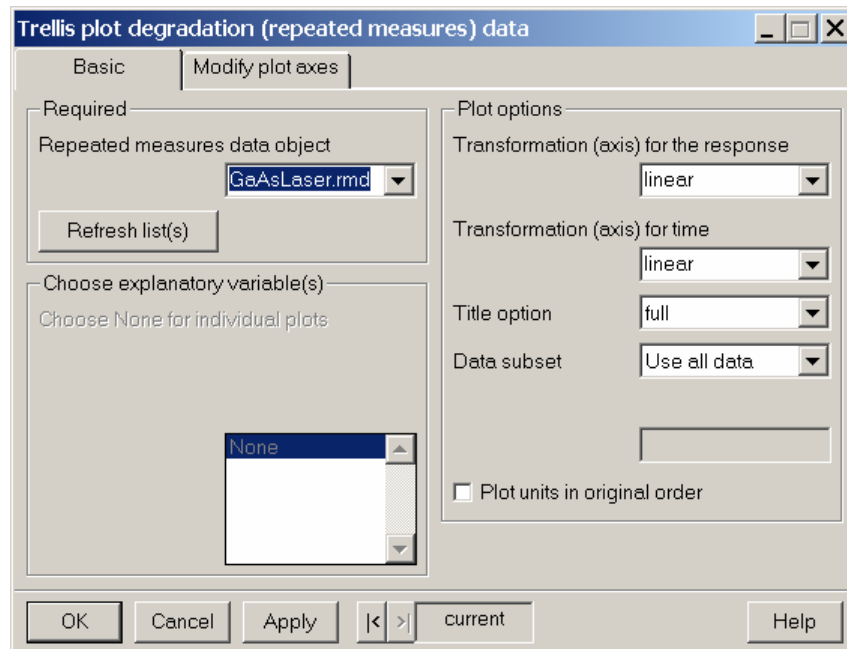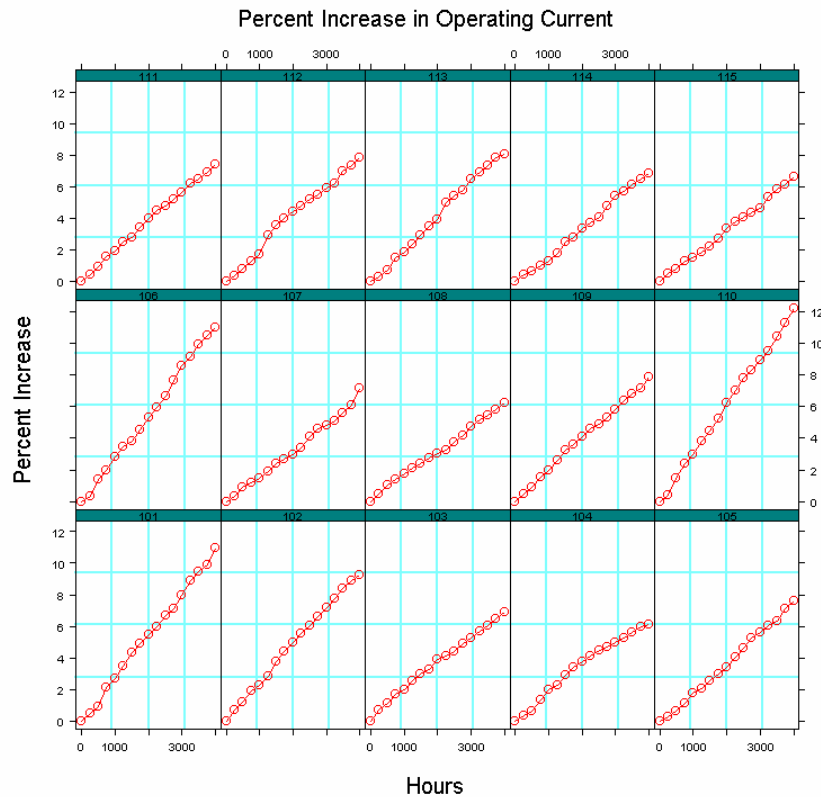trapolate data to failure-time data** dialog. It is necessary to specify the repeated measures (degradation) data object and the degradation level defining failure. For the GaAs lasers, failure was defined as the time at which 10 mA of current was needed to provide a specified (constant) amount of light output. It is also possible to specify a maximum time (a pseudo censoring time), beyond which no failures are extrapolated. For the GaAs lasers, focus was on the lower quantiles of the failure-time distribution and thus the paths were extrapolated only to 5000 hours, where 6 out of 15 lasers had failed (i.e., had predicted current greater than 10 mA). As in the dialog for plotting degradation paths, on has a choice of axis scales (e.g., linear, square root, or log). These same transformations are also used to determine the scale in which to fit the linear regression lines used to extrapolate the degradation paths to the compute the pseudo failure times. The effect of this choice can be quite pronounced (especially in the dangerous and discouraged practice where one is extrapolating far in time) and in the

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absence of strong physical motivation for one or the other, sensitivity analysis is strongly recommended.

Figure 135 Dialog requesting that the GaAs laser degradation date be mapped into pseudo failure time data using 10 mA as the definition of failure and a censoring time of 5000 hours.

After clicking on "OK" or "Apply," SPLIDA produces the plot in Figure 136 showing the degradation paths and extrapolations to the failure level of censoring time (which ever comes first) and the life data object GaAsLaserF10C5000.XLinear.YLinear.ld is available for doing failure-time analysis. Note that the default name (which could have been edited) reflects the failure definition level, pseudo censoring time, and the transformation/axes used in generating the pseudo faulure times. Such descriptive names are useful because analysts will often want to experiment with different choices for some or all of these and the automatic coding makes it easy to do such experiments and to compare the results of different analyses using all of the previously described tools for such data.
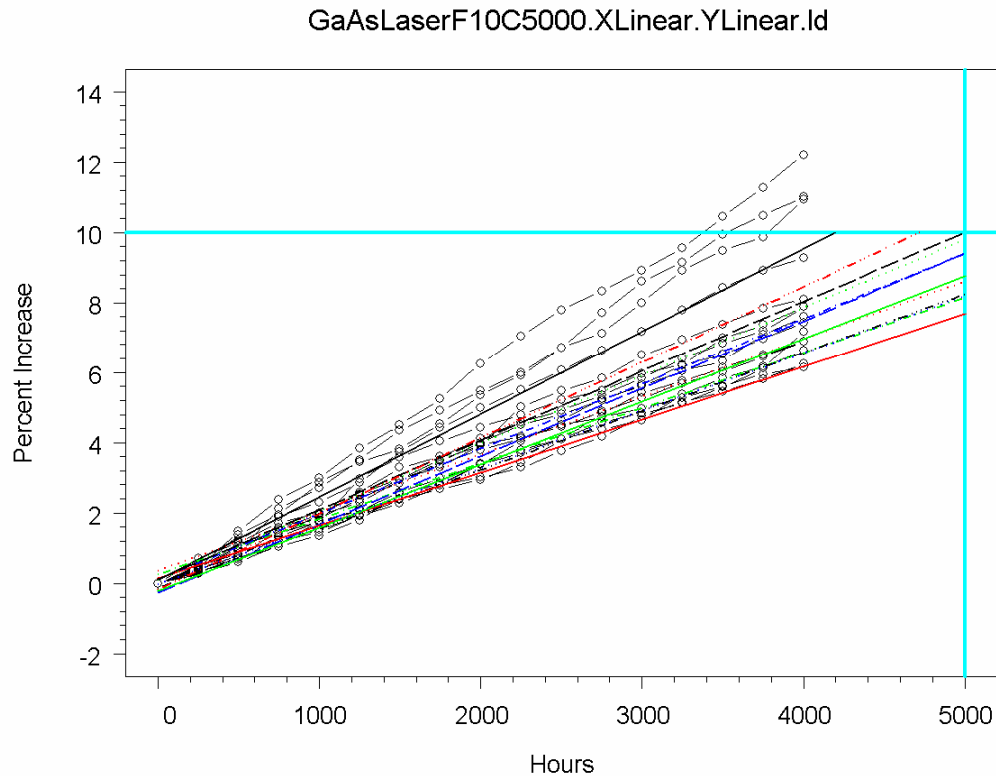
Figure 136 Plot showing the GaAs laser degradation data being mapped into pseudo failure time data using 10 percent as the definition of failure and a censoring time of 5000 hours.

Figure 137 is an event plot showing the six pseudo failure times and the nine units censored at 5000 hours in `GaAsLaserF10C5000.XLinear.YLinear.ld`. In this example, the amount of extrapolation is small (especially because no extrapolation was done beyond 5000 hours) and the degradation paths are well behaved. Thus, the pseudo failure times should provide reasonably good predictions for the actual failure times.

Figure 137 Event plot showing the predicted failure times for the GaAs laser degradation experiment.

14.5 Degradation residual trellis plot

We have emphasized the importance of having a solid basis for any model used in extrapolation of sample degradation paths. Whether or not such a model is available, it is important to assess the adequacy of the fitted models to the degradation paths. If the model does not even fit the data within the range of the data, careful consideration should be given to the usefulness of the data for purposes of predicting a failure-time distribution.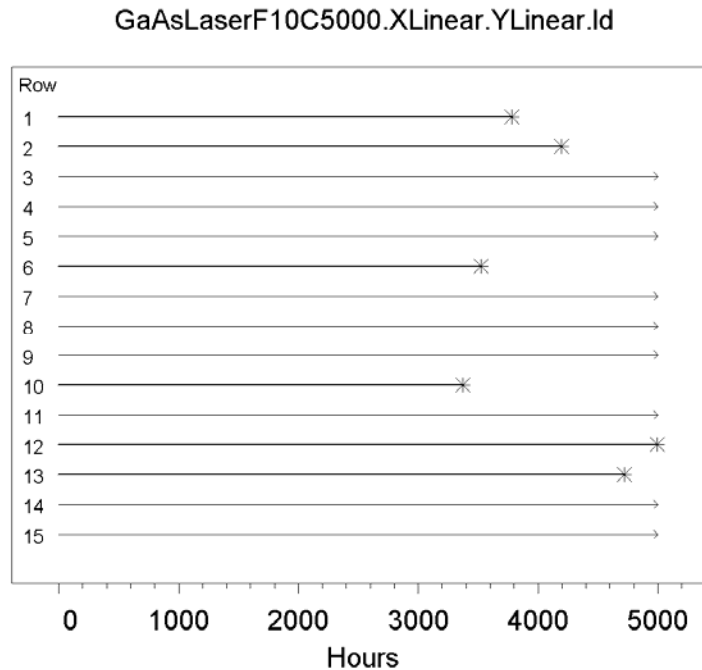 Of course, one should always remember that just because the model provides a good fit to the data and may be adequate for making predictions within the range of the data, there is no guarantee that the degradation process will continue to follow the fitted model outside of the range of the available data.

The dialog in Figure 138 allows the user to request a trellis plot of residuals for the straight lines fit to (possibly transformed) degradation data. Such plots help one to detect systematic departures from the fitted model. Figure 139 shows the residual trellis plot for the GaAs laser data. Although there were 15 tested lasers, Figure 139 shows residual paths for only 12 units. This is because three of the units had crossed the failure boundary, and thus required no extrapolation to obtain the failure time (simple linear interpolation was used instead). For this example, there appears not to be any systematic departures from the fitted straight-line model.

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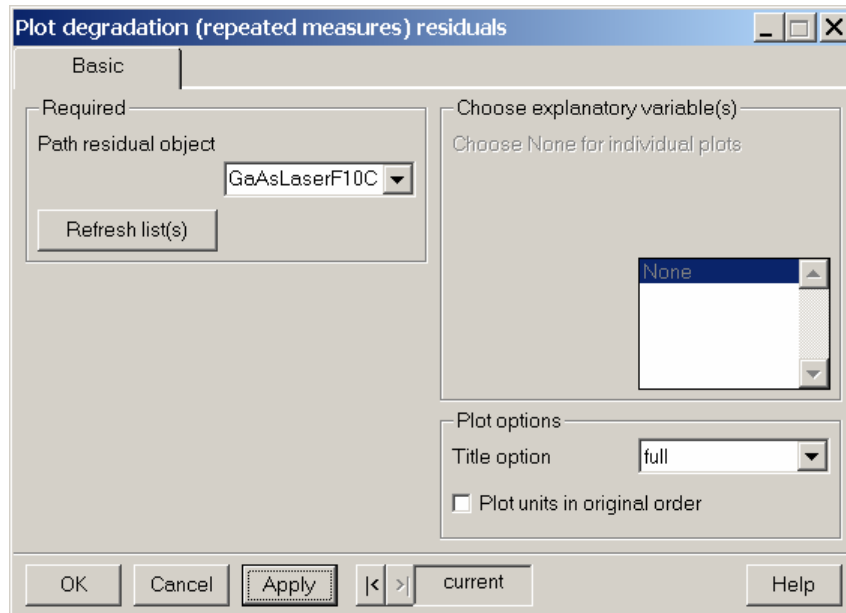
Figure 138 Dialog to request a residual trellis plot for each of the GaAs laser data regression lines.
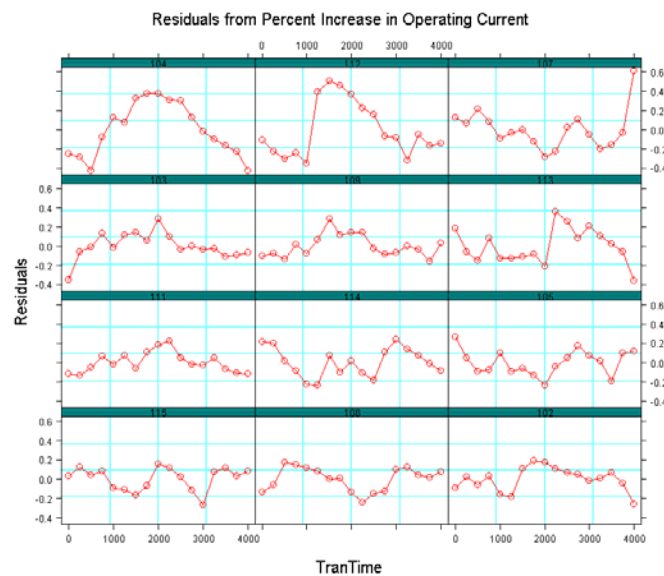
Figure 139 Trellis plot of residuals for each degradation path for the GaAs laser data.

15 Accelerated Test Repeated Measures Degradation Data Analysis

15.1 Creating an accelerated degradation data object

Many degradation experiments are run at accelerated conditions (e.g., high temperature) in order to accelerate the degradation process. A useful plot for such data is a cell-

average plot giving the average degradation as a function of time for the different level(s) of an explanatory variable.

The methods used in Chapter 14 can also be used for data with explanatory (or accelerating) variables. The primary difference is that one needs to specify the explanatory (accelerating) variable when creating the data object. Use **SPLIDA ►
Degradation data analysis ► Degradation data trellis plot to** brings up a dialog like that shown in Figure 140 This is used to create a repeated measures data object with an accelerating (explanatory) variable. The resistor example are repeated measures data on the resistance of carbon-film resistors, tested at different levels of temperature


Figure 140 Dialog to make a repeated measures degradation data object for a test with acceleration

15.2 Accelerated degradation cell-average plot

When an experiment is conducted with units at several levels of one or more explanatory variables, it is useful to compute averages across all units at the different combinations of the explanatory variables. The following example illustrates this with the resistor data in which three levels of temperature were used. The plot gives a good visual indication of the effect of temperature on degradation rate. Using **SPLIDA ► Degradation data analysis ► Accelerated degradation cell-average plot** brings up the dialog in Figure 141. In this case, the names of the most trecently created repeated measures data object (`Resistor.PercentIncrease.rmd`) was remembered by SPLIDA because it had been created in the same session.

Figure 141 Dialog requesting a plot of the average reading of all units at the different temperatures for the carbon-film resistor degradation data.

Figure 142 shows the resulting cell-average plot for the resistor degradation data.
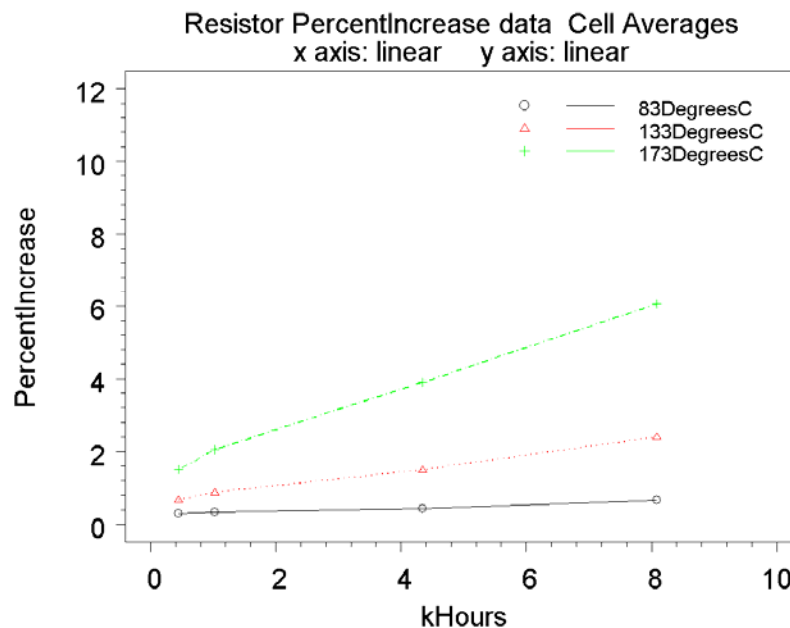
Figure 142 Plot of the average reading of all units at the different temperatures for the carbon-film resistor degradation data.

15.3 Other graphical analysis of the accelerated repeated measures degradation data

Use **SPLIDA ► Degradation data analysis ► Plot degradation (repeated measures) data** to obtain a dialog like that shown in Figure 143 that can be used to plot accelerated repeated measures degradation data
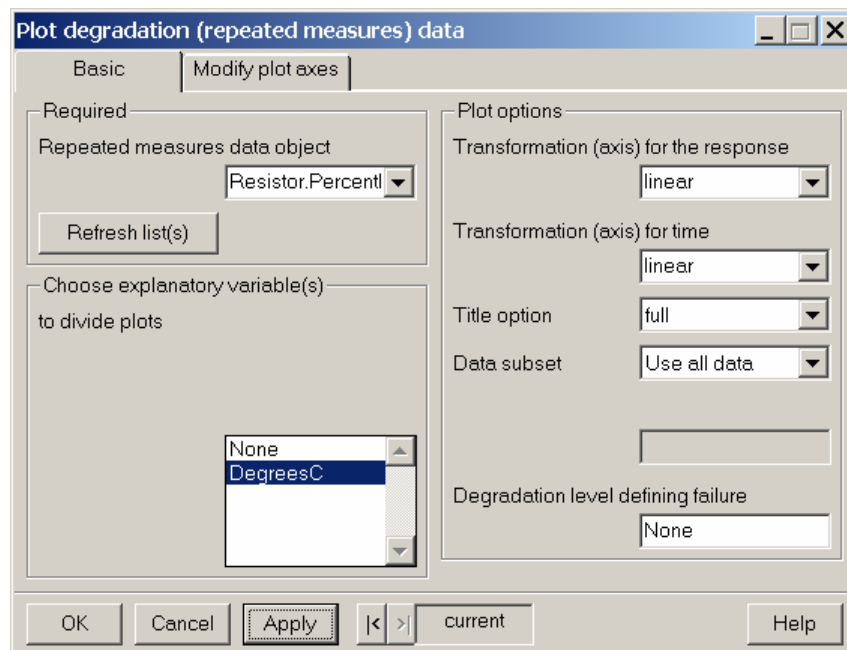
Figure 143 Dialog to request a plot of the resistor accelerated repeated measures degradation data

15.4 Other steps in the analysis of the accelerated repeated measures degradation data

Although the details are not presented here, the next steps in the analysis of the resistor data would be to convert the degradation paths into pseudo failure times and then to analyze these with the tools described in Chapter 9.

16 Accelerated Destructive Degradation Test Data Analysis

Accelerated tests are used widely in manufacturing industries to obtain timely information about the durability and reliability of components and subsystems. In a reliability test, a sample of units is put on test and either monitored over time or inspected at particular points in time. Acceleration is accomplished by testing at increased levels of accelerating variables like temperature, voltage, or stress. In accelerated *life* testing, one measures the time to failure. In accelerated *degradation* testing, one measures the amount of degradation or the degree of progression toward failure. In some applications, actual

physical degradation measurements are available (e.g., crack size or remaining strength). In other applications, performance or a surrogate response, such as power output, light output, or speed might be used instead. Generally, for purposes of physical modeling, it is better to have a response that is closely related to the degradation mechanism itself.

This section describes and illustrates SPLIDA's data analysis methods for accelerated destructive degradation test (ADDT) data. Figure 144 shows the SPLIDA menu along with the sub-menu for ADDT data analysis.
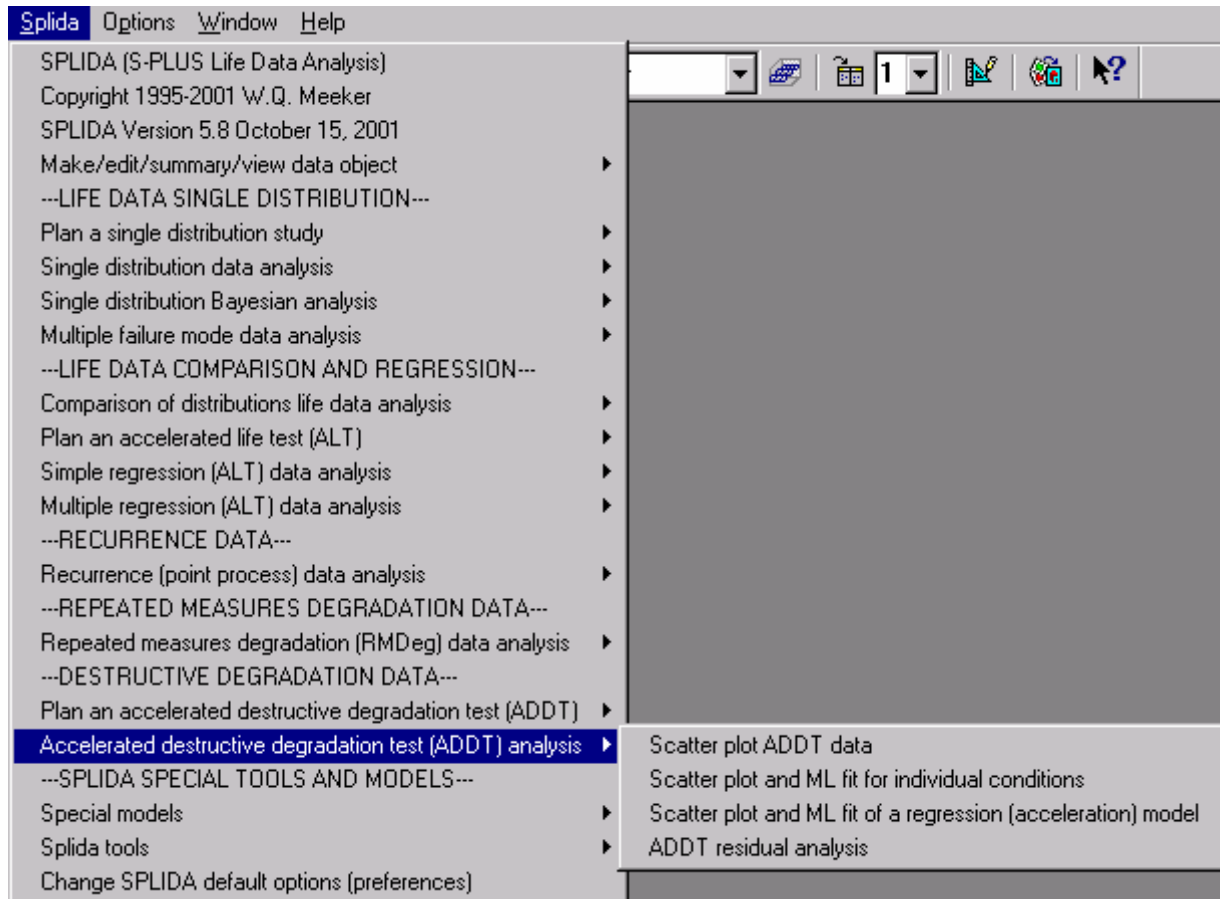
Figure 144 View of the SPLIDA menu for ADDT data analysis.

16.1 Making an accelerated destructive degradation data object

The first step in the analysis of ADDT data is to import the data into S-PLUS (usually from an Excel spreadsheet or a text file). As usual, the data set should consist of one row for each observation. The variables, corresponding to columns in the data set would include the measured degradation response (e.g., strength or size of a crack), the time of the measurement, and values of any explanatory variables. Optional columns could contain censoring information or case weights. The order of the columns is not important. Table 19 shows the variable names and ten rows of the AdhesiveBondB data set (this data set is one of SPLIDA's built-in examples).

```
DegreesC Weeks Newtons
```

```
50      0      70.1
50      0      76.7
50      0      84.5
50      0      88.0
50      0      88.9
50      0      90.4
50      0      91.9
50      0      98.1
50      2      77.8
50      2      78.4
```

Table 19 The first ten rows of the AdhesiveBondB data set.
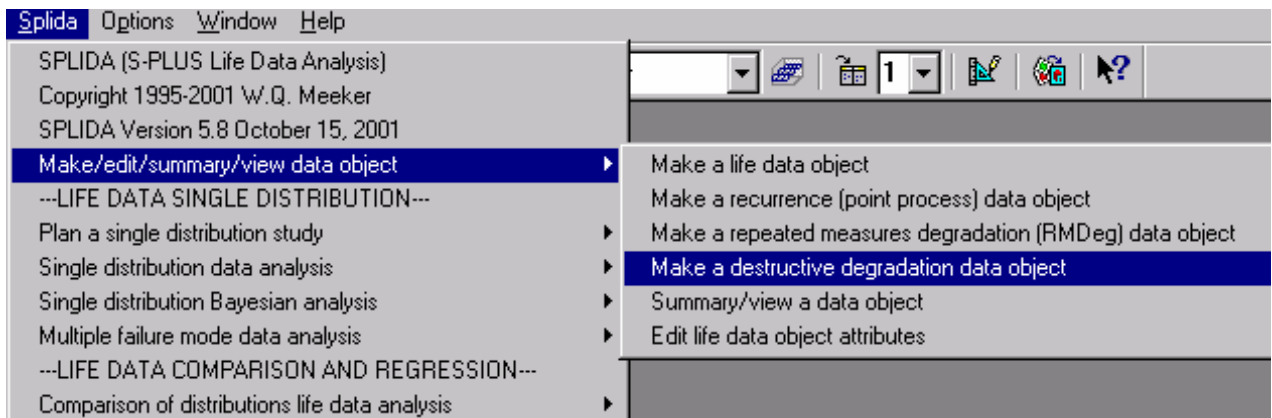
Figure 145 SPLIDA menu entry for making an ADDT data object.

As with other types of data, after the degradation data set has been imported into S-PLUS, it is necessary to use SPLIDA to create a destructive degradation data (.ddd) object. A data object contains the information about a data set. In particular, a data object defines the purpose of each variable and captures other information about the data set (e.g., units of time and the response, as well as a title for the data). This data object allows the analysis of the data set in various different ways without having to re-specify this information.

To make the destructive degradation data object, use **SPLIDA ►
Make/summary/view data object ► Make destructive degradation
data object** (see Figure 145) to bring up the dialog box in Figure 146. Select the required inputs (S-PLUS data set name, the response, and the time column). Then choose other possible variables, as needed, including the explanatory variables and the censoring variable (Status). For the AdhesiveBondB data, there is no weight variable. The names of the response and time columns serve as defaults for the units of these variables, so in this case these inputs need not be specified (SPLIDA tries to use available information as defaults whenever possible). After the input is entered into the dialog box, click the

Apply button to create the data object. A brief summary of the data object will be printed in the command window.

Figure 146 Dialog used to create a destructive degradation data object for the AdhesiveBondB data.

16.2 Degradation data scatter plot

After the data set has been created, it can be used as input to various data analysis procedures. Generally, data analyses begin with graphical analyses. For destructive degradation data, we start by plotting degradation versus time, using different symbols to represent the data at different levels of the explanatory variable(s). To do this, use **SPLIDA ► Accelerated destructive degradation test (ADDT) analysis ► Scatter plot ADDT data** to bring up the dialog box in Figure 147. Choose the AdhesiveBondB.ddd destructive degradation data object (SPLIDA will remember the most recently created .ddd object if it was created in the current S-PLUS session). Then clicking on Apply will produce a scatter plot. Different transformations for the response and time can be tried in an attempt to find a combination that provides approximately linear behavior in degradation over time. Generally, however, it would be preferable if this part of the model could be determined by physical knowledge of the degradation mechanism, and checked against data, as described in examples in Chapters 13 and 21 of Meeker and Escobar (1998).
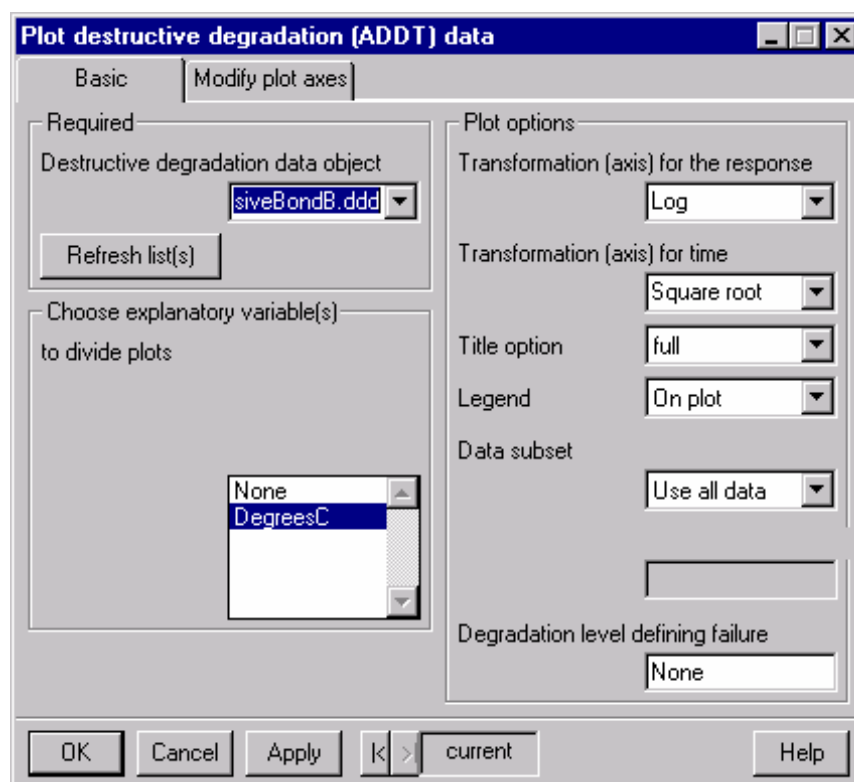
Figure 147 Dialog requesting a scatter plot of the AdhesiveBondB destructive degradation data.

For the AdhesiveBondB data, a log transformation for the response and a square root transformation for time resulted in an approximately linear relationship between degradation and time. The plot is shown in Figure 148 (also see Figure 150).
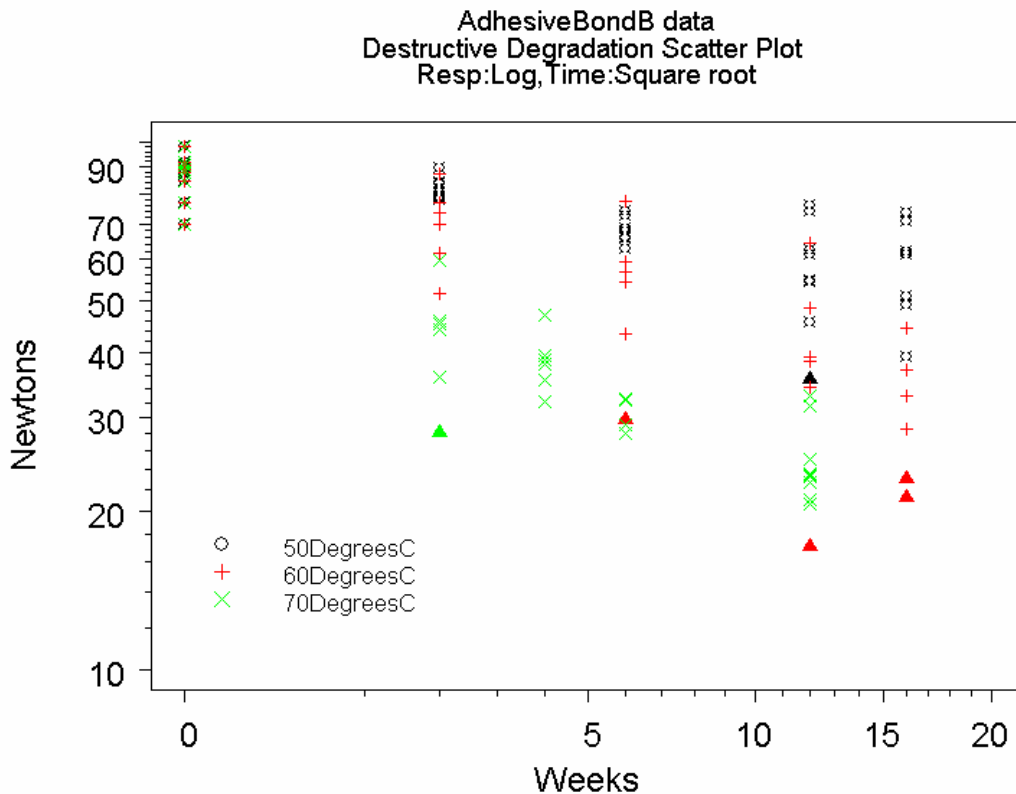
Figure 148 Scatter plot of the AdhesiveBondB destructive degradation data.

16.3 Individual scatter plot and ML fits for different accelerating variable levels

Extending the scatter plots described in Section 2.2, we now show how to plot fitted regression lines to the data at individual conditions (e.g., different levels of temperature). In general (to allow for censoring), the lines are fit by using maximum likelihood. Use **SPLIDA ► Accelerated destructive degradation test (ADDT) analysis ► Scatter plot and ML fit for individual conditions** to bring up the dialog box in Figure 149. Choose the transformations identified previously (or experiment with others). In this dialog, it is also necessary to specify a degradation distribution to describe the shape of the spread of the scatter of the response about the regression line (in the transformed scale). We choose to use the normal distribution because it provides a better description for the variability than the other distributions. There are also a number of optional inputs. For example, we have requested that ML lines be plotted and have entered 40 (Newtons) as the failure definition for the strength of the adhesive bond.
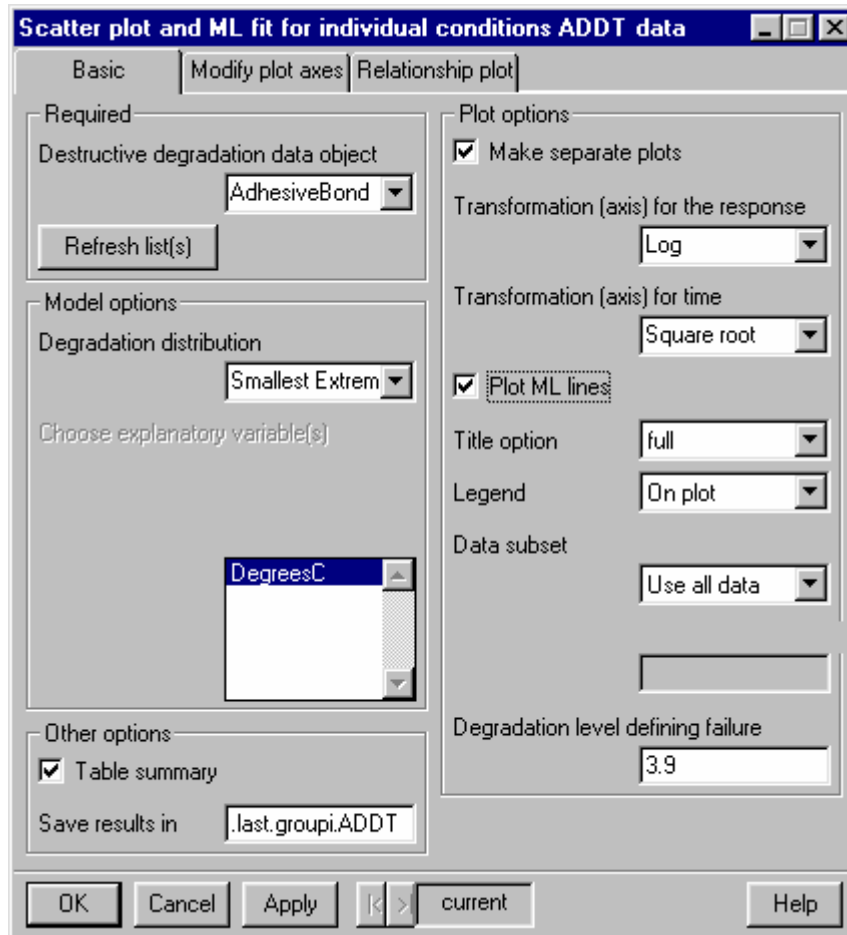
Figure 149 Dialog for individual scatter plots for the different levels of the explanatory variables.

After clicking on Apply, the plot in Figure 150 and the results in Table 20 are produced. The slopes in Table 20 are degradation rates that represent the change in log(Newtons) for a unit change in the square root of time for each of the temperatures. Figure 150 shows degradation versus time at each of the three different temperature conditions (on the transformed axes), along with the fitted regression lines (there is an option in the dialog to suppress plotting of the fitted lines). The horizontal failure-definition lines are also shown at 40 Newtons.

```
AdhesiveBondB data

Individual estimates and  95% confidence intervals for the slope at each
condition.

Normal distribution.

 DegreesC sample.size intercept   slope se.slope slope.lower slope.upper
       50          39     4.490 -0.1088  0.01494     -0.1424     -.08309

       60          32     4.489 -0.2089  0.02214     -0.2571     -.16969

       70          33     4.400 -0.3626  0.01944     -0.4028     -.32643
```

Table 20 Estimates, standard errors, and confidence intervals for the estimated slopes (degradation rates) for the individual levels of temperature.
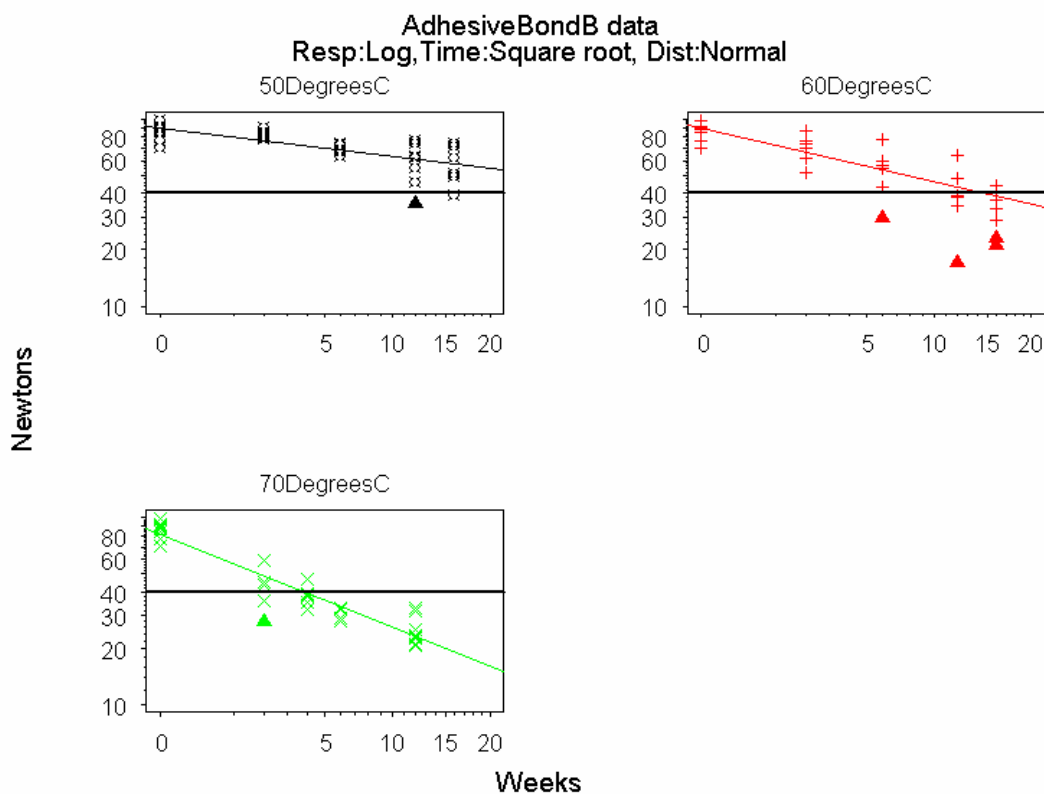
Figure 150 Individual scatter plots for the AdhesiveBondB destructive degradation data.

The default for the dialog in Figure 149 is to make a figure with a separate plot for each level (or combination of levels) of the explanatory variable(s). The check box in the upper right-hand side of the dialog allows the user to put all of the data and fitted lines on a single plot. The resulting plot is shown in Figure 151.
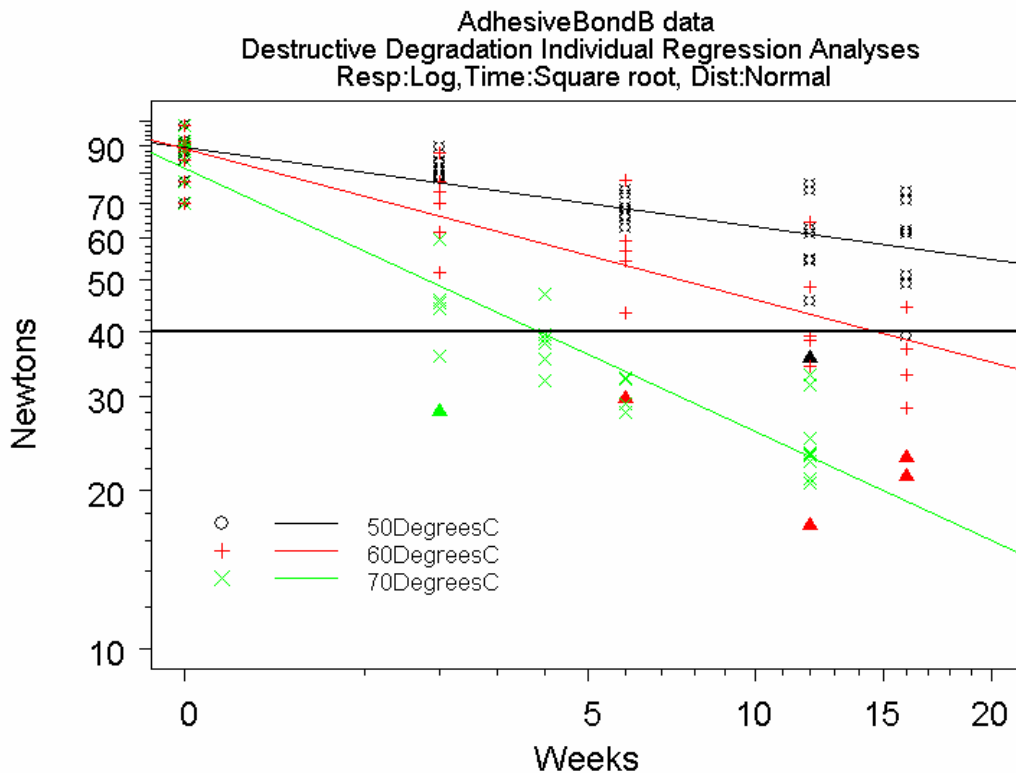
Figure 151 Scatter plot of the AdhesiveBondB data with individual ML lines.

The fitted regression lines in Figure 150 and Figure 151 were estimated individually for each level of temperature. For purposes of making inferences at other levels of temperature, (usually estimates of degradation rate or characteristics of the failure-time distribution at lower levels of temperature), it is necessary to fit a model relating degradation rate to temperature. As a first step in identifying such a model, the dialog in the Relationship plot page in the dialog box shown in Figure 149 allows one to request and control of some options for the "relationship model plot" of degradation rate versus the explanatory variable(s) (temperature for the AdhesiveBondB example). The underlying regression model assumes that the logarithm of degradation rate is linearly related to some transformation of the explanatory variable. When temperature is believed to affect the rate of a chemical reaction, the Arrhenius rate reaction model is commonly used to describe the relationship.

Figure 152 is a relationship model plot showing degradation rate (on a log axis) as a function of temperature (on a Arrhenius axis).
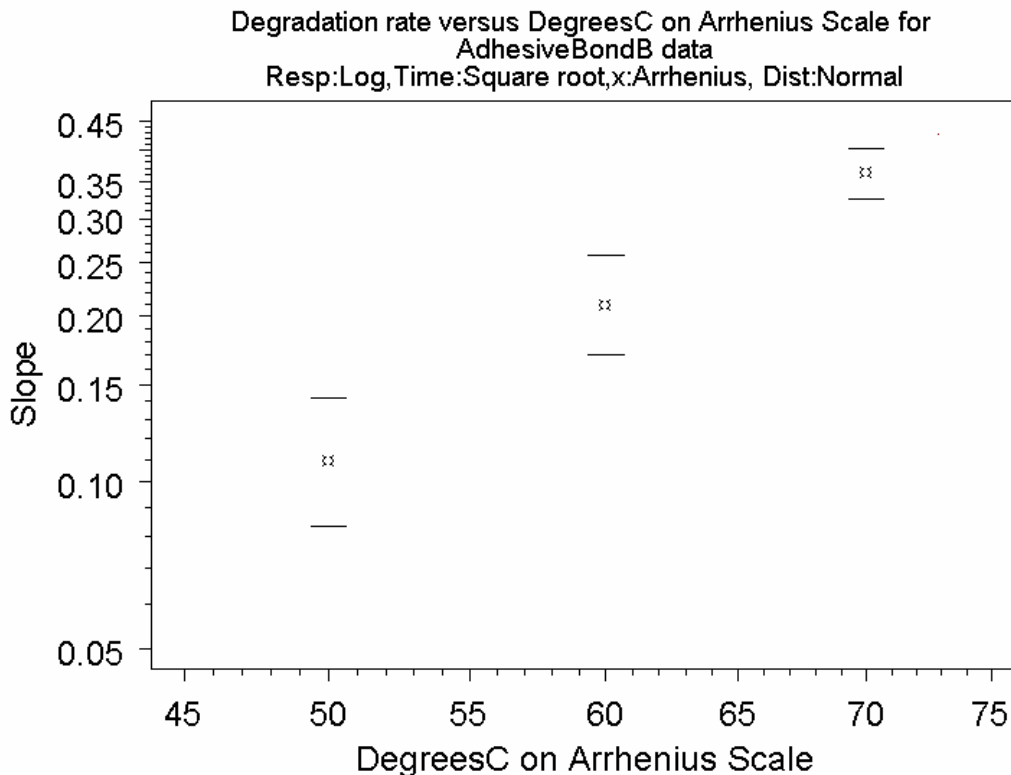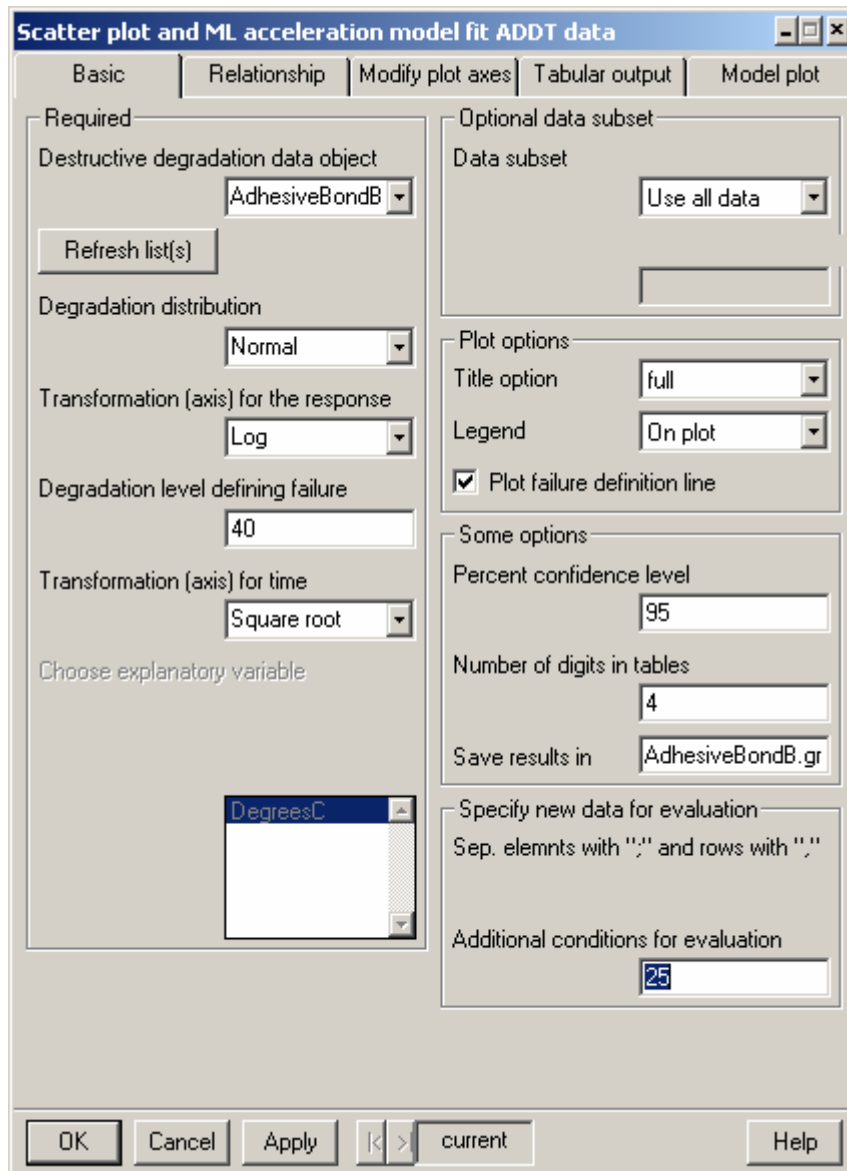
Figure 152 Relationship model plot showing degradation rate (log axis) as a function of temperature (Arrhenius axis) for the AdhesiveBondB destructive degradation data.

As shown in Meeker and Escobar (2002a), an Arrhenius relationship will plot as a straight line on Figure 152 (log axis for the rate or slope and an Arrhenius axis for temperature). The 95% (pointwise) confidence intervals (error bounds) for degradation rate in Figure 152 allow an assessment of statistical uncertainty. The line in Figure 152 was drawn using the line tool in the S-PLUS annotation tool bar. Although the points do not fall exactly on the straight line, the width of the error bounds, relative to the deviations from the straight line, suggest that the Arrhenius model could not be ruled out as a descriptor of the available data. Strictly speaking, one should use simultaneous confidence intervals to make this comparison, and these would be wider than the pointwise intervals displayed in Figure 152). The curvature exhibited in the three points suggests, however, that extrapolation to lower temperatures (like 25 degrees C) may not be justified. Generally, the justification for such extrapolation is not a statistical issue, but rather requires input from understanding of the physical/chemical aspects of the degradation-causing mechanism.

16.4 Scatter plot and ML fit of a regression (acceleration) model

To fit an ADDT model that includes the relationship between degradation rate and explanatory variables like temperature, use **SPLIDA ► Accelerated destructive degradation test (ADDT) analysis ► Scatter plot and ML fit of a regression (acceleration) model** to bring up the dialog box shown in Figure 153. In addition to the inputs required for the individual analyses described in Section 16.3, the user must now also specify the failure-definition level and the acceleration relationship (these were optional in previous dialogs). In addition, there are a number of optional inputs, as shown in Figure 153. For the AdhesiveBondB example, the product engineers wanted an assessment of degradation rate and the failure-time distribution at 25 degrees C. The results of the analysis will be saved, by default, in an object with the name AdhesiveBondB.groupm.ADDT.DegrArrh.normal.out (although it is generally not necessary, the user may edit and change this name). It is also necessary to visit the Relationship page (not shown here) to specify the relationship between log rate and the explanatory variable(s). For the AdhesiveBondB example the Arrhenius relationship was used.

Figure 153 Dialog box used to fit an acceleration model to the ADDT data.

As shown in Nelson (1990) and Meeker and Escobar (2002a), for a given failure-definition level, the degradation model induces a failure-time distribution. The properties of this distribution are usually of primary interest to product engineers. The Tabular output page allows the user to request tables of failure probabilities or quantiles of the induced failure-time distribution.

The Model plot page of the dialog allows the user to request a plot of the quantiles of the failure-time distribution versus the accelerating variable (temperature for the AdhesiveBondB example). This page of the dialog is shown in Figure 154.

Figure 154 Model plot page of the ADDT acceleration model-fitting dialog box.

After pressing the Apply button, SPLIDA will estimate the parameters of the degradation model and produce the plots in Figure 155 and Figure 156 as well as the information in Table 21.
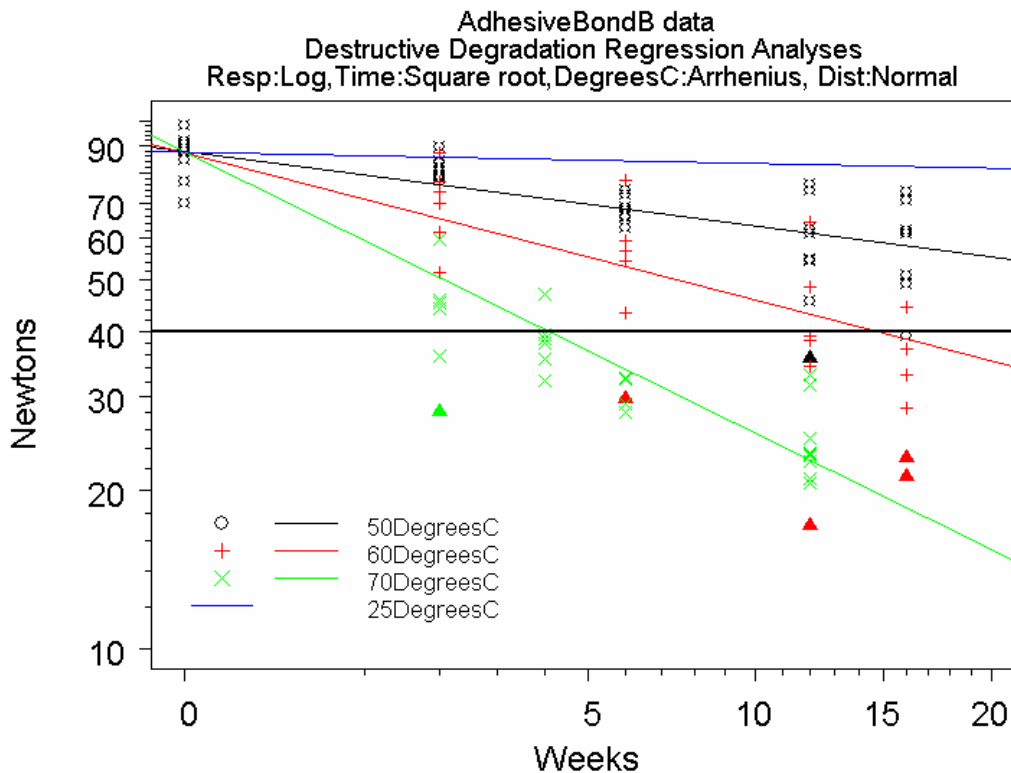
Figure 155 Plot of the ADDT normal-Arrhenius model for the AdhesiveBondB destructive degradation data.

In Figure 155, we see the fitted lines corresponding to the different levels of temperature, including the line for the specified use conditions 25 degrees C (obtained by extrapolation from the fitted Arrhenius model that relates degradation rate to temperature). Unlike the individual analyses displayed in Figure 151, all of the lines for the acceleration model cross at the intercept $\exp(\beta_0)=\exp(4.471) = 87.44$. The horizontal line at 40 Newtons allows one to visualize the failure-time distribution, at least at 70 Degrees C (the axes could be extended on the plot by using controls on the Modify plot axes page of the dialog box in Figure 153).

The model plot in Figure 154 shows estimates of the quantiles of the induced failure-time distribution as a function of temperature. As shown in Meeker and Escobar (2002a), a given quantile of the failure-time distribution is linear in the Arrhenius transformation of temperature. The .05, .5, and .95 quantile lines have been drawn (by default .10, .50, and .90 are used, but this was changed in the Figure 154 dialog) along with densities at the given levels of temperature (all levels of temperature in the data set plus additional levels entered on the Basic page of the dialog in Figure 153).
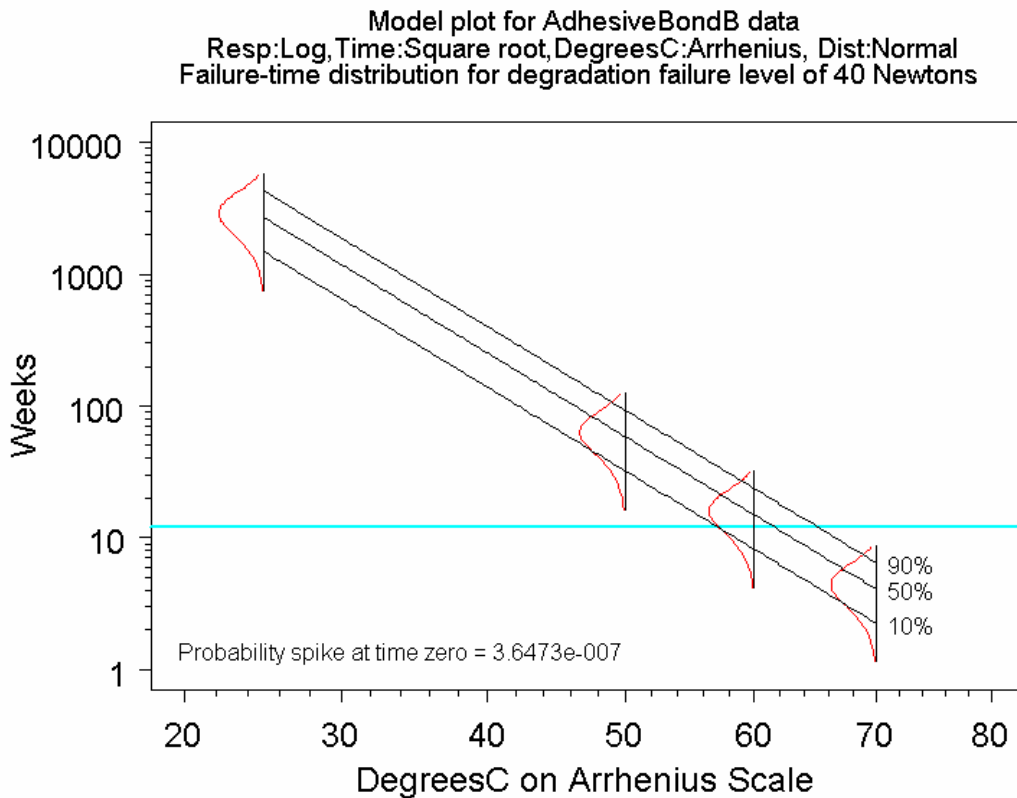
Figure 156 Model plot of failure-time distribution quantiles versus temperature for the AdhesiveBondB destructive degradation data.

Although the underlying degradation distribution is assumed to be a particular location-scale distribution, the form of the failure-time distribution is not simple. The form of the failure-time distribution depends on the assumed transformations on the response and on time. Also, as shown in Meeker and Escobar (2002a), the failure-time distribution induced by our degradation model has an "atom" of probability at time zero, corresponding to the probability that the degradation level is beyond the failure threshold at time 0. In Figure 154, the Probability spike at time zero = 0.011418 gives the ML estimate of the probability that a unit of AdhesiveBondB has a strength less than 40 Newtons at time 0 (note that this probability does not depend on temperature level).

The horizontal line at 84 hours is the time of the last inspection. Inferences beyond this point involve extrapolation in time. Figure 154 shows that the amount of extrapolation in time for this application is substantial (in both temperature and time).

Table 21 provides a summary of the ML estimation for the normal-Arrhenius model fit to the AdhesiveBondB destructive degradation data. Table 22 is a table of ML estimates and confidence intervals for several quantiles of the failure-time distribution at 25 degrees C, with a failure definition of 40 Newtons (obtained with the controls on the Tabular output page of the dialog in Figure 154.

```
Degradation

Response units: Newtons
Time units: Weeks
Normal Distribution

Relationship: Resp:Log,Time:Square root,DegreesC:Arrhenius

Maximum likelihood estimation results
Appears to have converged; relative function convergence
Log likelihood at maximum point: 34.97

                MLE    Std.Err.   95% Lower   95% Upper
```

Table 21 Summary of ML estimation for normal-Arrhenius model fit to the AdhesiveBondB destructive degradation data.

```
Parametric ML Failure-time Quantile Estimates
at 25. DegreesC for failure defined at a level of 40 Newtons
Pointwise Approximate 95% Confidence Intervals

 Dead-on-Arrival-Normal Distribution

    p    Weeks Std.Err. 95% Lower 95% Upper
 0.01  757.42   370.37    290.48    1975.0
 0.05 1201.38   596.53    453.97    3179.3
 0.10 1479.71   742.95    553.09    3958.8
 0.50 2692.88  1402.10    970.56    7471.5
 0.95 4778.27  2572.08   1663.72   13723.
```

Table 22 Table of ML estimates of quantiles of the AdhesiveBondB failure-time distribution

16.5 ADDT model residual analysis

After fitting an ADDT model, it is important to assess the adequacy of the fit of the model by plotting the residuals in various ways. To do this in SPLIDA for ADDT data, use **SPLIDA ► Accelerated destructive degradation test (ADDT) analysis ► Residual analysis** to bring up the dialog box in Figure 157. The dialog will recall the most recent ADDT model output that was created in the current S-PLUS session (if any). The default for this dialog is to get all possible residual plots. It is easy, however, to request one or some of the available plots.

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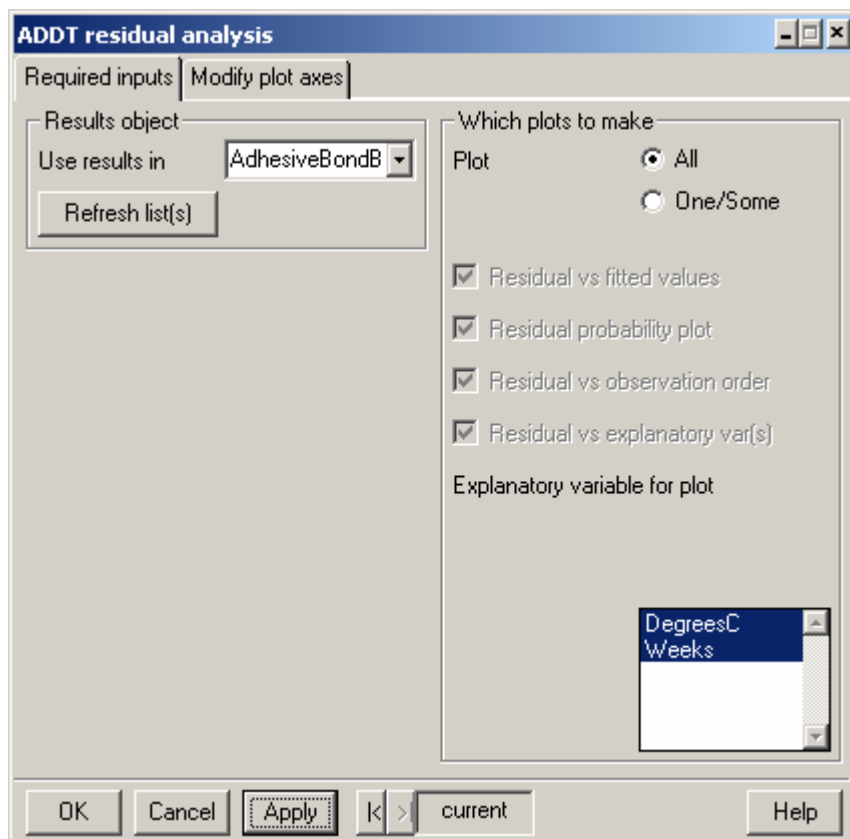
Figure 157 ADDT residual analysis dialog

Figure 158 is a plot of the residuals versus fitted values for the normal-Arrhenius plot fit to the AdhesiveBondB data. This plot suggests that the amount of variability does not differ importantly as a function of temperature.
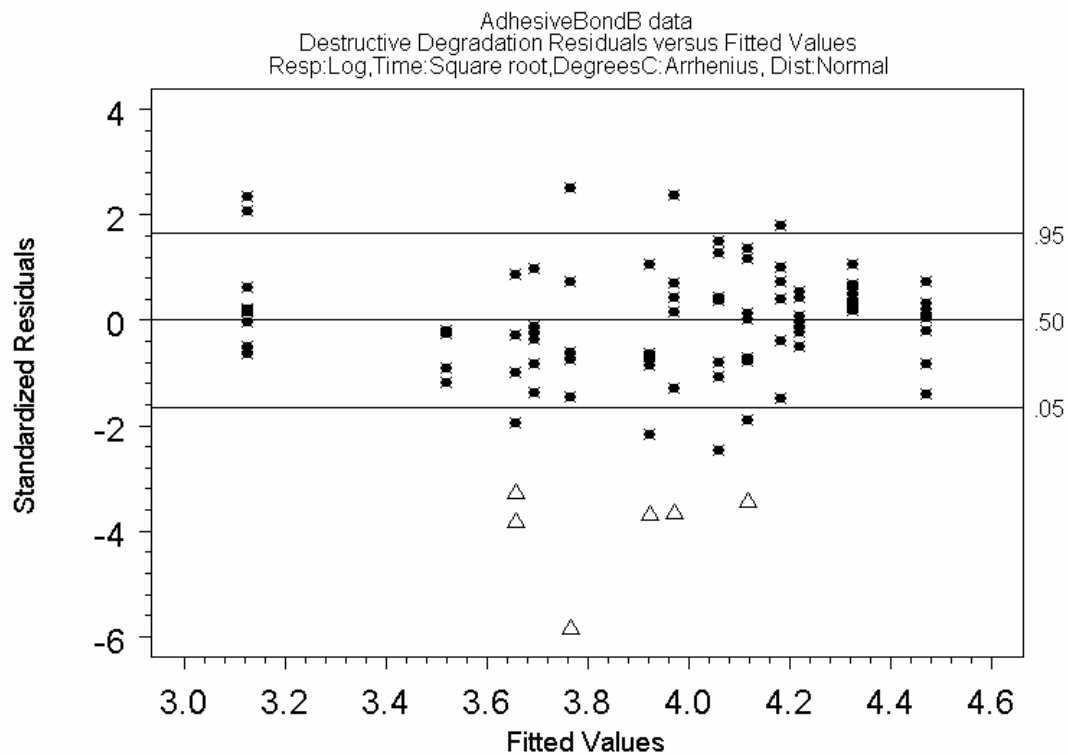
Figure 158 Plot of the residuals versus fitted values for the Arrhenius plot fit to the AdhesiveBondB data.

Figure 159 is a probability plot of the residuals for the normal-Arrhenius plot fit to the AdhesiveBondB data. This plot suggests that the normal distribution provides a reasonably good fit to the residuals. This is in contrast with the smallest extreme value probability plot in Figure 160 (based on fitting a model in which the residuals were assumed to have a smallest extreme value distribution) where there is some deviation in the lower tail of the distribution.
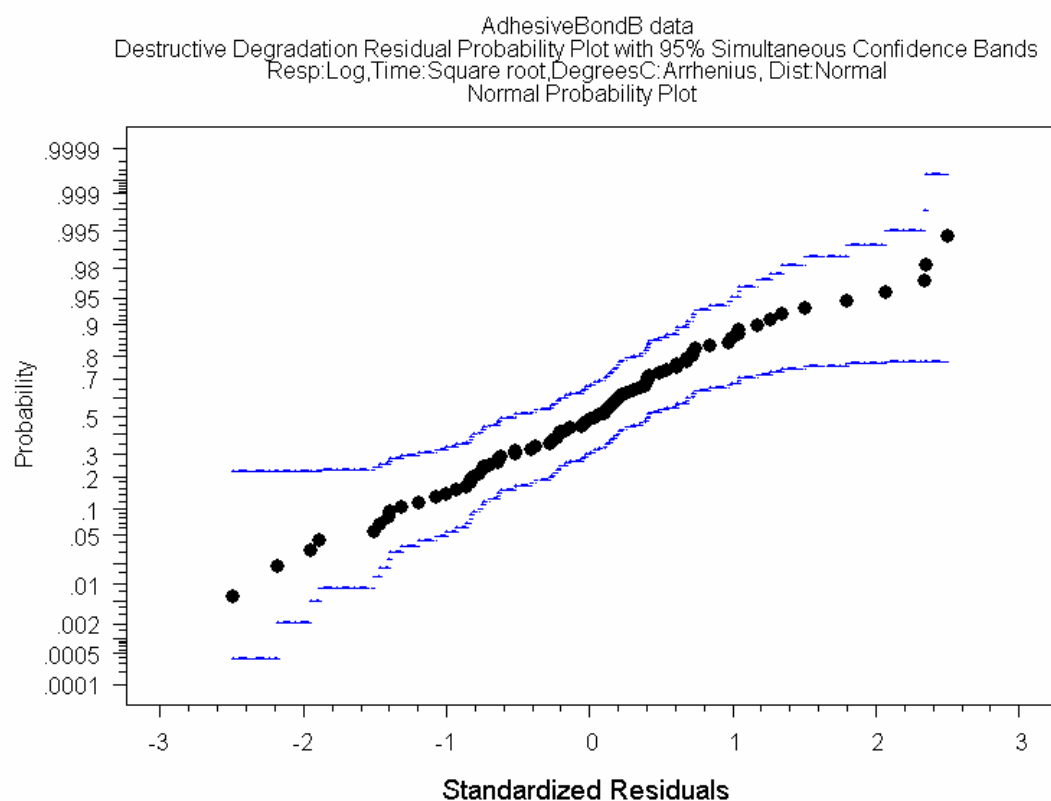
Figure 159 Normal distribution probability plot for the Arrhenius model fit to the AdhesiveBondB data.
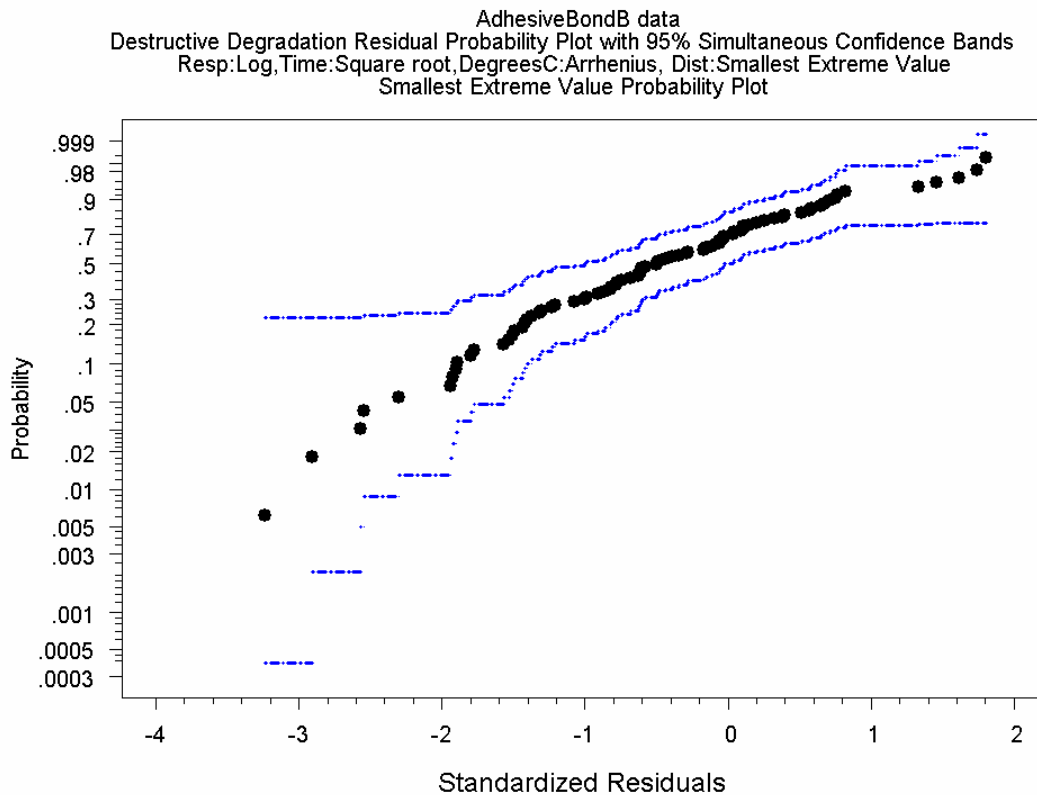
Figure 160 Normal distribution probability plot of the residuals for the Arrhenius model fit to the AdhesiveBondB data.

16.6 Analysis of destructive degradation data with two accelerating variables

ADDT Data Object

This section shows how to use SPLIDA to analyze a set of ADDT data with two (or more) experimental factors (or accelerating variables). Because of the similarities with the one variable analysis methods described in Sections 16.1 to 16.5, this section presents an example and focuses particularly on concepts and tools that differ from those used in the single-variable analysis.

In this section, we use the AdhesiveBondD destructive degradation data from an experiment on the strength of an adhesive bond tested at different levels of temperature and humidity. The data available in the SPLIDA destructive degradation data object AdhesiveBondD.ddd. To look at the data, use **SPLIDA ►**
Make/edit/summary/view data object ► Summary/view a data object
to bring up the dialog box. Select destructive degradation objects for type of dataset objects, choose the AdhesiveBondD.ddd data object. The default optional output is to print a summary of the data but you can also check "Print data" to view all the data. Then click Apply. This produces the data summary; see Table 23 for the first few lines of the summary for the AdhesiveBondD data. In addition to information about each of

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the experimental factor (Weeks, DegreesC, RH), the summary displays information about the response and number of observations at each of the unique experimental factor level combinations.

The experiment design has a 3x2x8 factorial arrangement with three levels of temperature (50, 60, 70 DegreesC), two levels of relative humidity (20 and 80 percent RH), and eight levels of time (0, 1, 2, 4, 6, 8, 12, 16 weeks). The response is strength of the adhesive measured in Newtons; an adhesive strength below 25 Newtons is considered a failure.   There are 288 observations with 6 observations for each combination of temperature, relative humidity, and time. Note that, as in the single-accelerating variable example, the time factor is treated in a special manner. In particular, one objective of the experiment is to assess the life distribution of the adhesive at use conditions of 25 DegreesC and 50% RH.

```
Number of rows in data matrix= 288
Time units:  Weeks
Response units:  Newtons
Response minimum:  11
Response maximum:  56.8
Number of cases in data set= 288
Number of exact failures in data set= 288

Summary of numeric columns in X matrix:
         min max      mean         sd        cv
   Weeks   0  16     6.125     5.264    0.8594
DegreesC  50  70    60.000     8.179    0.1363
      RH  20  80    50.000    30.052    0.6010


unique X values (or combinations)
List of unique X values (or combinations)
  Weeks DegreesC RH min-Resp max-Resp #exact
 1      0        50 20      31.5      50.1       6
 2      1        50 20      33.1      45.7       6
 3      2        50 20      29.8      48.6       6
 4      4        50 20      28.6      47.8       6
 5      6        50 20      27.6      43.2       6
 6      8        50 20      28.6      42.6       6
```

Table 23 Summary of the AdhesiveBondD data set.

Scatter Plot

To get a scatter plot of the response versus time, use **SPLIDA ► Accelerated destructive degradation test (ADDT) analysis ► Scatter plot and ML fit for individual conditions** to bring up the dialog box similar to the one in Figure 149. Specify the AdhesiveBondB.ddd as the "destructive degradation data object." For

this example, choose the log transformation for the response and the square root transformation for time. Specify normal for the degradation distribution and enter 25 (Newtons) as the failure definition for the strength of the adhesive bond. In the "Relationship plot page," verify that DegreesC is the Focus variable and that the Relationship is Arrhenius (these are the defaults). Then click on Apply. This generates the scatter plots in Figure 161, the relationship model plot Figure 162, and a summary of the individual ML fits, as shown in Table 24.
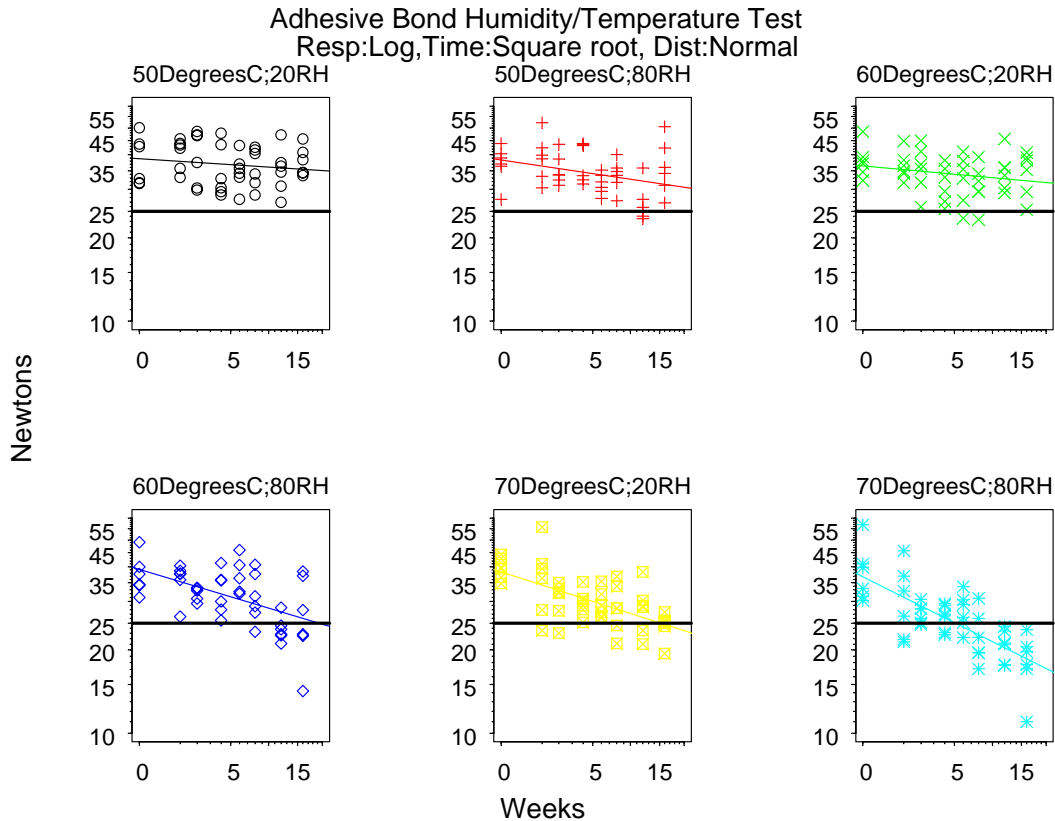
Figure 161 Individual scatter plots for the AdhesiveBondD destructive degradation data.

Figure 161 is similar to Figure 150, showing strength versus time at the distinct experimental factor level combinations of DegreesC and RH. The fitted lines are individual fits using the normal distribution. Table 24 provides information on the regression lines fitted to each factor combination. The sample size in the table corresponds to the six units at each of the eight time points. One can get a single plot with an overlay of all the scatter plots by deselecting the "Make separate plots" in the "Plot options" of the dialog box.

```
Adhesive Bond Humidity/Temperature Test
Individual estimates and  95% confidence intervals
for the slope at each condition.
 Normal degradation distribution.
```

```
DegreesC   RH sample.size    intercept     slope   se.slope slope.lower slope.upper
      50   20           48        3.658  -0.02242    0.01982     -0.1267    -0.003966
      50   80           48        3.646  -0.05104    0.02035     -0.1115    -0.023361
      60   20           48        3.597  -0.03124    0.01937     -0.1053    -0.009271
      60   80           48        3.667  -0.10221    0.02292     -0.1586    -0.065864
      70   20           48        3.645  -0.10885    0.01960     -0.1549    -0.076486
      70   80           48        3.604  -0.17038    0.02249     -0.2207    -0.131549
```

Table 24 Estimated slopes (degradation rates) for the individual combinations of DegreesC and RH for the AdhesiveBondD data.

Degradation rate versus DegreesC on Arrhenius Scale for Adhesive Bond Humidity/Temperature Test
Resp:Log,Time:Square root,x:Arrhenius, Dist:Normal
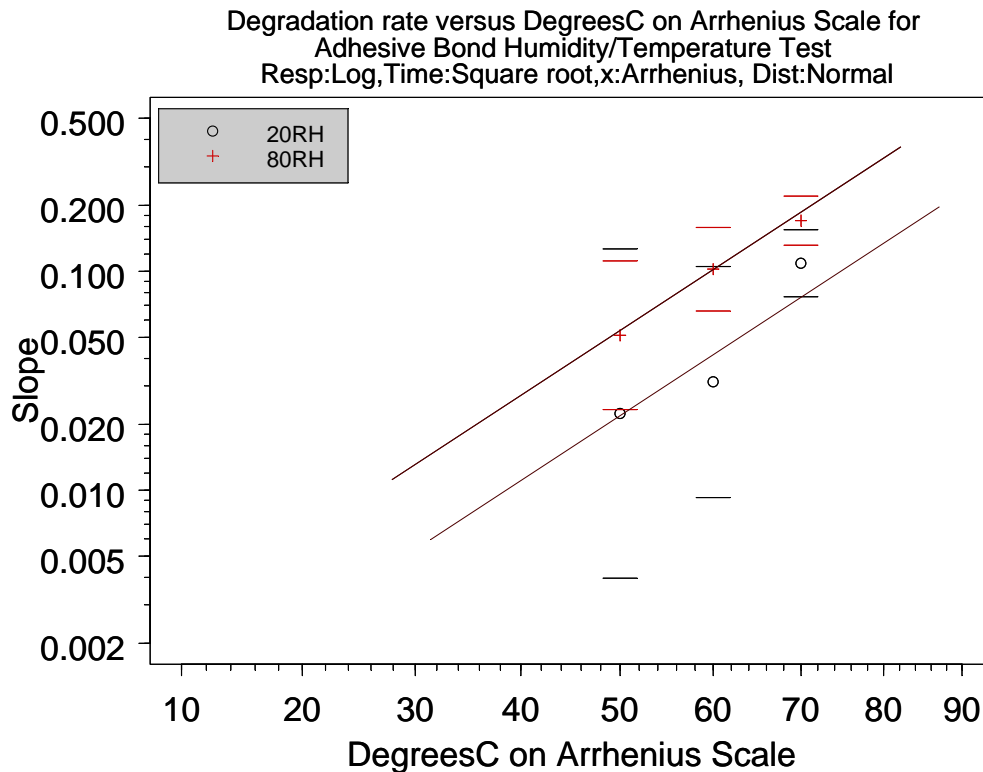
Figure 162 Relationship model plot showing degradation rate (log axis) as a function of temperature (Arrhenius axis) for each level of RH for the AdhesiveBondD data.

Relationship plots like the one in Figure 162 are used to assess relationship between the degradation rate (degradation versus time slope) and a chosen accelerating variable (focus variable). The crosses in Figure 162 indicate the degradation rate estimates for different levels of temperature (the chosen focus variable) when RH is 80%. Similarly, the open circles indicate the degradation rate estimates for different levels of temperature when RH is 20%. The horizontal segments depict approximate 95% pointwise confidence intervals for the degradation rates. As shown in Meeker and Escobar (2002a) for a model without interaction, for each fixed level of RH, the degradation rate estimates should fall approximately along a straight line and the lines for different values of RH should be approximately parallel. The lines in Figure 162 were drawn using the line tool

in S-PLUS annotation tool bar. The estimates for 20% RH show some curvature and extrapolations to low levels of temperature and RH may be seriously incorrect. To make a definite assessment of the apparent departure from linearity one would need a more elaborate statistical procedure. A similar relationship plot for RH is obtained by choosing RH as the focus variable and Humidity for the relationship in the dialog box. This plot (not shown here) shows degradation rate estimates as a function of RH for each fixed value of temperature. Because, for these data, there are only two levels of RH for each fixed level of temperature, the model plot for RH is only informative to assess the parallelism of the lines for different levels of temperature.

Fitting an Acceleration Model

To fit an ADDT model that includes the relationship between degradation rate and the accelerating factors DegreesC and RH, use **SPLIDA ► Accelerated destructive degradation test (ADDT) analysis ► Scatter plot and ML fit of a regression (acceleration) model to** bring up the dialog box shown in Figure 163. In addition to the inputs required for the individual analyses described above, specify the use conditions 25;50 for "Additional conditions for evaluation," then go to the "Relationship page" (see Figure 164) click on "Relationships" and choose Arrhenius for DegreesC and Humidity for RH. Then go to the "Model plot" page and check the "Make a model plot" option. Click on "Choose" to specify the fixed value of 50 for RH and then click on "Apply." SPLIDA generates Figure 165 and Figure 166, and the ML estimates for the model parameters. Table 25 shows part of the output generated by SPLIDA.

Figure 163 Dialog box used to fit a two variables acceleration model.
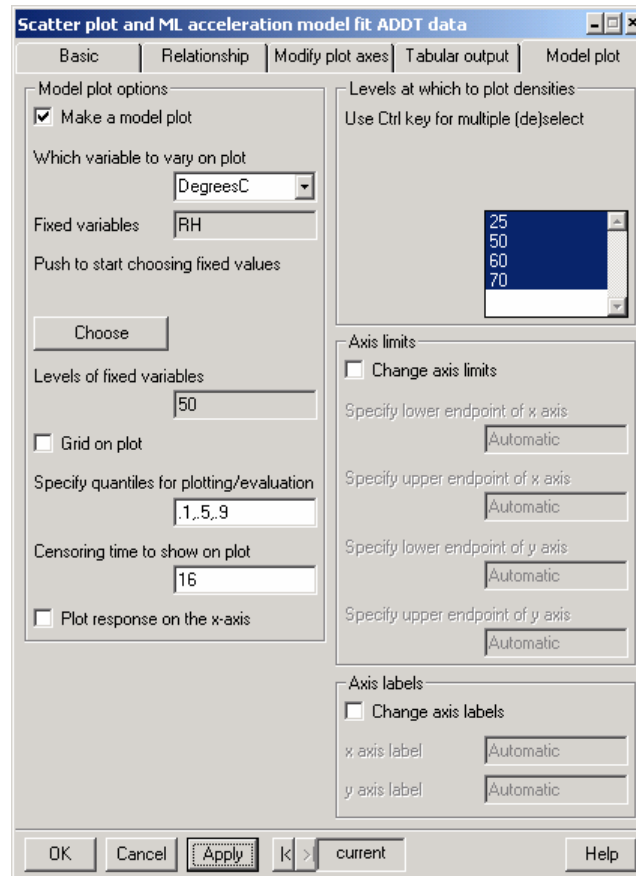
Figure 164 Model plot page of the dialog box used to fit a two variables acceleration model.

Figure 165 shows the strength versus time data in a scatter plot as well as the fitted lines corresponding to different combinations of DegreesC and RH, including the extrapolated line for 20 DegreesC and 50% RH. Figure 166 shows estimates of the quantiles of the induced failure-time distribution as a function of temperature when RH is fixed to 50%. One can get a similar plot in which the focus variable is RH, where DegreesC is fixed to a given value. To generate such a plot, go back to the "Model plot page" in Figure 164 and pick RH as the variable to vary on the plot at the same time specify the fixed value of DegreesC to use in the plot.
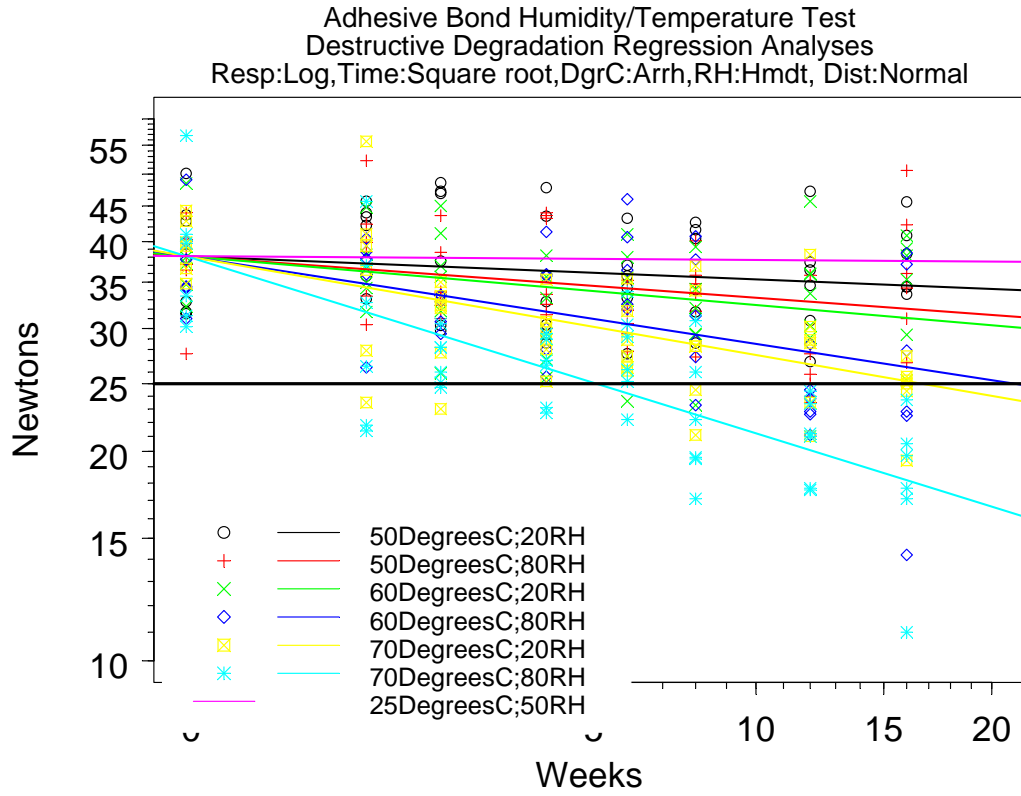
Figure 165 Plot of the ADDT normal-Arrhenius-Humidity model for the AdhesiveBondD.
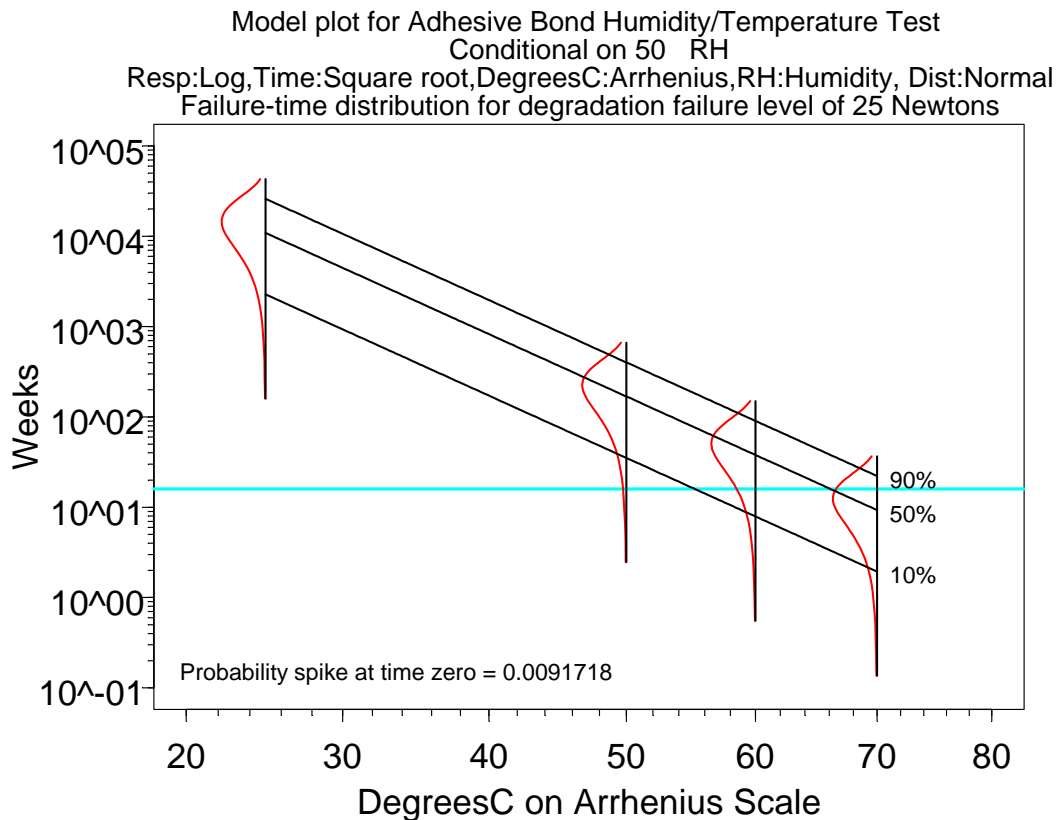
Figure 166 Model Plot of failure-time distribution quantiles versus temperature when RH=50% for the AdhesiveBondD data.

```
Response units: Newtons

Time units: Weeks

Normal DistributionRelationship: Resp:Log,Time:Square
root,DegreesC:Arrhenius,RH:Humidity

Maximum likelihood estimation results

Log likelihood at maximum point: 86.92

 Interpretation parameter MLEs
 beta0      beta1  beta2  beta3  sigma
 3.641 -2024525684 0.6921 0.2105 0.1789
```

Table 25 ML estimates of the normal-Arrhenius-RH model for the AdhesiveBondD data.

Residual analyses provide checks on model adequacy.   To get the residual plots for the fitted normal-Arrhenius-RH model use **SPLIDA ► Accelerated destructive degradation test (ADDT) analysis ► Residual analysis** to bring up the residual dialog box and request the plots.   SPLIDA remembers the name of the most recent ADDT model output object that was created during the current S_PLUS session. Otherwise you need to choose the output from the available list in the dialog box. The default output object name corresponding to the output in Figure 163 for the normal-Arrhenius-RH analysis is

AdhesiveBondD.groupm.ADDT.DgrCArrh.RHHmdt.normal.out, but the name is editable in the analysis dialog (shown in Figure 163) allowing one to choose a shorter and more convenient name.

17 Planning an Accelerated Destructive Degradation Test

This section describes methods for planning accelerated destructive degradation tests (ADDTs). Figure 167 shows the menu for SPLIDA's ADDT planning capabilities. As with other SPLIDA sub-menus, one generally starts at the top item and progresses through the ordered sequence of steps. To provide an example for test planning we will revisit the AdhesiveBondB example, and explore the possibility of using a test plan that is different from the one in the original example.
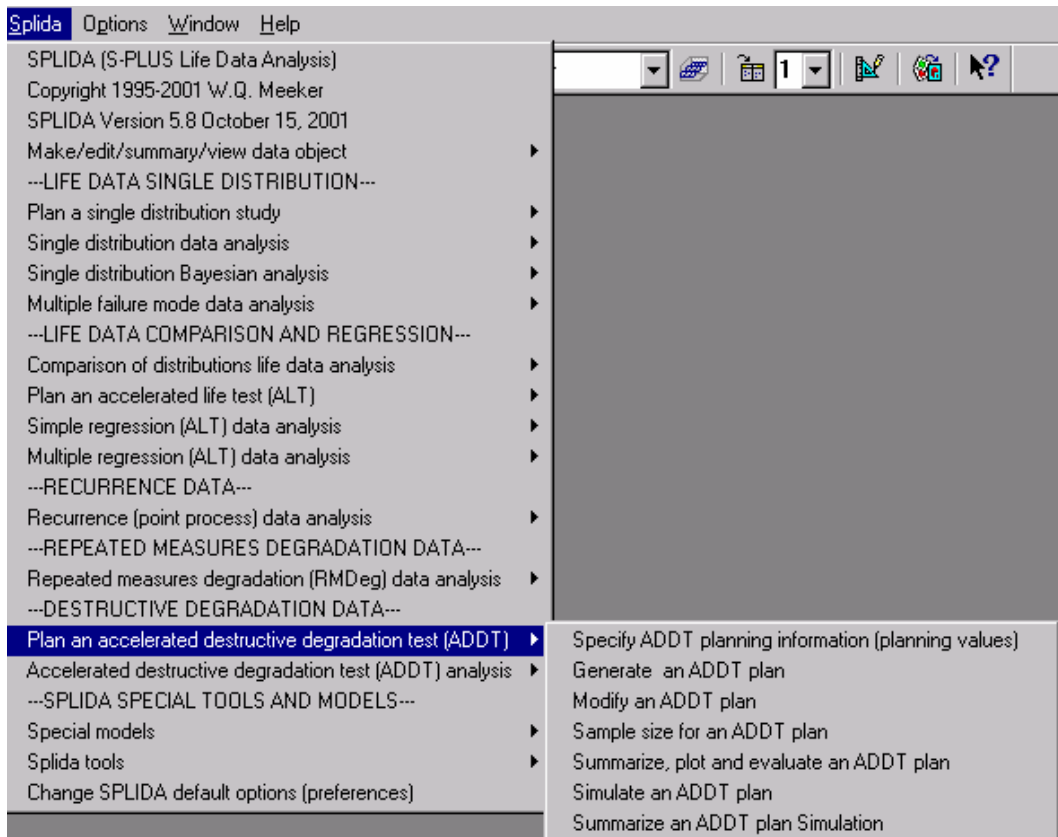
Figure 167 View of the SPLIDA menu for ADDT planning.

17.1 Specifying ADDT test planning information (planning values)

The first step in planning an ADDT is to elicit information about the testing situation. The needed planning information consists of the model and the model parameters. Such information is usually obtained from previous experience with similar materials or products, pilot tests, or engineering judgment. In the example presented here, we will use the parameter estimates from the analysis in Section 16.4 as our planning values.

The planning information is provided to SPLIDA using the dialog box obtained with **SPLIDA ► Plan a destructive degradation test (ADDT) ► Specify ADDT test planning information (planning values).** This will bring up the dialog box in Figure 168. As usual, the required inputs are at the top/left in the dialog. When making the test planning values object, it is generally useful to provide most of the optional inputs (e.g., time and response units), to make the planning output easier to understand. The degradation level defining failure and the use conditions need not be specified here, but it is generally convenient to do so. Otherwise, these values will have to be specified in various places later in the planning process (it is, of course, possible to alter these values in subsequent dialogs). The default name for the planning values object is last.Distribution.ADDTpv, where "Distribution" is replaced by the chosen distribution. Generally, however, it is a good idea to edit this to something more meaningful. For the example, we use AdhesiveBondB.Normal.ADDTpv.

Figure 168 Basic information page of the dialog box for providing ADDT planning values.

Figure 169 shows the Parameter page of the dialog box for providing ADDT planning values. There are two options in this page. One can specify the basic parameters β_0, β_1, β_2, and σ, as defined in Meeker and Escobar (2002b). A problem with this approach is that β_1 has no practical interpretation and it generally has a rather large magnitude (e.g., on the order of 10^{11}). The alternative is to specify the slope of the regression line at a given level of the accelerating variable (e.g., slope at 50 degrees C) in place of β_1. This can be done because of the relationship $\upsilon^W = \beta_1^W \exp(\beta_2^W x)$ where υ^W is the slope at transformed temperature $x = -11605/(\text{degrees C} + 273.15)$. For the example, the planning value for the slope at 50 degrees C is $\upsilon^W = \beta_1^W \exp(\beta_2^W x) = -8.641e+008*\exp(-.6364*35.912) = -0.10257$.

Figure 169 Parameter page of the dialog box for providing ADDT planning values.

After clicking on Apply, a small summary table of the planning information is given.

17.2 Generating an ADDT test plan

SPLIDA has a tool to generate a simple ADDT test plan. Subsequently the plan can be modified to have a more complicated structure, if needed. Use **SPLIDA ► Plan a destructive degradation (ADDT) test ► Generate an ADDT test plan** to generate the dialog box in Figure 170. Fill the accelerating variable name and then push the Choose levels button to choose the levels of the accelerating variable (50,60,70 for temperature in this case). The time units given here should agree with what is given in the planning values object. Then enter a list of times (integers separated by commas) at which inspections are to be conducted. Finally, enter the number of units per time/level combination. Usually this number would be somewhat greater than the default of one. It is generally a good idea to provide a meaningful name for the test plan object to be created. The default is last.ADDTplan, but we will use AdhesiveBondB.ADDTplan. After providing the required and the desired inputs, click on Apply. The plan shown in Table 26 will be printed.

Figure 170 Dialog box to generate a simple ADDT plan.

	Weeks0	Weeks2	Weeks4	Weeks6	Weeks12	Weeks16	Weeks32
50Temp	8	8	8	8	8	8	8
60Temp	8	8	8	8	8	8	8
70Temp	8	8	8	8	8	8	8

Table 26 Simple test plan generated from the dialog in Figure 170.

17.3 Modifying an ADDT test plan

The test plan generated in Section 17.2 (shown in Table 26) provides a convenient starting point for creating other test plans. The dialog box launched by using **SPLIDA ► Plan a destructive degradation (ADDT) test ► Modify an ADDT test plan,** shown in Figure 171, allows the user to choose subsets of the rows or columns in the original plan and to change the allocations in the individual cells. First, one chooses an existing test plan object from those on the database. One option is to replace the original plan. Alternatively, the original plan can be left intact by specifying a new name for the modified plan. One can then choose a desired subset of the times and levels of the explanatory variables.
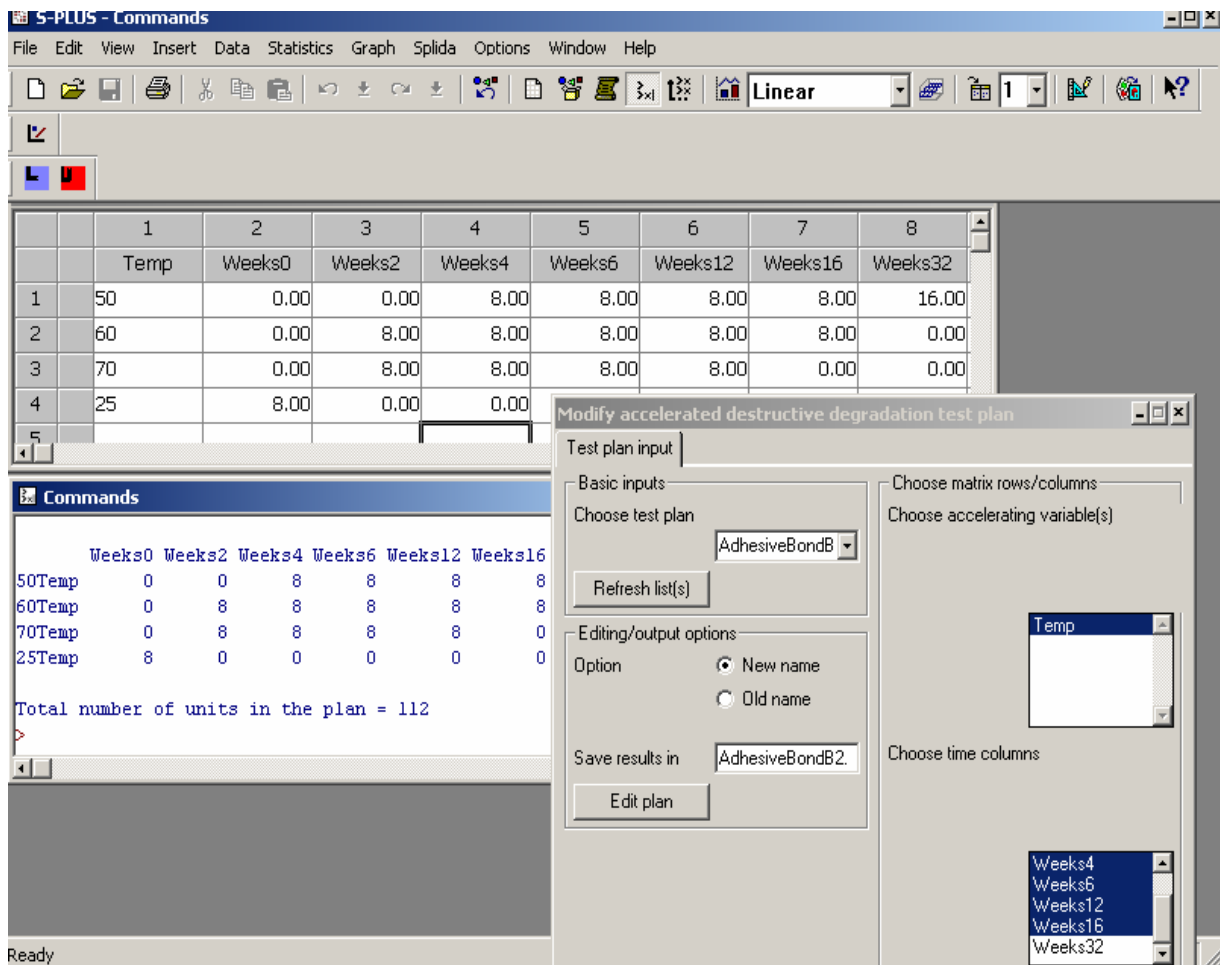
Figure 171 Screen shot showing the modify ADDT plan dialog and the pop-up spreadsheet for editing allocations.

As an alternative to the original plan, the allocations for cells on the extremes of the matrix (NW and SE corners) were set to zero (by editing the spreadsheet). It was thought that very little degradation would be observed early in the experiment at low temperatures and that too much degradation would have occurred by the end of the test at high levels of temperature. Note that it is possible to add rows to the test plan matrix by editing the spreadsheet, but that to add a new column (time point) one must return to the dialog in Figure 170. It will also be of interest to add the inspection point at 32 weeks

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(with 16 units at 50 degrees C) to see if this will improve estimation precision. Then, for comparison, the inspection at 32 weeks was de-selected on the dialog, so that the range of times would be the same as the original plan.

17.4 Sample size for an ADDT test plan

SPLIDA has a tool that allows the user to determine the sample size needed to produce a desired degree of estimation precision for a particular quantity of interest. The sample size depends on the specified planning values and corresponds to a given experimental layout, as defined by an ADDT plan object. To use the sample use tool, use **SPLIDA ► Plan a destructive degradation (ADDT) test ► Sample size for an ADDT test plan** to launch the sample size tool dialog shown in Figure 172. Choose the desired plan values object and test plan object (as in other dialogs, SPLIDA will remember and use as defaults the most recently created objects if they were created in the current session). Clicking on Apply will produce the plot in Figure 173.

Figure 172 Dialog box for the ADDT sample size tool.

We quantify the precision in terms of the "precision factor" R. When a confidence interval is computed for a positive quantity θ using the normal-approximation, it has the form $[\hat{\theta} \times R, \quad \hat{\theta}/R]$ where $\hat{\theta}$ is the ML estimate of θ and R>1. Thus the upper endpoint of the confidence interval is 100(R-1)% larger than the estimate of θ and the lower endpoint is 100(R-1)/R% smaller than the estimate of θ.

The plot in Figure 173 gives the needed sample size as a function of the precision factor R for the test plan that includes the 32 Weeks inspection point. The precision factor reflects the width of the confidence interval that one can expect to achieve approximately 50% of the times in repeated tests like the one being planned. The rectangle tool from the

annotate tool bar was used to mark the sample size needed to obtain a precision factor of R=1.5. The total required sample size is approximately 220. The computation assumes that the 220 units would be allocated in approximately the same proportion as the units in the test plan matrix. The sample size is considerably larger than the original plan because of the desire for more precision (From Table 22, the original plan had an R factor of about 3179.3/1201.38=2.64 for the .05 quantile of the failure time distribution at 25 degrees C).

The sample size tool was also used for the test plan without the 32 Week inspection and a total sample size closer to 250 units would be required to achieve the same degree of precision. The needed sample size is, in general, highly dependent on the range of time and temperature, but not too dependent on the allocation within that range.
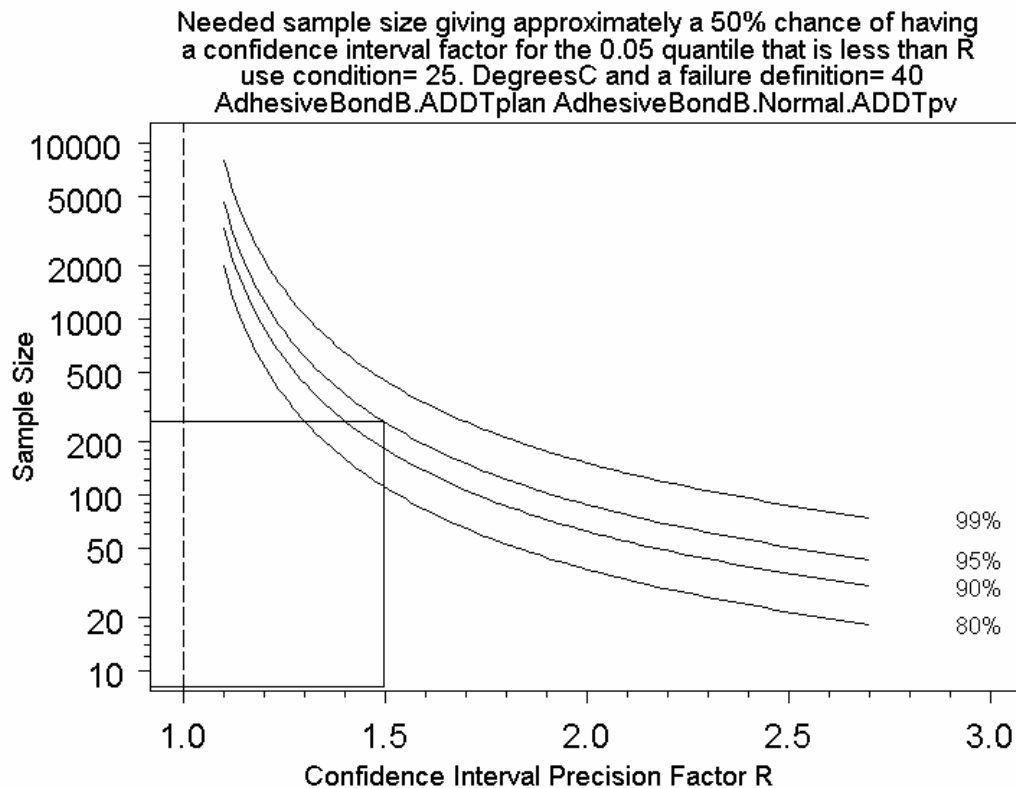
Figure 173 Output for the ADDT sample size tool for the modified plan.

17.5 Summarizing and evaluating an ADDT test plan

To get a quick evaluation of the precision for a given test plan, use **SPLIDA ► Plan a destructive degradation (ADDT) test ► Summarize, plot, and**

evaluate an ADDT test plan to request the dialog box in Figure 174. By default, SPLIDA will evaluate the properties of the estimates of the .10, .50, and .90 quantiles of the failure-time distribution, but we have asked for .05, .5, and .95. As with the sample size tool, SPLIDA will remember the most recently created ADDT test plan and planning value objects, if they were created in the current S-PLUS session. Then clicking on Apply will produce the results in Table 27.

Figure 174 Dialog box to obtain a summary and evaluation of a given ADDT plan for a given set of planning values.

```
With a failure definition of 40 Newtons,
the large sample approximate standard deviation
of the 0.1  quantile at 25. DegreesC  = 442.3 Weeks and the
corresponding 95% confidence precision factor of R= 1.8

With a failure definition of 40 Newtons,
the large sample approximate standard deviation
of the 0.5  quantile at 25. DegreesC  = 851.3 Weeks and the
corresponding 95% confidence precision factor of R= 1.861

With a failure definition of 40 Newtons,
the large sample approximate standard deviation
of the 0.9  quantile at 25. DegreesC  = 1396 Weeks and the
corresponding 95% confidence precision factor of R= 1.902


Quantile ML estimate precision information
 Quantile  Weeks     Ase R-Factor
     0.1 1474.9  442.31   1.8000
```

```
0.5 2685.4  851.28   1.8614
0.9 4255.8 1396.23   1.9022
```

Table 27 Summary of the large-sample approximate standard errors and approximate precision factors for the modified test plan.

17.6 Simulating an ADDT test plan

The sample size and evaluation tools described in Sections 17.4 and 17.5, respectively, are useful for doing initial specification and evaluation of a proposed ADDT plan. It is useful, however, to supplement these tools by further evaluation of a test plan with simulation.

Using **SPLIDA ► Plan a destructive degradation (ADDT) test ► Simulate an ADDT test plan** will bring up the dialog box in Figure 175. In this case, because SPLIDA remembered the most recent planning values object and test plan object, and because we entered the degradation-defining level and the use conditions into the planning values object, there is no addition required information that needs to be specified. In the Output options part of the dialog, one can choose the number of simulations to run. Because the simulations are computationally intensive, the number of simulations used in an initial evaluation should usually be small (20 is the default). For an accurate evaluation of test plan properties, at least 200 to 400 simulations should be used. It is also possible to request to see the results of the analyses for some of the individual simulated samples, and looking at four or five of these is useful.

Figure 175 Dialog box to request a simulation of an ADDT plan.

After clicking on Apply, the simulations will begin, with information being printed to indicate progress, including the amount of memory being used by S-PLUS (some versions of S-PLUS will require additional memory for each iteration and it is useful to

monitor this "buildup"). Figure 176 is an example of the graphical output for the analysis of one of the simulated samples. In some cases, after viewing several of the plots like Figure 176, it is useful to use the Modify plot axes page of the dialog in Figure 175 to "lock in" the minimum and maximum values of the y-axis of these plots. This makes it easier to compare visually the sample-to-sample variability in the estimates of the degradation model.
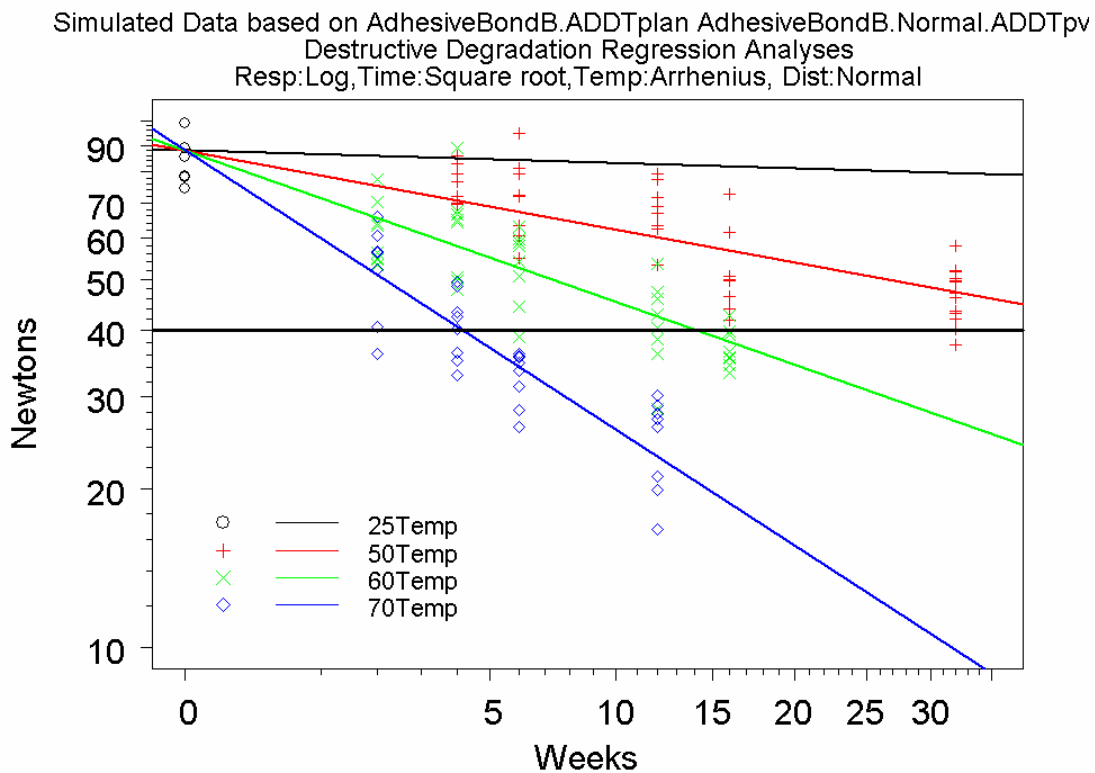
Figure 176 Detailed view of one of the simulated ADDT data sets and ML estimates.

After the final simulation, SPLIDA will make a simple summary plot like the one in Figure 177 of the ML estimates of the .05 quantile of the failure-time distribution as a function of temperature. The longer dark solid line is the .05 quantile line implied by the model and the planning values. The first quantile listed in the Figure 175 dialog is used as the quantile to be plotted in this summary plot. The shorter, lighter lines are the sample estimates of this quantile line. This plot provides an initial evaluation of the precision of estimates from the proposed ADDT plan and allows one to visualize the approximate width of confidence intervals for this quantile as a function of the explanatory variable (temperature in Figure 177).

The simulation results also are saved in an object (the name of the object combines the names of the planning values object and the test plan object, but can be modified in the box Save results in of the dialog box shown in Figure 175). Then, using this object, other evaluations of the results of the simulation (e.g., focused on particular quantities of interest) can be done as described in the next section. In addition, the last simulated data

set is always saved as the degradation data object last.sim.ADDT.ddd. When in the test planning stage, it is sometimes useful to do a more complete analysis of such a data set, using the data analysis tools described in Section 16 to get a better feeling for the characteristics of the results that might follow from a particular experimental program.

Accelerated destructive degradation test simulation based on
AdhesiveBondB.ADDTplan AdhesiveBondB.Normal.ADDTpv
Failure time 0.05 quantile vs DegreesC for failure definition 40 Newtons
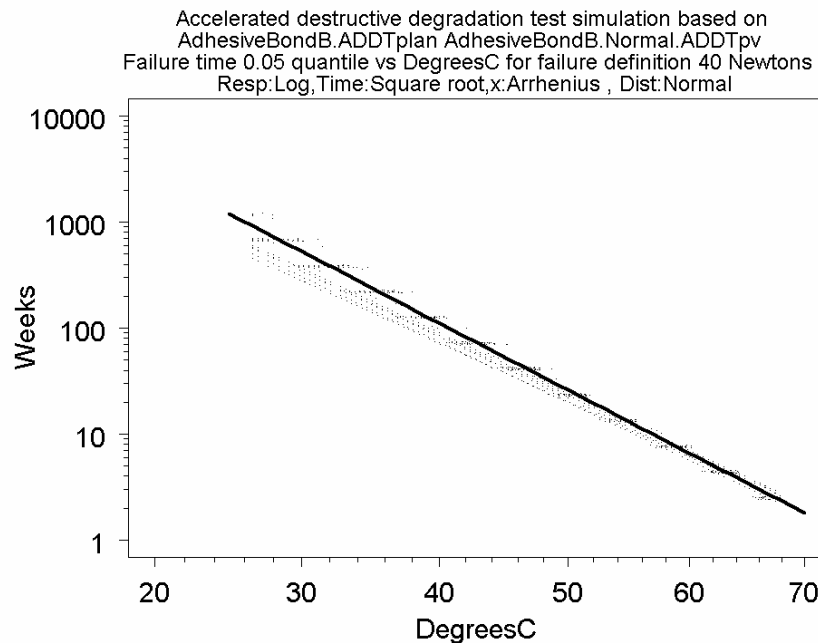Resp:Log,Time:Square root,x:Arrhenius , Dist:Normal

Figure 177 Initial default summary of an ADDT plan simulation showing the true and estimates of the .05 quantile from simulated accelerated tests, as a function of temperatures.

17.7 Summarization of ADDT test plan simulation results

The results of the simulations described in Section 17.6 are saved in a destructive degradation simulation results object. The results of the simulation can then be used to generate various summaries that focus on particular quantities of interest (especially characteristics of the failure-time distribution of the product or material under study). Using **SPLIDA ► Plan a destructive degradation (ADDT) test ► Summarize ADDT test plan simulation results** will launch a dialog box like the one in Figure 178. In this dialog, one must first choose a simulation results object. Then there are various choices for graphical outputs (and in some cases there are corresponding tabular summaries are generated). First there are options to get three overall summary plots: a particular quantile of the failure-time distribution versus an accelerating variable, degradation versus time at the use conditions for the accelerating variable, and fraction failing versus time at the use conditions for the accelerating variable. The first option is the same plot given when the simulation is actually run (e.g., Figure 177), and so is not shown again. Examples of the other two plots, based on a simulation like that used in Section 17.6, but with 200 simulation trials, are given in Figure 179 and Figure 180, respectively. It is possible to control the axes of these plots

with the controls on the Modify plot axes dialog in Figure 178, if just one plot at a time is requested. Otherwise the Modify plot axes page of the dialog is inactivated.

Figure 178 Dialog box to request additional evaluation(s) or the results of an ADDT simulation.

Accelerated destructive degradation test simulation based on
AdhesiveBondB.ADDTplan AdhesiveBondB.Normal.ADDTpv
0.5 quantile of degradation versus Weeks at 25. DegreesC
Resp:Log,Time:Square root,x:Arrhenius, Dist:Normal
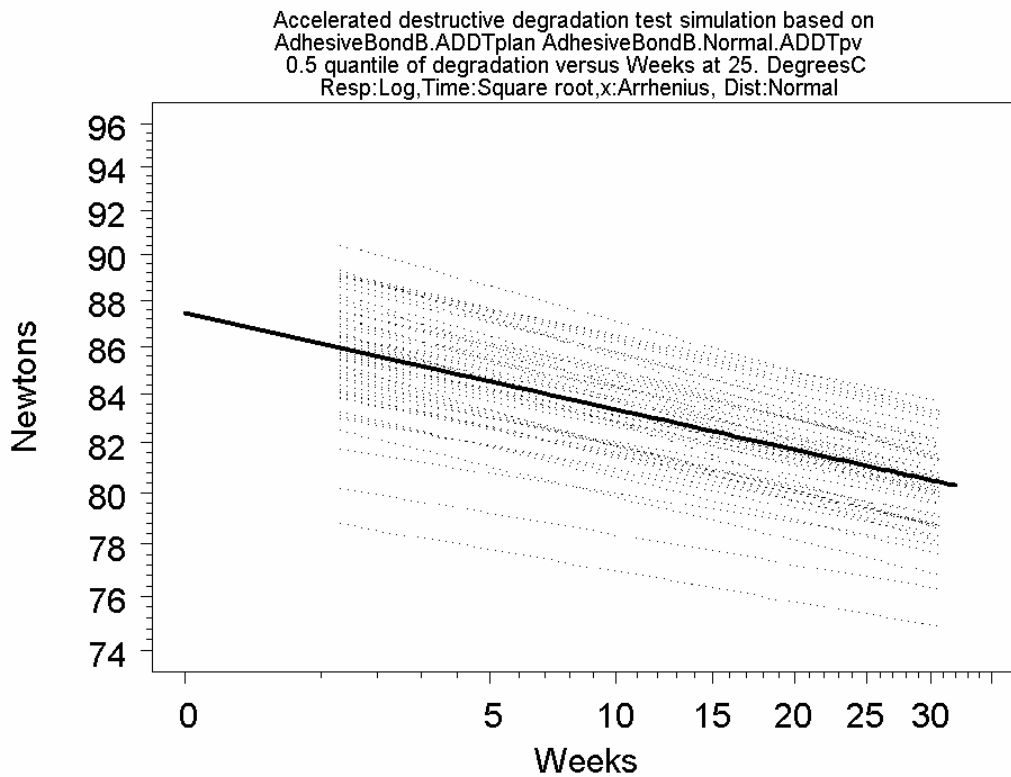
Figure 179 Simulation summarization giving 200 estimates of the relationship between strength (in Newtons) versus time (Weeks) at 25 degrees C.

Accelerated destructive degradation test simulation based on
AdhesiveBondB.ADDTplan AdhesiveBondB.Normal.ADDTpv
Fraction failing versus Weeks for 40 Newtons at 25. DegreesC
Resp:Log,Time:Square root,x:Arrhenius, Dist:Normal
Lognormal Probability Plot
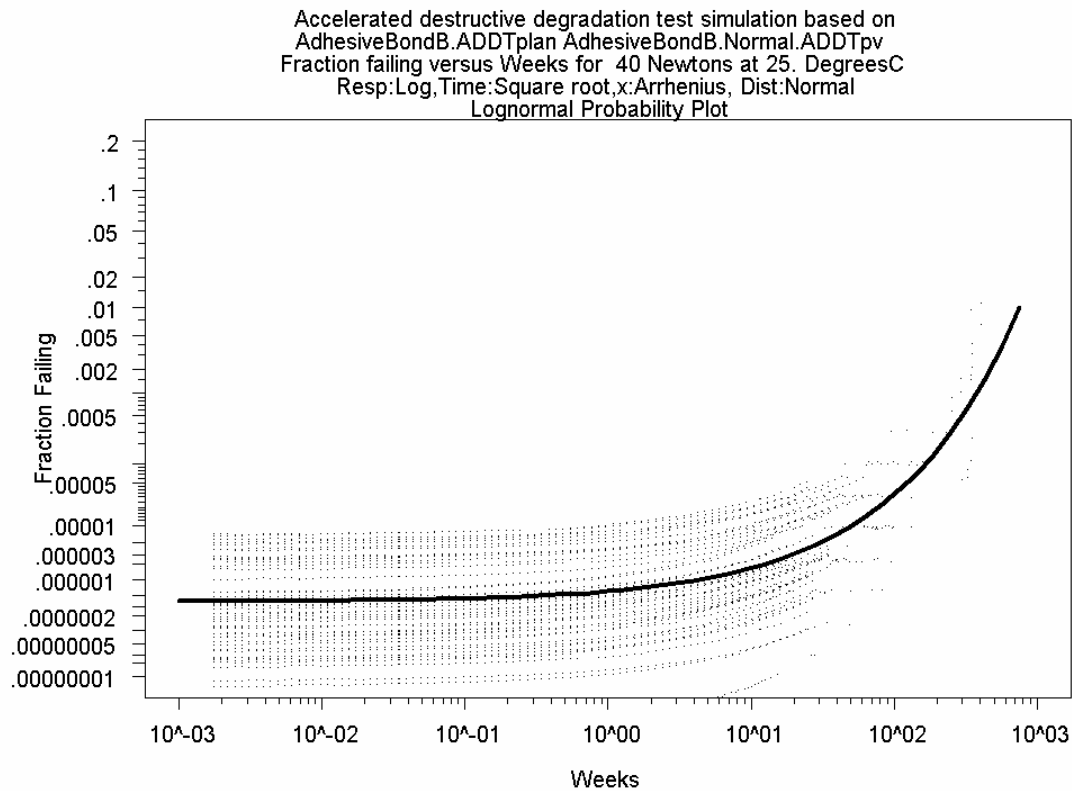
Figure 180 Simulation summarization giving 200 estimates of fraction failing versus time (Weeks) at 25 degrees C.

In addition to these summary plots, it is possible to request plots of the joint and/or marginal distributions of particular "focus quantities." Choose the "Focus quantities" page. Two different focus quantities can be chosen. The particular focus quantities can be the basic parameters of the model, a particular quantile of the failure-time distribution (at specified use conditions) or a probability of failure (for a specified time and use conditions). Figure 182, for example, shows the joint distribution and marginal distributions of the .05 quantile of the failure-time distribution at use conditions (25 degrees C) and for a failure definition of 40 Newtons of strength. As with the simple summary plots described in Section 17.6, these summary plots allow one to visualize the repeated-sampling variability in estimates of quantities of interest and to approximately gauge the width of confidence intervals.

Figure 181 Focus quantities page of the Summarize ADDT simulation result dialog box
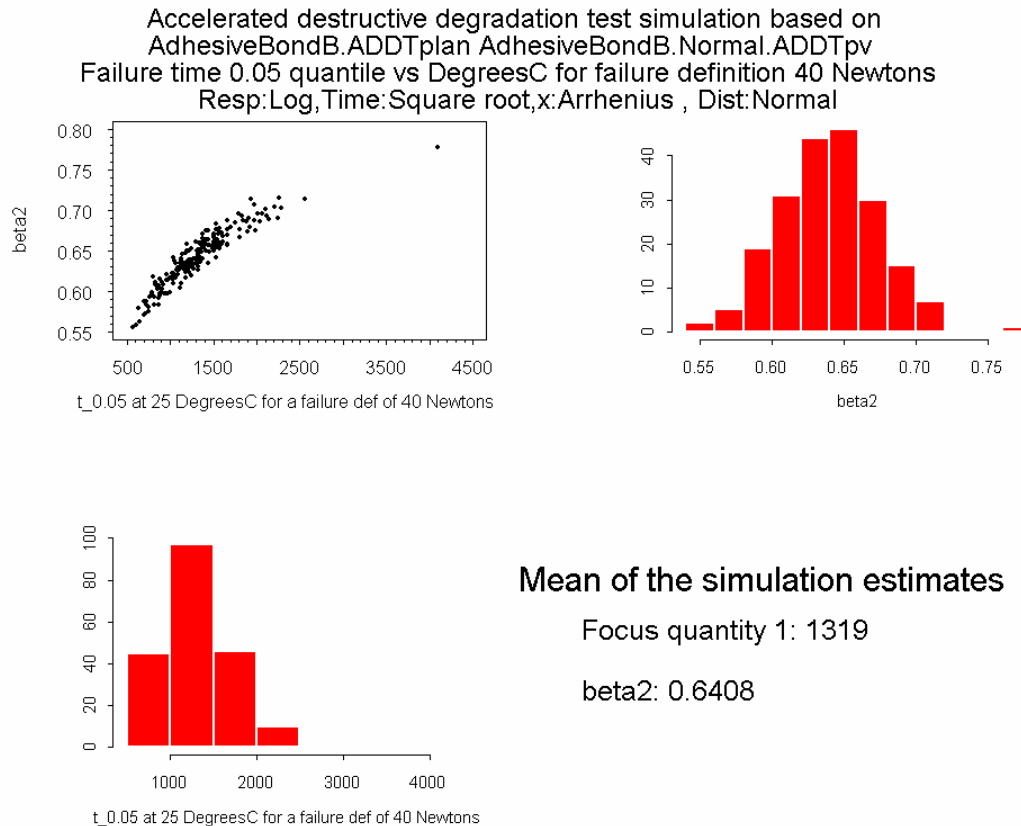
Figure 182 Simulation summarization giving the joint and marginal distributions of 200 estimates of the .05 quantile of the failure-time distribution at the use conditions (25 degrees C) for the failure definition of 40 Newtons and the degradation path acceleration parameter β₂.

17.8 Planning an ADDT study with two accelerating variables

This section shows how to use SPLIDA to plan a test for ADDT data with two accelerating variables. Because of the similarity with the test planning methods for a single variable ADDT data described in Sections 17.1-17.7, we present an example and focus on concepts and tools that differ from those used in the single-variable planning.

The Test Plan Problem and Preliminary Information

We evaluate, perturb, and simulate a test plan for a temperature-humidity experiment to assess the reliability of an adhesive strength. This experiment is similar to the experiment analyzed in Section 16.6 and the following planning information is available. Use the log transformation for the response and the square root transformation for the time. The transformed data can be reasonable modeled with the normal distribution. The logarithm of the degradation rates can be modeled with the [Arrhenius, Humidity (logit)]

relationship. The failure definition is an adhesive strength below 25 Newtons. There are 216 units for testing, the test can last up to 32 weeks, but there is interest in evaluating a shorter 16 weeks long test.

Specifying ADDT test planning information

To specify the planning information, use **SPLIDA ► Plan a destructive degradation test (ADDT) ► Specify ADDT test planning information (planning values).** Choose normal for the distribution, Log for the "Transformation for the response," and Square root for " Transformation for time." Type in 2 for the "Number of accelerating variables," Newtons for "Response units," 25 for "Degradation level defining failure," and 25,50 for "Use condition levels." Edit the results object name in "Save results in" to read AdhesiveBondD.Normal.ADDTpv. Click on "Relationships" and choose Arrhenius for the "Relationship 1" and RH for "Relationship 2." Select the "Parameters" dialog page and type in the parameter planning values as shown in Figure 183.

Figure 183 Parameters page of the "Specify ADDT model information (planning values)" dialog box.

In Figure 183, the entry "ADDT beta2 values" refers to model parameters (β_2, β_3). The planning values for beta0, beta1, beta2, and sigma are obtained directly from Table 25. Similar to Section 17.1, we use as planning values, υ^W, the slope of the log degradation rate at (50 DegreesC, 80% RH) rather than the planning value for β_1. In this case,

$$\upsilon^W = \beta_1^W \exp(\beta_2^W x_1 + \beta_3^W x_2) \text{ with } x_1 = -11605/(\text{DegreesC} + 273.15), \text{ and } x_2 = \log[\text{RH}/(100 - \text{RH})]$$

where RH is expressed as a percent. For the example, the planning value for the slope at 50 degrees C and 80% RH is equal to `-2.0245e+009*exp(-`
`.6921*35.912+.2105*1.3863) = -0.0435`.

Generating an ADDT test plan

To generate a test plan with duration of 32 weeks, use **SPLIDA ► Plan a destructive degradation (ADDT) test ► Generate an ADDT test plan.** Specify DegreesC and RH for the "Accelerating variable name(s)." Click on "Choose levels," enter 50, 60, 70 for "DegreesC" levels and click on " OK." Then enter 20, 80 for RH levels and click on " OK" again. Specify Weeks for "Time units." Enter 0, 1, 2, 4, 6, 8, 12, 16, and 32 for "Inspection times." Enter 4 for "Number of units per time/level combination." Edit the name of the object under "Save results in" to read AdhesiveBondDLong.ADDTplan.   Click on "Apply."   Then SPLIDA generates the plan given in Table 28; observe that the test is 32 weeks long and it has 216 observations.

	Weeks0	Weeks1	Weeks2	Weeks4	Weeks6	Weeks8	Weeks12	Weeks16	Weeks32
50DegreesC;20RH	4	4	4	4	4	4	4	4	4
60DegreesC;20RH	4	4	4	4	4	4	4	4	4
70DegreesC;20RH	4	4	4	4	4	4	4	4	4
50DegreesC;80RH	4	4	4	4	4	4	4	4	4
60DegreesC;80RH	4	4	4	4	4	4	4	4	4
70DegreesC;80RH	4	4	4	4	4	4	4	4	4

Table 28 A 32 week long ADDT test plan for the temperature-humidity study.

Modification, sample size computations, and evaluation of the test plan.

 After the ADDT test planning information AdhesiveBondD.Normal.ADDTpv and the test plan AdhesiveBondDLong.ADDTplan for the two accelerating variable have been defined, the modification, sample size computations, and evaluation of the test situation are very similar to one variable procedures described in Sections 17.3 to 17.5. For example,
To modify the test plan above such that the new test plan is only 16 weeks long with 4 observations at each factor level combination, we proceed as in Section 17.3. Use **SPLIDA ► Plan a destructive degradation (ADDT) test ► Modify an ADDT test plan** to open the "Modify accelerated destructive degradation test plan" dialog box. Then select AdhesiveDLong.ADDTplan for the "Choose test plan" and edit the name in "Save results in" to read something like AdhesiveDShort.ADDTplan. Go to the "Time columns" window and unselect "Week32." The dialog box should look like Figure 184. Now click on Apply. SPLIDA generates the modified plan given in Table

29. One can also edit the plan to change the allocations to the different factor level combinations, the editing process is exactly as shown in Section 17.3.
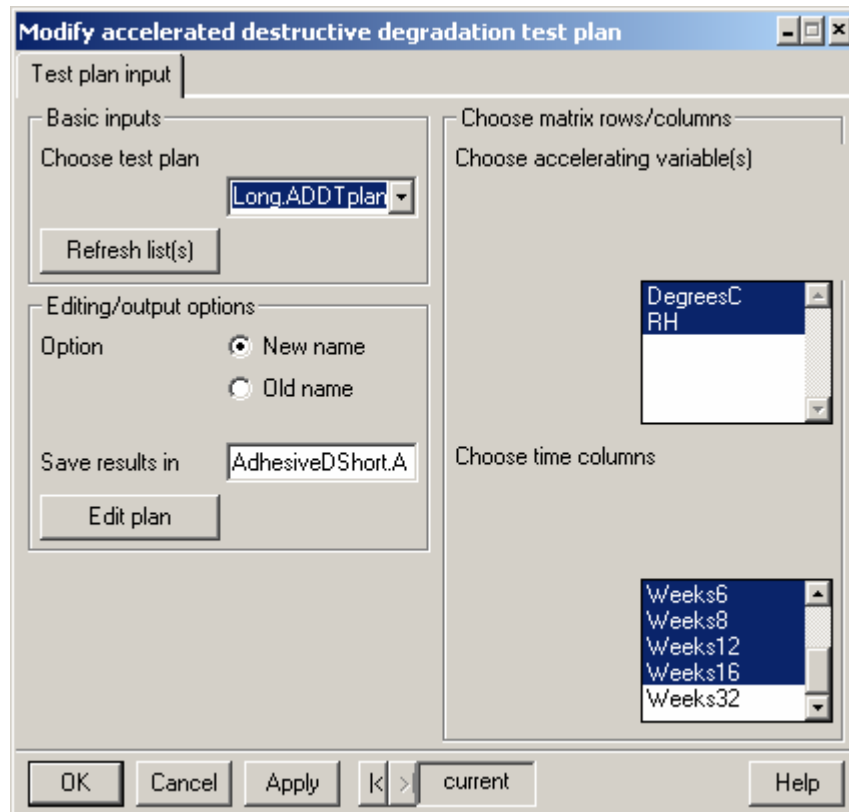
Figure 184 ADDT dialog to modify the 32 weeks long temperature-humidity ADDT plan

```
               Weeks0 Weeks1 Weeks2 Weeks4 Weeks6 Weeks8 Weeks12 Weeks16
50DegreesC;20RH     4      4      4      4      4      4       4       4
60DegreesC;20RH     4      4      4      4      4      4       4       4
70DegreesC;20RH     4      4      4      4      4      4       4       4
50DegreesC;80RH     4      4      4      4      4      4       4       4
60DegreesC;80RH     4      4      4      4      4      4       4       4
70DegreesC;80RH     4      4      4      4      4      4       4       4
```

Table 29 A 16 weeks long ADDT temperature-humidity study.

Simulation of the test plan.

To simulate and evaluate the specified 32-week test plan , use **SPLIDA ► Plan a destructive degradation (ADDT) test ► Simulate an ADDT test plan.** SPLIDA remembers the most recent "Plan values object" and "Test plan object" if they were created in the current S-PLUS session. Make sure that those object names agree

with the names you assigned in previous steps. For this example, they are
AdhesiveBondD.Normal.ADDTpv and AdhesiveBondDLong.ADDTplan, respectively.
Change "Number of simulations" from the default value 20 to 200 and "View detail for
how many samples" to 0. SPLIDA keeps the results of the simulation in the object listed
under "Save results in," keep track of this name for future reference, but edit the object
name if you prefer a different name. Then click Apply. SPLIDA generates a figure (not
shown here) similar to Figure 177.
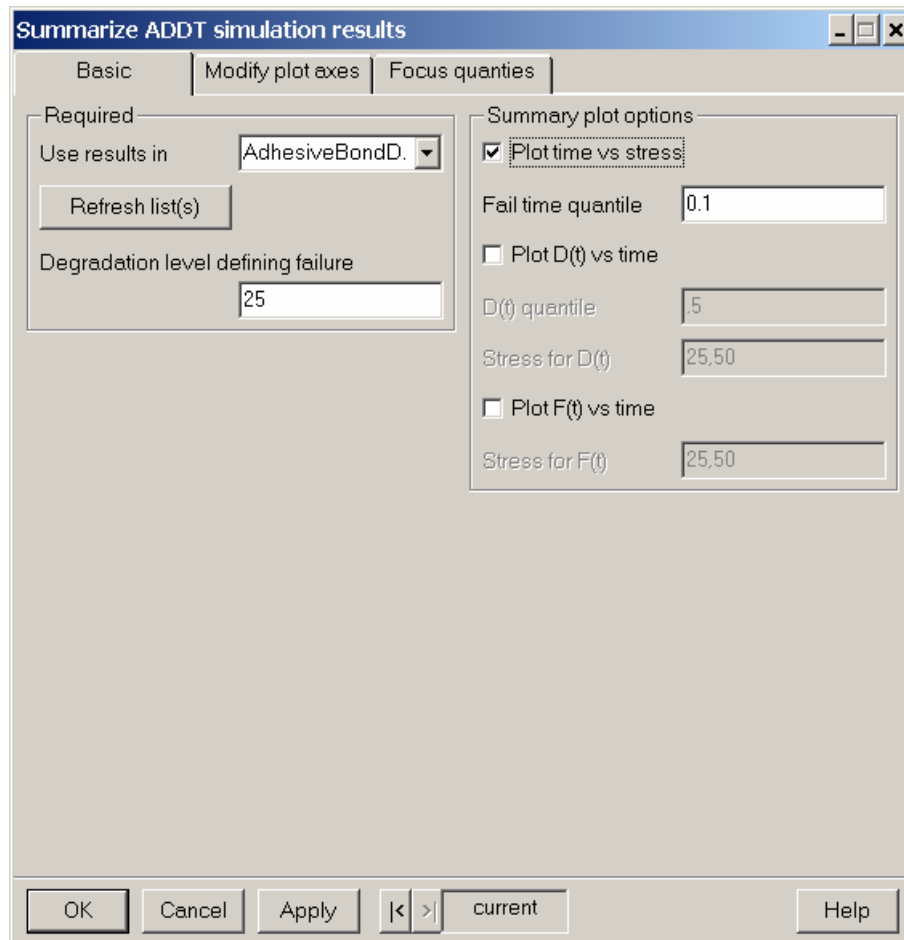
Figure 185 Dialog box to request the conditional plots of life versus accelerating variables.

Now we use the simulation results to evaluate the test plan. Use **SPLIDA ► Plan a destructive degradation (ADDT) test ► Summarize an ADDT plan simulation.** Verify that that the name under "Use results in" agrees with the name given to the simulation results in a previous step. Check the "Plot time vs stress" plot option. Click on Apply and SPLIDA generates the conditional plots given in Figure 186 and Figure 187. Figure 186 is the conditional plot of life, the .10 quantile, as function of DegreesC with RH fixed at 50% according to the use conditions. Similarly, Figure 187 is

the conditional plot of life versus RH when temperature is fixed at the use condition of 25 Degrees. These use conditions were specified as inputs for the simulation through the plan values object AdhesiveBondD.Normal.ADDTpv
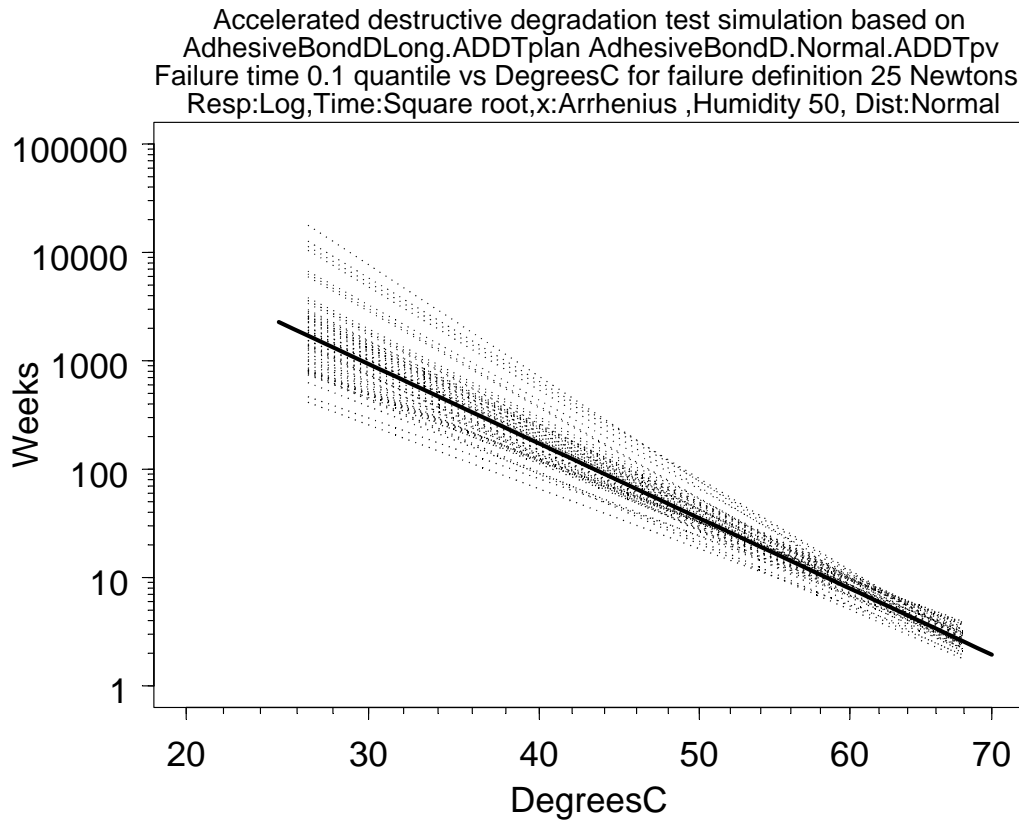

Figure 186 Conditional plot of life versus DegreesC when RH is fixed at 50%
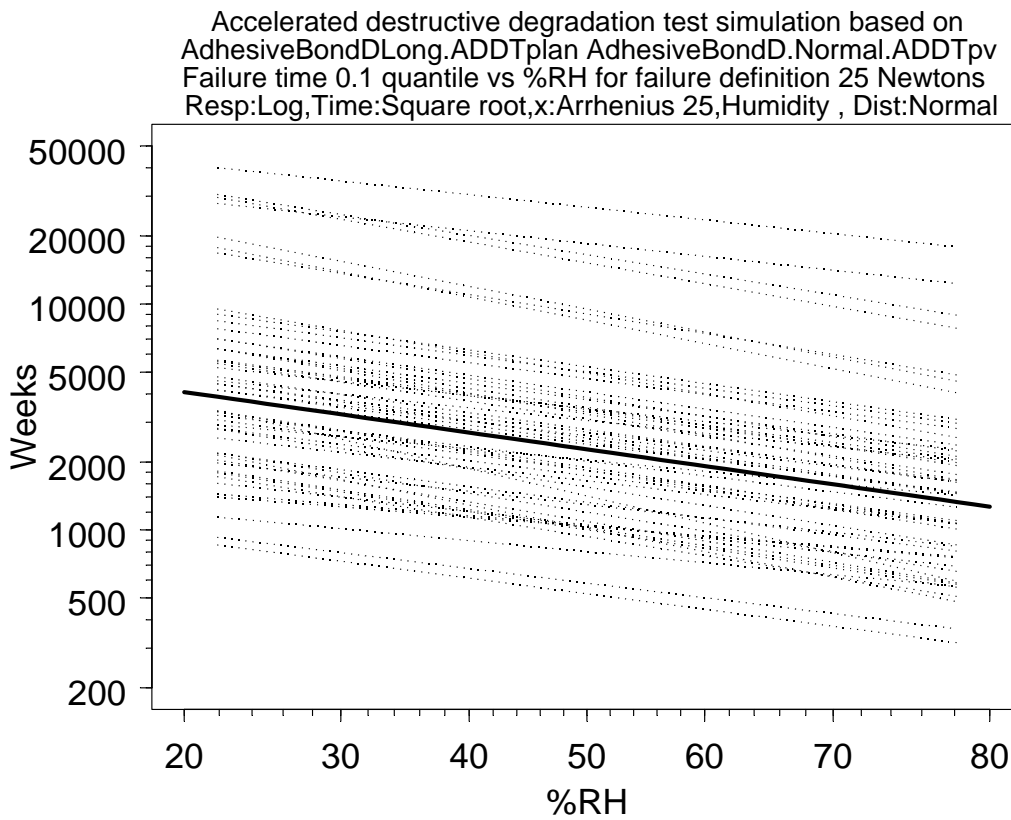
Figure 187 Conditional plot of life versus RH versus DegreesC is fixed at 25.

Additional simulation summaries for quantiles, failure probabilities, and model parameters estimates can be obtained using the "Focus quantities" page of the "Summarize ADDT simulation results" dialog box. The steps to request those summaries are very similar to the ones described in Figure 181 of Section 17.7 and we don't show them here.

18 Special Models

SPLIDA contains a collection of functions that were developed to allow a user who knows SPLUS programming to write additional S-PLUS functions that will allow one to relatively easily fit special models to data. Dialogs have been developed to provide an interface to some of these models. Some other models are available only at the command line. Examples of these models can be found in the files echapter11.q and echapter11a.qof the SPLIDA folder.

18.1 Generalized gamma distribution

SPLIDA ▶ Special models ▶ Generalized gamma distribution launches the dialog in Figure 188. Presently, when launching this dialog, you will get a warning

message. This is because there is some change, especially with data with heavy censoring, that the likelihood will not be well behaved, and that there might be some problems with the algorithm that is trying to maximize the likelihood. Also, asking to do the profile plot for the lambda parameter will take a large amount of computer time.
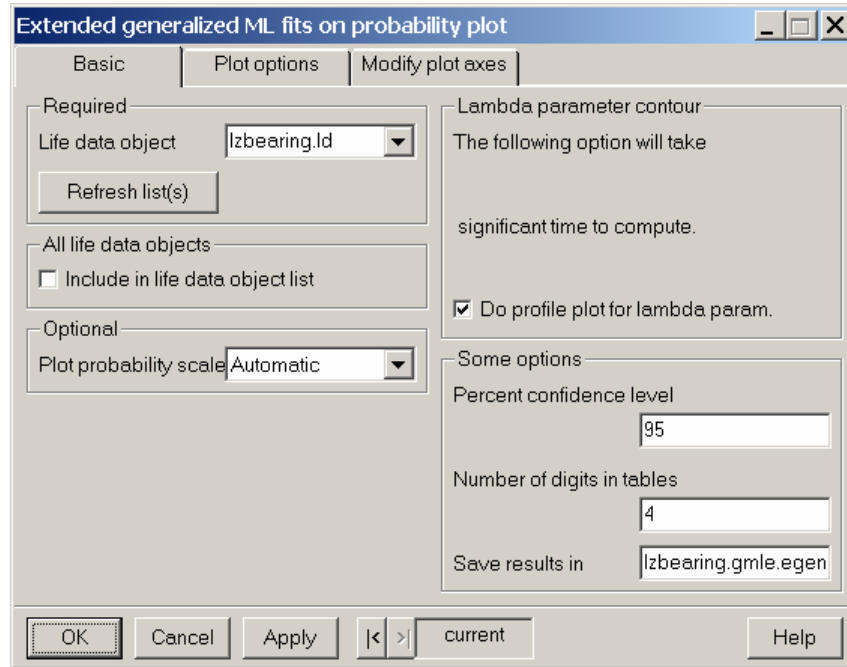
Figure 188 Dialog for fitting the generalized gamma distribution

Figure 189 is a Weibull probability plot showing the generalized gamma MLE on Weibull probability paper and Figure 190 shows the profile plot for the λ parameter. Table 30 shows a summary of the ML estimates, because the confidence interval for λ contains the values 0 and 1, the data are consistent with both the Weibull and the lognormal distribution.
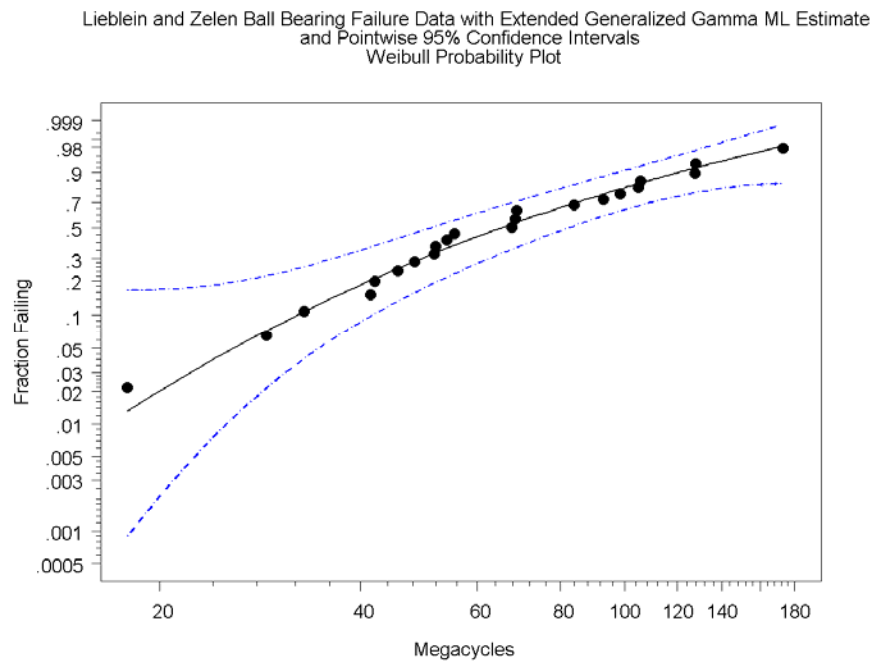
Figure 189 Weibull probability plot showing the ML estimate of the generalized gamma distribution fit to the Ball Bearing data.
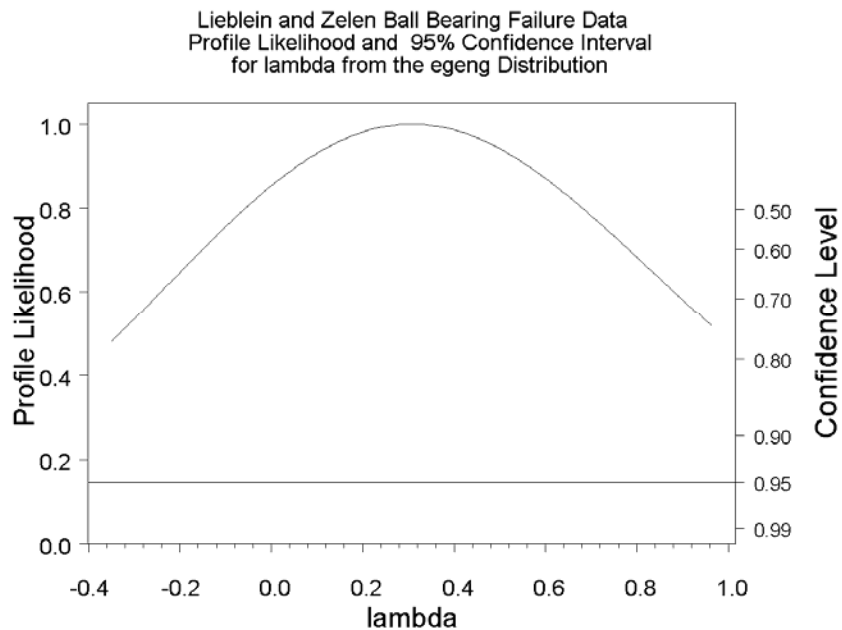
Figure 190 Profile plot of the generalized gamma distribution lambda parameter.

```
Lieblein and Zelen Ball Bearing Failure Data
Response units: Megacycles
Extended Generalized Gamma Distribution
Maximum likelihood estimation results

Appears to have converged; relative function convergence

Log likelihood at maximum point: -17.51

             MLE Std.Err. 95% Lower 95% Upper
    mu 4.2301  0.17705    3.8831    4.5771
 sigma 0.5100  0.07934    0.3545    0.6655
lambda 0.3076  0.54861   -0.7676    1.3829
```

Table 30 Summary of the ML of the generalized gamma to the Ball Bearing data.

18.2   Fit other special distributions

SPLIDA ▶ Special models ▶ Generalized gamma distribution
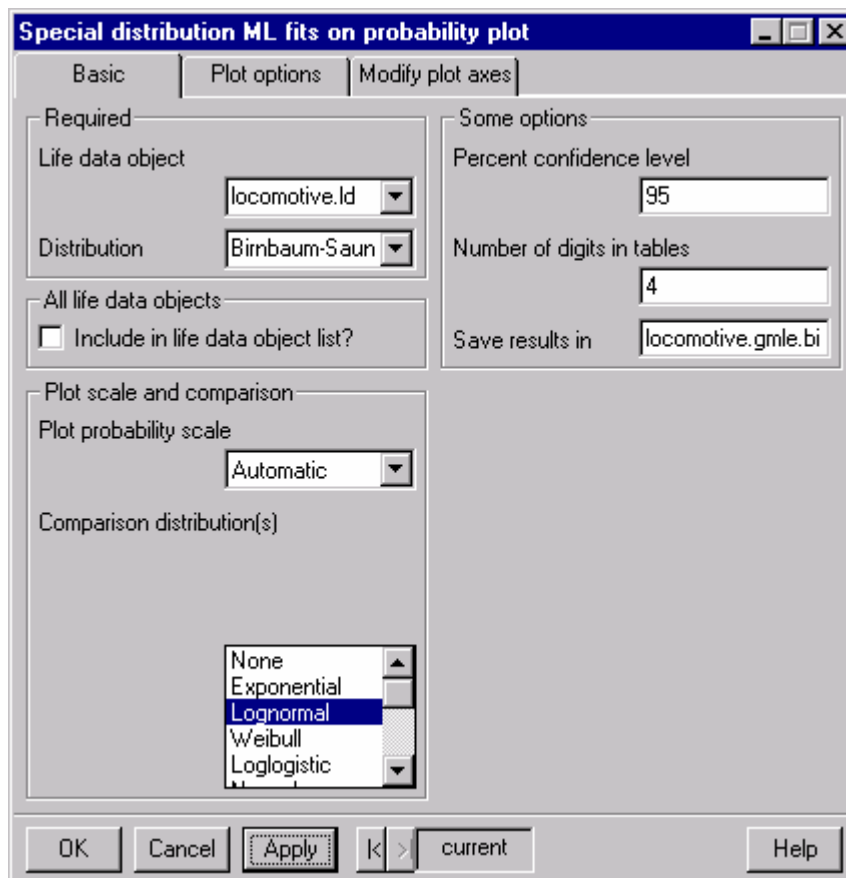
Figure 191 Dialog for requesting a comparison between the Birnbaum-Saunders and lognormal distributions on lognormal probability paper.
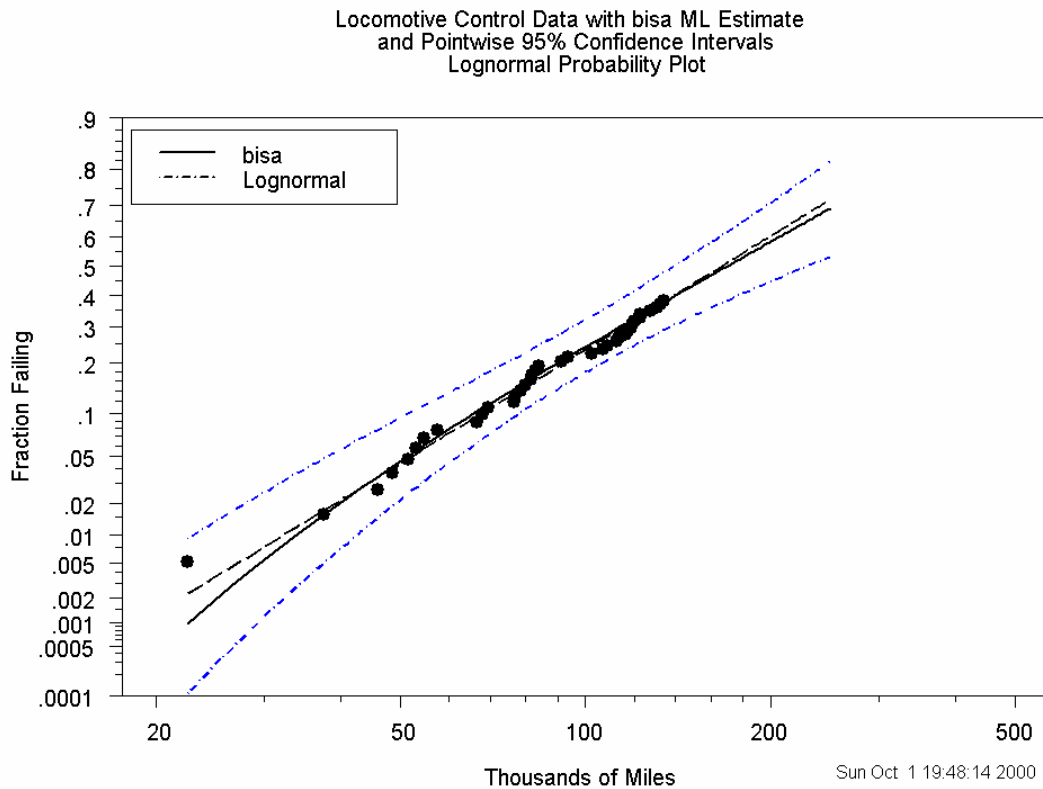
Figure 192 Comparison of the ML estimates for the Birnbaum-Saunders and lognormal distributions on lognormal probability paper.

18.3 Limited failure population model

SPLIDA ► Special models ► Limited failure population model

(not yet available in theGUI)

18.4 Generalized (two failure modes) limited failure population model

SPLIDA ► Special models ► Generalized (two failure modes) limited failure population model

(not yet available in the GUI)

18.5 Random fatigue limit model

SPLIDA ► Special models ► Random fatigue limit model

(not yet available in the GUI)

18.6 Two-dimensional warranty bivariate model

SPLIDA ► Special models ► Two-dimensional warranty bivariate model

(not yet available in the GUI)

19 SPLIDA Tools

This section describes some special tools that are available in SPLIDA.

19.1 Delete SPLIDA objects

As one uses SPLIDA, objects of different kinds are saved in the users workspace (folder SplidaUser, by default). These include data objects, analysis results objects, simulation results objects, etc. The amount of time it takes to launch or refresh some dialogs is proportional to the number of objects (because the list of objects must be searched to create lists for appropriate objects in the dialogs). Thus, especially if one notices bothersome slowdown, it is a good idea to delete unneeded objects from time to time. Although the S-PLUS object browser might be used for this purpose, using **SPLIDA ► Special tools ► Delete SPLIDA objects** will launch the dialog in Figure 193, allowing the user to choose the object type. This provides a listing of all objects of that type, from which the user can choose the particular object or objectss to delete.
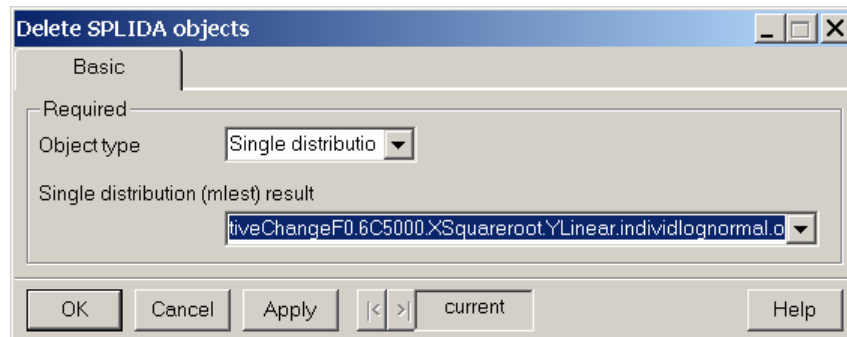
Figure 193 Dialog to allow deletion of selected SPLIDA objects

19.2 Custom blank probability/relationship paper

SPLIDA has powerful tools for creating special graphics that are useful in reliability data analysis. These plots have special nonlinear axes. These include probability plots with axes that linearize the cumulative distribution function, and relationship or model plots that linearize the relationship between a response (e.g. time to failure) and an explanatory variable (like temperature). SPLIDA allows the under to create blank plotting paper with these axes. These are useful for making plotting paper on which data can be plotted by hand. Alternatively, on could create a blank plotting page and then use S-PLUS commands to add additional information (e.g. plotted points) to the plot. Using **SPLIDA ► Special tools ► Custom probability paper** will launch a dialog like that in Figure 194.

Figure 194 Dialog to request a blank Weibull probability plot.

Several options, including requesting a grid (useful for plotting by hand or reading off numbers from a probability plot) and changes in the range of the axes have been exercised. Clicking on `Apply` results in the plot in Figure 195.
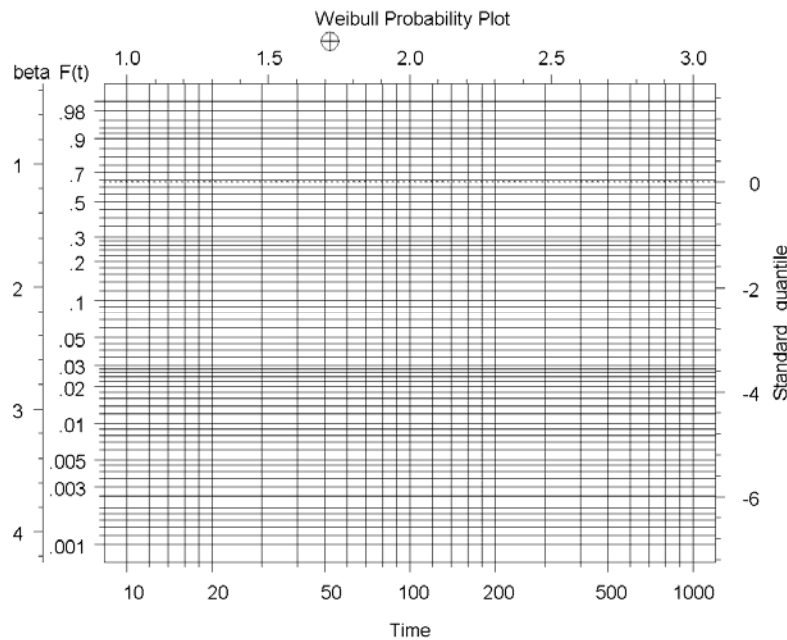
Figure 195 Custom blank Weibull probability paper

20 SPLIDA Setup and Operation

SPLIDA is distributed as a self-extracting executable file SPLIDA.exe. When SPLIDA.exe is executed, a Winzip window like the one in Figure 196 appears and when the unzip option is chosen, Winzip creates a folder (`c:\Splida` by default) that contains other folders and files. This is known as the "`SPLIDA` folder." A snapshot of the inside of this folder is given in Figure 197.
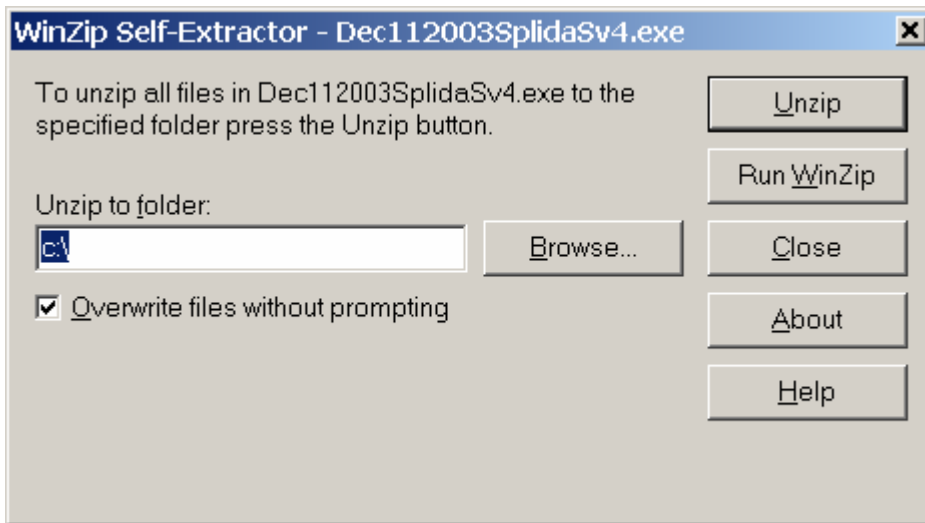
Figure 196 Winzip window to extract SPLIDA
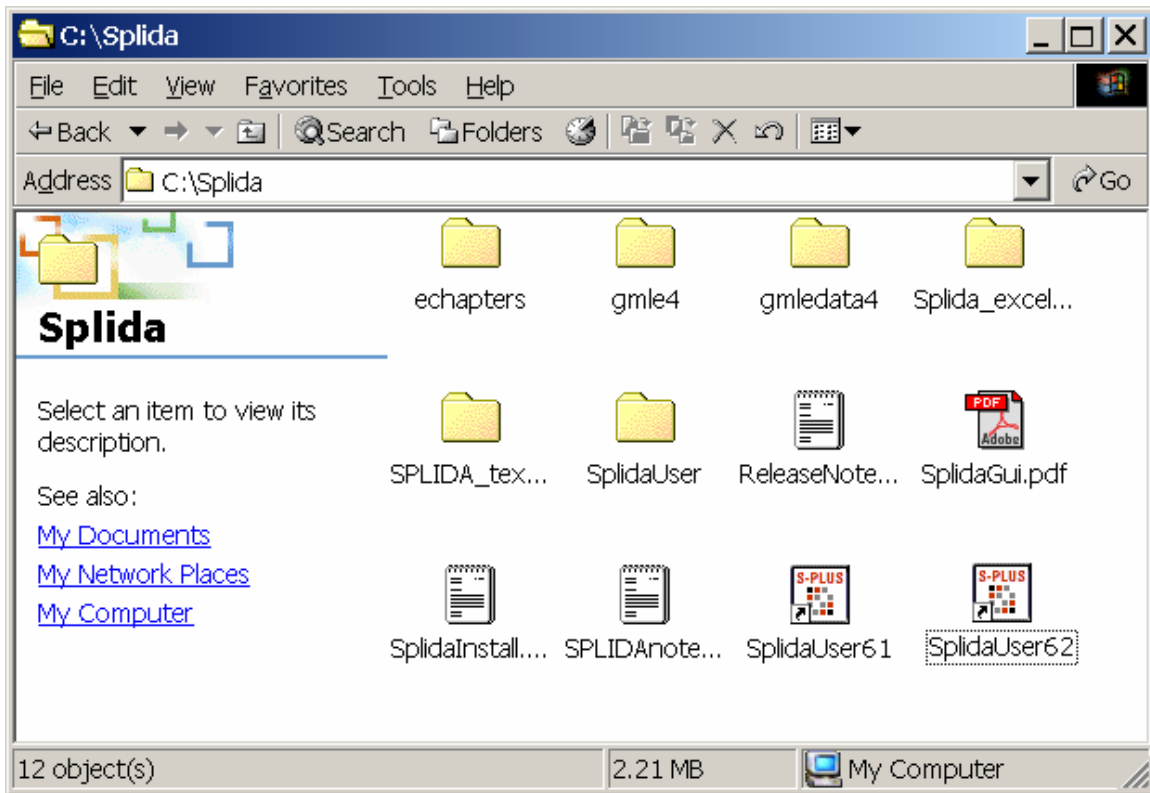
Figure 197 View inside the SPLIDA folder

20.1 Simple setup for SPLIDA

To install a new version of SPLIDA over an existing version, you should *always* delete the old **SPLIDA** folder before unzipping the new SPLIDA folder. Generally, you should not have to make any manual changes in any **SplidaUser** folder (some automatic changes are made when a new version of SPLIDA is installed).

The following description assumes that you are running Splus 6.2 and that it is installed in the default location **C:\Program Files\Insightful\splus62\cmd**. If you are using some other version if Splus (6.0 or higher), the approach is similar, but you should use the alternate shortcut.

To install SPLIDA using default locations (the simple recommended setup):

1. Move the SPLIDA.exe file to some location (like your Desktop) and double-click to execute the file. Do not change any defaults, but click on "Unzip" (see Figure 196) to make the c:\Splida folder shown in Figure 197. You should get a message saying that xxx files have been unzipped (where xxx is 147 for the December 2003 version of SPLIDA).

2. Make a *copy* of the **SPLIDAUser62** shortcut in a convenient place (e.g., the desktop).

3. Make a *copy* of the SPLIDAUser *folder* to `C:\SPLIDAUser`. This is the place where user-created objects will be stored.

4. To complete the setup, click on the **SplidaUser** shortcut. This will bring up the warning in Figure 198. Click "OK." S-PLUS will create a `.Prefs` folder in the SplidaUser folder and SPLIDA will create the SPLIDA menu items (this could take a few minutes).
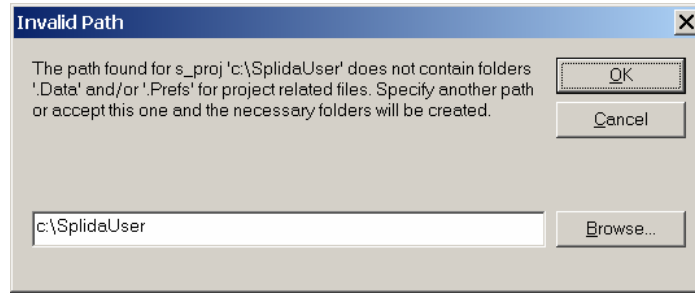
Figure 198 S-PLUS Invalid path warning

When done, SPLIDA will be one of the items on the S-PLUS menu bar. To preserve the setup, exit S-PLUS and then again, click on the **SplidaUser** shortcut.

5. SPLIDA is now ready for use. To test SPLIDA, from the S-PLUS menu bar use **Splida ▶ Single distribution analysis ▶ Probability plot with parametric ML fit**. Then choose life data object BearingCage.ld and distribution lognormal. Click "Apply."

It may be desirable (on Windows 2000 and XP, where it is possible) to install SPLIDA in a write-protected folder to reduce any chance that files in the installed folder will become corrupted.

20.2 Installing SplidaUser folders in other places or with other names

Some S-PLUS users like to use separate startup folders for different projects, so that created objects do not get confused from one project to another. Similarly, if different users work on the same computer usually they would want to operate using separate S-PLUS/SPLIDA startup files. If you copy a **SplidaUser** folder to other than **C:** or use a name other than **SplidaUser** for the folder, then the **SplidaUsr62** shortcut properties must point to both the location of S-PLUS and the **SplidaUser** folder

a. Make a *copy* of the **SplidaUser** *folder* (do not just move it) in a convenient place (e.g., in c:\ or in a project folder elsewhere) and, if desired, give the folder a new name, say corresponding to a user's name or a project name.

b. Make a *copy* of the **SplidaUsr62** shortcut (do not just move it) in a convenient place (e.g., the desktop) and, if desired, give the shortcut a new name.

c. Right-click on the shortcut, choose properties, and go to the shortcut tab. On the right-hand side of the target line, edit the S_PROJ="**C:\SplidaUser**" so that it points to the location of your particular **SplidaUser** folder (or what ever name the folder now has) and its location.

20.3 Installing Splida in another place

In some cases, it may be necessary to install the **Splida** folder (as it is called now) in a place other than C:\. This can be done by changing the Winzip option to some other location (e.g., **D:\Program Files\Splida**). If the **SPLIDA** folder is installed in a place other than the default **c:\SPLIDA**, the file Path.txt in any SplidaUser folder should be modified (e.g., using Notepad) to point to the chosen location of SPLIDA. Again, may be desirable to install SPLIDA in a write-protected folder to reduce any chance that files in the installed folder will become corrupted.

An alternative to using the Path.txt file to identify the location of SPLIDA is to set an environmental variable in Windows. In particular SplidaHome should be set to the SPLIDA path (e.g., **D:\Program Files\Splida**). Doing this is particularly useful if you move a **SplidaUser** folder from one computer to another and have SPLIDA located in different places on those computers.

20.4 Modifying the shortcut when S-PLUS is installed in other than the default location

If S-PLUS is installed somewhere other than the default location for S-PLUS 6.2 (i.e., **C:\Program Files\Insightful\splus62\cmd**), the **SplidaUsr62** shortcut provided with the distribution must be modified to point to the actual location of S-PLUS. Right-click on the shortcut, choose properties, and go to the shortcut tab. Edit the target line so that the first part points to S-PLUS (looking at an existing shortcut to S-PLUS will show how to do this). Make sure that the right-hand side of the target line still points to the SplidaUser folder (S_PROJ="**C:\SplidaUser**" by default).

20.5 Creating a SPLIDA/S-PLUS shortcut from scratch

 For most purposes, the **SplidaUsr62** shortcut (perhaps modified as described in Sections 20.2 or 20.4) provided with the SPLIDA distribution should be all that is

needed. For completeness, however, we describe how to create such a shortcut from scratch.

Copy or create a new shortcut to S-PLUS. Use right-click **New ► Shortcut** to bring up the Windows Shortcut Wizard. Browse to the location of SPLUS and select SPLUS.exe (C:\Program Files\Insightful\splus62\cmd\SPLUS.exe for S-PLUS 6.2. Click next and choose a name for the shortcut (e.g. SplidaProjX). Then click Finish. This creates a standard S-PLUS shortcut. Now to make it into a SPLIDA shortcut, right click on the new shortcut and choose "properties" under "shortcut." Set S_PROJ on the "Target" command line (note: do this on the "Target" and not"Start in" command line) of this shortcut to read:

" C:\Program Files\Insightful\splus62\cmd\SPLUS.exe "   S_PROJ="c:\SplidaUser"

where " C:\Program Files\Insightful\splus62\cmd\SPLUS.exe " is the existing path to S-PLUS 6.2 and depends on the version of S-PLUS and the option used when S-PLUS was installed. Do not change the first part. Only add the S_PROJ="`c:\Splida`" to the end of the line. ***Make sure there is at least one space*** before the S_PROJ="`c:\SplidaUser`". After typing the S_PROJ="`c:\SplidaUser`", click on "apply" and "Close."

20.6 Manually making or remaking the SPLIDA menu structure and startup (`.Prefs`) folder

SPLUS uses a `.Prefs` folder to define user preferences and the graphical under interface (GUI). When you attempt to use S-PLUS in a target folder that does not contain either the `.Prefs` or the _Data folder, they are generally created automatically by S-PLUS (S-PLUS first asks if you want to do this). At the time of distribution, the **SplidaUser** folder does not contain `.Prefs` folders, but it is created in the setup described in 20.1. This is because each version/release of SPLUS may have a different `.Prefs`. SPLUS will generate the `.Prefs` folder automatically, appropriate for your version of SPLUS if a `.Prefs` folder is not detected in your **SplidaUser** folder. When SPLIDA is started, it will make the necessary changes to install or update (after you install a new version of SPLIDA over an older version)..

If you are using the "professional" version of S-PLUS and, for some reason, you would like to regenerate the SPLIDA `.Prefs` folder from scratch, you can follow the following steps (you need the Professional version of S-PLUS to do this):

   a. Delete the .Prefs folder in the SPLIDA target folder.
   b. Fire up S-PLUS using a shortcut that points to the target folder (e.g., **SplidaUser**).
   c. When S-PLUS warns you that .Prefs and/or `.Data` are missing, click OK.
At the command line, give the command `CreateSplidaPrefs()`.

Depending on the speed of your processor, it may take from 1 to 3 or 4 minutes to complete and you will see something like the following output:

```
SPLIDA Version xxx
Loading advanced SPLIDA life data analysis symbols.
Advanced SPLIDA life data analysis symbols successfully dyn.loaded.
Removing the old SPLIDA menu structure.
SPLIDA menu was not detected---nothing to remove.
Remaking all of the SPLIDA GUI properties and dialogs---wait.
Finished remaking all of the SPLIDA GUI properties and dialogs.
Remaking the SPLIDA menus.
Done remaking the SPLIDA GUI.
You should exit and restart S-PLUS now, in order to preserve the changes.
```

20.7 Comments on using SPLIDA (and S-PLUS) on Windows

a. If S-PLUS seems to be hanging or taking too long to execute, you can sometimes type <ESC> to break out of a command.
b. If graphics window becomes corrupted, kill the graphics window (by clicking the x in the upper right-hand corner of the graphics window). A fresh graphics window will be started. It is also necessary to do this to make newly-selected graphics options (e.g., switching from color to black and white) take effect.
c. As you use S-PLUS over time, memory usage can build up, and it may be necessary (especially if you are only working with 128MB or less memory) to restart S-PLUS. The command check.memory( ) will allow you to monitor this. If you can afford it, install more memory. We have noticed a big improvement when going from 128MB to 512 MB, and now we use 256 MB or better. If you run short on memory and are using an old version of Windows (98 or ME), S-PLUS may hang, Windows can crash, and it may be necessary to reboot your machine.

21 Outline of the SPLIDA Menu Structure

The following table contains an outline of the SPLIDA menu structure. The GUI dialogs for the items shown in regular-faced type are still in development.

SPLIDA ►
    Make/summary/view data object ►
        Make a life data object
        Make recurrence (point-process) data object
        Make degradation (repeated measures) data object
        Summary/view a data object
        Edit a life data object
    Plan single distribution study ►
        Specify life test planning information (planning values)

Plot life test planning information (planning values)
Plot of approximate required sample size
Simulate a life test
Probability of successful demonstration
Single distribution life data analyses ►
Plot nonparametric estimate of cdf and confidence bands
Probability plot with nonparametric confidence bands
Probability plot with parametric ML fit
Compare distribution ML fits on probability plot
Likelihood contour plot and confidence intervals
Bootstrap confidence intervals
Threshold parameter probability plot with parametric ML fit
Single distribution Bayes analysis life data analyses ►
Specify a prior distribution
Make a posterior distribution
Summarize a posterior distribution
Multiple failure mode life data analysis ►
Individual modes
Combined modes (with one or more omitted)
Make life data objects for individual modes
Comparison of distributions life data analysis ►
Probability plot and ML fit: different shapes (slopes)
Comparison likelihood contour plot
Probability plot and ML fit: common shapes (slopes)
Make life data objects for individual groups
Probability of correct selection for specified test plans
Plan an accelerated life test (ALT) ►
Specify an ALT model information (planning values)
Specify an ALT test plan
Plot, summarize, and evaluate an ALT test plan
Simulate an ALT test
Simple regression (ALT) data analysis ►
Censored data scatter plot
Probability plot and ML fit for individual conditions
Prob plot and ML fit for indiv cond: common shapes (slopes)
Probability plot and ML fit of a regression (acceleration)
Make life data objects for individual groups
Multiple regression (ALT) life data analysis ►
Censored data pairs plot
Censored data scatter plot
Probability plot and ML fit for individual conditions
Prob plot and ML fit for indiv cond: common shapes (slopes)

Probability plot and ML fit of a regression (acceleration)
Conditional model plot
Sensitivity analysis plot
Make life data objects for individual groups
Regression residual analysis ►
Residuals versus fitted values
Residuals versus possible explanatory variables
Residuals versus observation order
Residual probability plot
Recurrence (point process) data analysis ►
Recurrence event plot
Mean cumulative function plot
Compare two mean cumulative functions plot
Convert renewal recurrence data to life data
Degradation (repeated measures) data analysis ►
Degradation data plot
Degradation data trellis plot
Accelerated degradation cell-average plot
Convert/extrapolate degradation data to life data
Degradation residual plot
Destructive degradation data (ADDT) analysis ►
Scatter plot of ADDT data
Scatter plot and ML fit for individual conditions
Scatter plot and ML fit of a regression (acceleration) model
Residual analysis
Plan a destructive degradation (ADDT) test ►
Specify ADDT test planning information (planning values)
Generate an ADDT test plan
Modify an ADDT test plan
Sample size for an ADDT test plan
Summarize, plot, and evaluate an ADDT test plan
Simulate an ADDT test plan
Summarize ADDT test plan simulation results

Special models ►
Generalized gamma distribution
Fit other special distributions
Limited failure population (LFP) model
Dead on arrival (DOA) population model
Generalized (two failure modes) limited failure population model
Random fatigue limit (RFL) model
Two-dimensional warranty bivariate model
Change SPLIDA default options (preferences)

d.

22 SPLIDA Data Set Index

This Chapter describes the source and, when not included in Meeker and Escobar (1998), a little background for some of the data sets supplied with SPLIDA. The first part of the file does this by data set name. The second is organized according to the numbered Examples and Exercises in Meeker and Escobar (1998, page 684), making it easy to find data sets used in that book (if the name is not obvious).

Complete references not given below (e.g. Nelson 1982) are given at the end of Meeker and Escobar (1998). Also see the "Examples" entry in the index of Meeker and Escobar (1998).

AlloyC.txt   Meeker and Escobar (1998), Example 11.16, page 276.

Appliance01.txt   Nelson (1982) page 121

BearingCage.txt   Abernethy, Breneman, Medlin, and Reinman, (1983) also Meeker and Escobar (1998), Example 10.12, 12.10, 14.1, 14.2, 14.3, 14.4, 14.5

Concrete.txt   Holmen (1979), analyzed in Castillo and Hadi (1995) and discussion of Pascual and Meeker (1999)

ConnectionStrength.txt   Nelson (1982), page 111.

DeviceA.txt   Hooper and Amster (1990) and also Meeker and Escobar (1998), Examples 12.9, 19.1, 19.2, 19.3, 19.4, 19.5, 19.6, 19.7, 19.8, 19.9

DeviceB.txt   Meeker, Escobar, and Lu (1998) also Meeker and Escobar (1998), Chapter 21

DeviceC.txt   Meeker, Unpublished ALT data use in Meeker and Escobar (1998), Exercise 19.2

DeviceG.txt   Meeker and Escobar (1998), Example 15.6, page 383

DeviceH.txt   Doganaksoy, Hahn, and Meeker (2000)

DiskBer.txt   Murray (1993) and also Meeker and Escobar (1998), Exercise 13.9

Fan.txt   Nelson (1982), pages 133, 318 and also Meeker and Escobar (1998), Examples 7.12, 7.13, 11.3, 11.17

GaAsLaser.txt   Meeker and Escobar (1998), Example 13.10

HeatExchanger.txt   Meeker and Escobar (1998), Example 3.1, 3.2, 3.3, 3.4, 3.6, 3.11, 6.4, 9.8, 9.9, 9.10, 9.11

LaminatePanel.txt   Shimokawa and Hamaguchi~(1987) and also Pascual and Meeker (1999)

LocomotiveControl.txt   Nelson (1982), page 33

MachineH.txt Meeker and Escobar (1998), Example 16.6

MetalWear.txt   Meeker and Escobar (1998), Example 21.6

NiCdBattery.txt   Brown and Mains (1979) and Meeker and Escobar (1998), Example 1.9

NewSpring.txt   Meeker, Escobar, and Zayac (2003), A factorial experiment to compare the life times of springs as a function of a processing temperatire and amount of displacement in the spring test (stroke).

PartA.txt   Unpublished Meeker (1999) An experiment to compare the life times of units tested by three different operators.

PhotoDetector.txt   Weis et al. (1986), also Meeker and Escobar (1998), Exercise 3.12

PrintedCircuitBoard.txt   Meeker and LuValle (1995), and also Meeker and Escobar (1998), Examples 1.8 and and 18.7

RepairTimes.txt   Unpublished Meeker (1997) Recorded times to repair a particular kind of electronic system, without regard to failure mode.

RocketMotor.txt   Olwell,D. H. and Sorell, A. A. (2001)

ShockAbsorber.txt   O'Connor (1985), also Examples 3.8, 3.9, 3.10, 6.3, 8.1, 8.3, 8.4, 8.5, 8.6, 8.7, 8.8, 8.10, 8.11, 8.12, 8.13, 8.14, 8.15, 9.4, 9.5, 9.6, 9.7, 9.12 from Meeker and Escobar (1998)

Snubber.txt   Nelson (1982) also Meeker and Escobar (1998) Examples

17.13, 17.14, 17.15

TurbineDevice.txt   Unpublished Meeker (1999)

ValveSeat.txt Nelson, W. (1995a) also Meeker and Escobar (1998), Examples 16.3, 16.4

WorkStation.txt   Meeker and Escobar (1998), Exercise 16.1

ZelenCap.txt   Zelen (1959) also Meeker and Escobar (1998), Example 17.11

at7987.txt   Meeker and Escobar (1998), Examples 6.2, 6.6, 6.7, 11.18

berkson10220.txt   Berkson (1966) also Meeker and Escobar (1998), Examples 7.1-7.4

berkson20.txt Berkson (1966) also Meeker and Escobar (1998), Examples 7.1-7.4

berkson200.txt   Berkson (1966) also Meeker and Escobar (1998), Examples 7.1-7.4

berkson2000.txt   Berkson (1966) also Meeker and Escobar (1998), Examples 7.1-7.4

bkfatigue10.txt   Bogdanoff and Kozin (1985) also Meeker and Escobar (1998), Example 11.4

bleed.txt   Abernethy, Breneman, Medlin, and Reinman (1983) also Meeker and Escobar (1998), Example 6.9

bulb.txt   Davis (1952) and Nash (19xx)

cirpack5.txt   Hooper and Amster (1990), Table 8.2

cirpack6.txt   Meeker and Escobar (1998), Table 1.3, Example 1.3, 11.8

comptime.txt   Meeker and Escobar (1998), Example 17.1, 17.3, 17.4, 17.5, 17.6, 17.7

cylinder.txt   Nelson and Doganaksoy (1989) also Meeker and Escobar (1998), Example 16.5

doatrun.txt   Meeker (1996), unpublished Data are truncated because an unknown number of DOA's were removed from the data set.

electro_mech.txt   Hahn and Meeker (1982a)

grampus.txt   Lee (1980) and also Ascher, H., and Feingold, H. (1984) also Meeker and Escobar (1998), Examples 16.9, 16.10, 16.11, 16.12

grids1.txt   Doganaksoy, N., and Nelson, W. (1998) also Meeker and Escobar (1998), Example 16.7

grids2.txt   Doganaksoy, N., and Nelson, W. (1998) also Meeker and Escobar (1998), Example 16.7

halfbeak.txt   Ascher, H., and Feingold, H. (1984) also Meeker and Escobar (1998), Example 16.7

icdevice02.txt Meeker and Escobar (1998), Example 19.11

inconel.txt   Shen (1994) also Pascual and Meeker (1999)

lfp1370.txt   Meeker (1987) also Meeker and Escobar (1998), Examples 3.5, 11.5, 11.6, 11.7

lzbearing.txt   Lieblein and Zelen (1956), also Lawless (1982) and Meeker and Escobar (1998), Examples 1.1, 6.8, 8.16, 11.1, 11.2, 12.2, 12.3, 12.4, 12.5, 12.6

muliple_fail_classh.txt   Nelson (1990), page 32

mylarpoly.txt Kalkanis and Rosso (1989) also Meeker and Escobar (1998), Examples 18.5, 19.10

piccioto.txt   Picciotto (1970)

piccioto_sub.txt   subset of data in piccioto.txt

Resistor.txt   Suzuki, Maki, and Yokogawa (1993) also Meeker and Escobar (1998), Example 18.2 and Table C.3

superalloy.txt Nelson (1990) also Meeker and Escobar (1998), Examples 17.2, 17.8, 17.9, 17.10

tantalum.txt   Singpurwalla, Castellino, and Goldschen (1975) also Meeker and Escobar (1998), Examples 19.11, 19.12

titanium.txt

titanium01.txt Meeker and Escobar (1998), Exercise 6.7

turbine.txt   Nelson (1982) also Meeker and Escobar (1998), Examples 1.7, 3.x

v7tube.txt   Davis (1952) also Meeker and Escobar (1998), Example 1.6, 6.10

Meeker and Escobar (1998) examples and exercises using data

Example   1.1   lzbearing.txt
Example   1.2   lfp1370.txt
Example   1.3   cirpack6.txt
Example   1.4   Fan.txt
Example   1.5   HeatExchanger.txt
Example   1.6   v7tube.txt
Example   1.7   turbine.txt
Example   1.8   PrintedCircuitBoard.txt
Example   1.9   NiCdBattery.txt
Example   1.10   AlloyA.txt
Example   3.1   HeatExchanger.txt
Example   3.2   HeatExchanger.txt
Example   3.3   HeatExchanger.txt
Example   3.4   HeatExchanger.txt
Example   3.5   lfp1370.txt
Example   3.6   HeatExchanger.txt
Example   3.7   HeatExchanger.txt
Example   3.8   ShockAbsorber.txt
Example   3.9   ShockAbsorber.txt
Example   3.10   ShockAbsorber.txt
Example   3.11   HeatExchanger.txt
Example   3.12
Example   3.13   turbine.txt
Example   6.2   at7987.txt
Example   6.3   ShockAbsorber.txt
Example   6.4   HeatExchanger.txt
Example   6.5   turbine.txt
Example   6.6   at7987.txt
Example   6.7   at7987.txt
Example   6.8   lzbearing.txt
Example   6.9   bleed.txt
Example   6.10   v7tube.txt
Example   7.3   berkson20.txt
Example   7.12   Fan.txt

Example   19.10   mylarpoly.txt
Example   19.11   icdevice02.txt
Example   19.12   icdevice02.txt
Example   20.1
Example   21.1   DeviceB.txt
Example   21.2   DeviceB.txt
Example   21.3   DeviceB.txt
Example   21.4   DeviceB.txt
Example   21.5   DeviceB.txt
Exercise 19.2   DeviceC.txt

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