Financial engineering

Introduction to financial time series

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Chapter outline

1. Financial returns and their statistical properties.

2. Empirical characteristics of financial returns.

Financial returns and their statistical properties

- **Financial time series analysis** (FTSA) gives practical and theoretical understanding of data collected on financial markets, such as stock and commodity prices, exchange rates or bond yields.

- FTS are usually a time series of prices of an asset for a given period of time. However, FTSA use to consider asset returns, which measures the relative changes in prices, as they have more attractive statistical properties.

- There are several reasons to analyze FTS:

  1. Investors and fund managers who understand the dynamic behavior of asset prices are more likely to have realistic expectations about future prices and the risks to which they are exposed.
  2. Quantitative analysts need to understand asset price dynamics to calculate competitive prices for derivative securities.
  3. Finance researchers who explore hypothesis about capital markets need to consider the implications of price dynamics.

Financial returns and their statistical properties

- There is a key feature of asset prices: prices move relatively slowly when conditions are calm, while they move faster when there is more news and uncertainty.

- The **volatility** of prices refers to the rate at which prices change. This is a main concern in FTSA because high volatility imply a higher chance of a large adverse price change. Usually, volatility is measured in terms of the conditional variance of the returns.

- Predictions concerning future prices are obtained from conditional probability distributions that depend on recent prices. Three prediction problems arise:

  1. Which way will the price go, up or down? It is very difficult to obtain a satisfactory answer.
  2. Which is the probability distribution of future prices? This can be answered by Monte Carlo simulation of the assumed price dynamics.
  3. How volatile will prices be in the future? The level of volatility can be measured and predicted with some success using historical asset prices.

Financial returns and their statistical properties

- The appropriate frequency of observations in a price series depends on the data available. Very often, selecting daily prices will be both appropriate and convenient. A series of daily prices contains useful information that may be missing in a series of weekly or monthly prices.

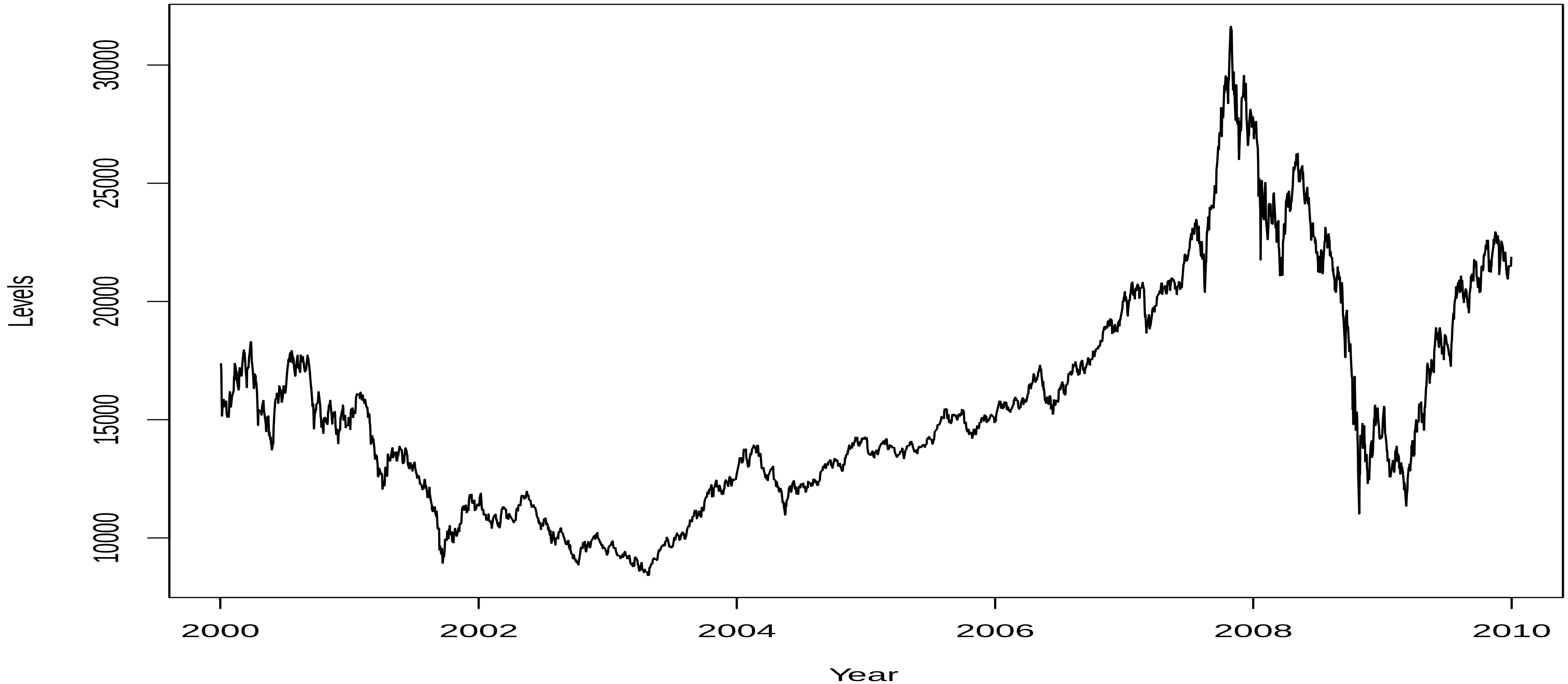
- Analysis of prices recorded more frequently than once a day must take account of uneven flow of transactions during the day, which creates intraday effects.

- Examples of financial time series includes:

  1. Daily closing prices of Apple stock.
  2. Daily closing level of Hang Seng index.
  3. Quarterly earnings of IBM.
  4. USA monthly interest rates.
  5. Daily exchange rates between US Dollar vs Yen.

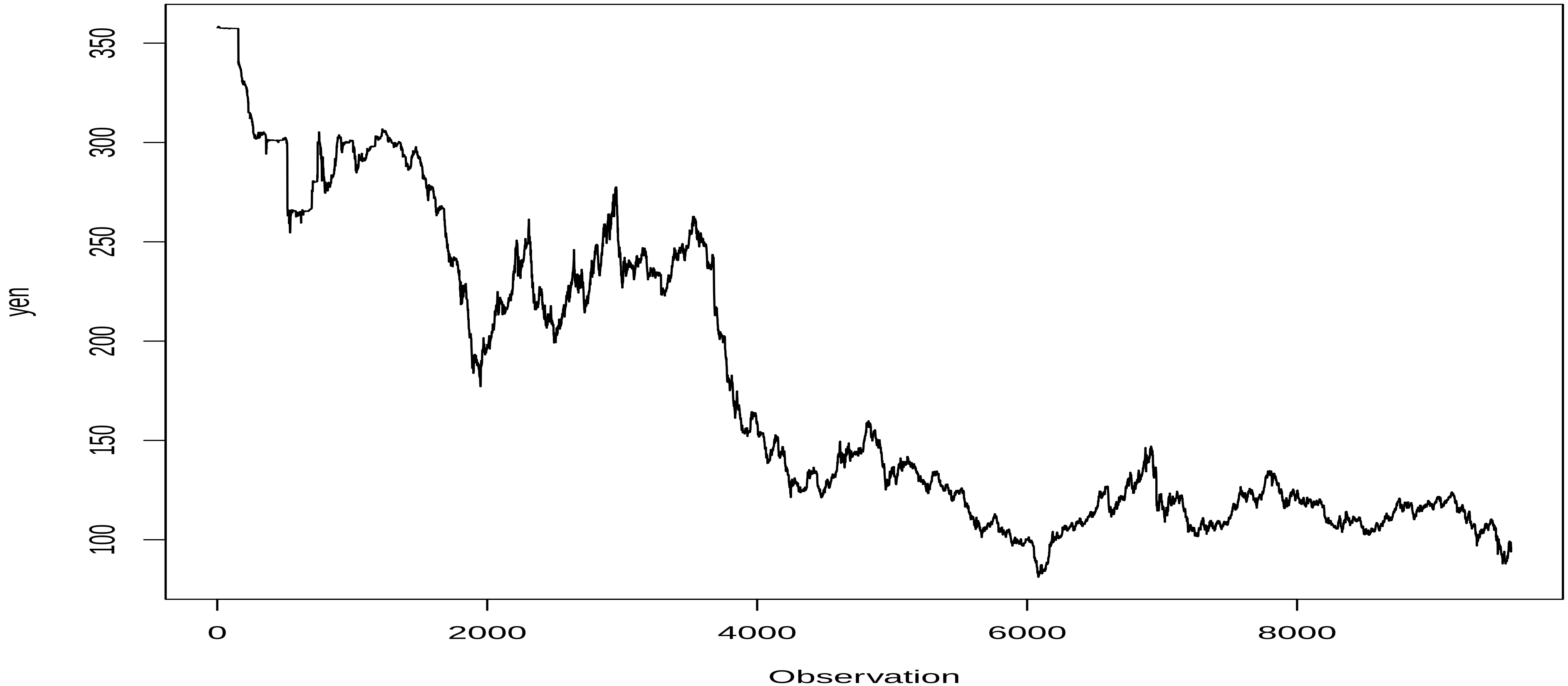
Financial returns and their statistical properties

The figure shows the time plot of the daily level of Hang Seng index from January 2000 to December 2009 (no observations for Saturdays, Sundays and holidays).



Financial returns and their statistical properties

The figure shows the time plot of the exchange rate between US Dollar vs Yen from January 1971 to March 2009 (no observations for Saturdays, Sundays and holidays).



Financial returns and their statistical properties

- These series of prices are clearly non-stationary with several ups and downs. Moreover, the velocity of increases and decreases vary at different periods.

- Most financial studies involve **returns**, instead of prices, of assets. There are at least two reasons:

  1. Return series are easier to handle than price series because prices are highly correlated while returns have very little correlation, if any.
  2. The return of an asset is a complete and scale-free summary of the investment opportunity.

- In what follows, P_t denotes the price of an asset at time t. Given the series of prices, there are several definitions of an asset return. We review the two most important ones.

Financial returns and their statistical properties

1. The **k-period simple return** of an asset is the relative change of the price at periods t and $t+k$, i.e.:

$$R_{t+k} = \frac{P_{t+k} - P_t}{P_t}$$

   In particular, the **one-period simple return** corresponds to $k = 1$.

2. The **k-period log return** of an asset is the logarithm of the ratio between the prices at periods $t+k$ and t, i.e.:

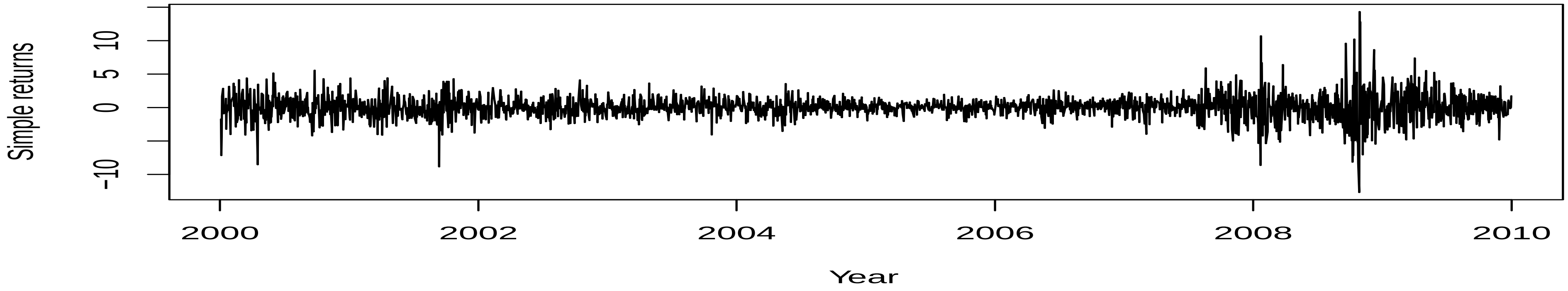
$$r_{t+k} = \log \frac{P_{t+k}}{P_t} = \log \left[\frac{P_{t+k}}{P_{t+k-1}} \times \cdots \times \frac{P_{t+1}}{P_t} \right] = \sum_{i=1}^{k} \log \frac{P_{t+i}}{P_{t+i-1}} = \sum_{i=1}^{k} r_{t+i}$$

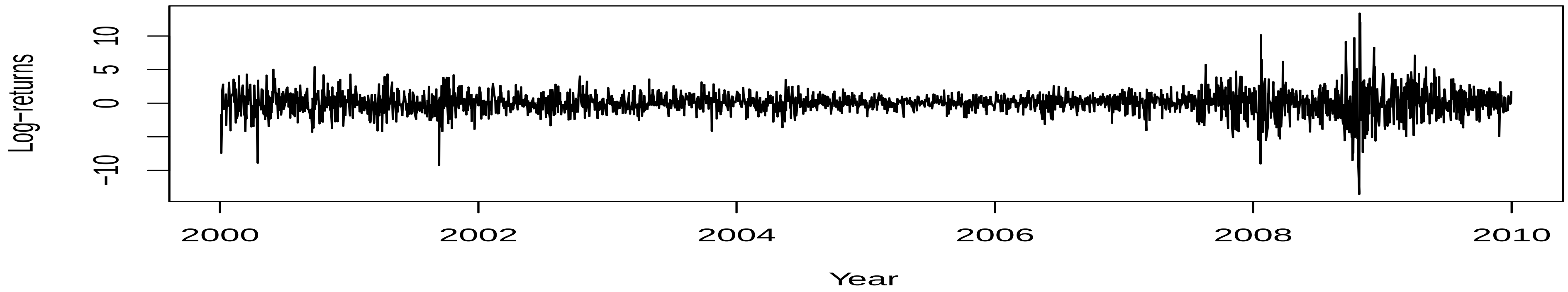
   In particular, the **one-period log return** corresponds to $k = 1$. Note that $r_{t+k} = \log(1 + R_{t+k})$.

- Sometimes, asset returns are given in percentages. In this case, the previous returns are multiplied by 100.

Financial returns and their statistical properties

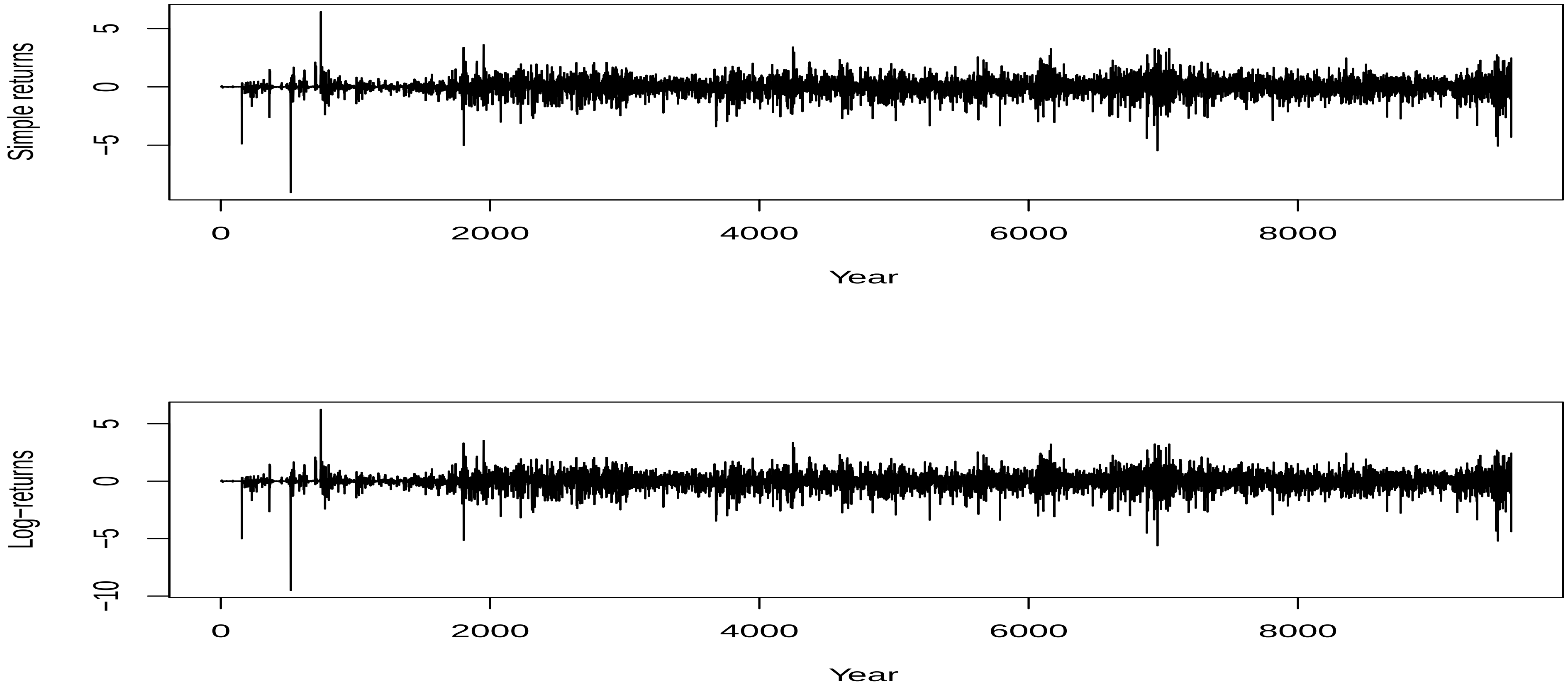
The figure shows the time plot of simple and log returns of Hang Seng index from January 2000 to December 2009.





Financial returns and their statistical properties

The figure shows the time plot of simple and log returns of the exchange rate between US Dollar vs Yen from January 1971 to March 2009.



Empirical characteristics of financial returns

- Financial returns usually exhibit several interesting and complicated features, usually called **stylized facts**.

1. Returns appear to vary around the mean levels, which are close to zero. Thus, they are mean stationary, at least at certain periods of time.
2. The volatility is not constant over time. This is because, during periods of time in which economic crises, wars or political disorders happen, returns fluctuate strongly, while in tranquil periods, returns fluctuate weakly. This effect is called **volatility clustering**.
3. Returns usually take several large positive and negative values. More precisely, extreme negative returns are more frequent than extreme positive returns. As a consequence, the unconditional distribution of returns is known to be negatively skewed and heavy-tailed.
4. Although the serial correlation of returns is very small, if any, squared and absolute returns show strong serial correlation.

Empirical characteristics of financial returns

- The next table provides with some descriptive statistics of simple and log returns for a U.S. market index (S&P) and two individual stocks (IBM and Intel). The returns are for daily sample intervals and are in percentages. The data spans and sample sizes are also given in the table.

	Security	Start	Size	Mean	St. Dev.	Skew.	Ex. kurt.
	S&P	02/01/70	9845	0.029	1.056	-0.73	22.81
Sim. ret.	IBM	02/01/70	9845	0.040	1.693	0.06	9.92
	Intel	15/12/72	9096	0.108	2.891	-0.15	6.13
	S&P	02/01/70	9845	0.023	1.062	-1.17	30.20
Log ret.	IBM	02/01/70	9845	0.026	1.694	-0.27	12.17
	Intel	15/12/72	9096	0.066	2.905	-0.54	7.81

Empirical characteristics of financial returns

- Several observations arise from the table:

  1. The mean of daily return series is close to 0.
  2. The market index has smaller standard deviation than individual stocks.
  3. Returns usually have a slightly negative skewness.
  4. Daily returns tend to have high excess kurtosis. Specially, the market index.
  5. The descriptive statistics show that the difference between simple and log returns is not substantial.

Empirical characteristics of financial returns

- Conditional distributions are more relevant that marginal distributions of asset returns. However, the marginal distributions may still be of some interest.

- Several statistical distributions has been proposed in the literature for the marginal distributions of asset returns, including normal distribution, lognormal distribution, stable distribution and scale mixture of normal distribution, among others.

- A traditional assumption is that the simple returns are independent and identically distributed (iid) as normal. This assumption makes statistical properties of asset returns tractable but it is not supported by empirical analysis, as seen in the previous example.

Empirical characteristics of financial returns

- Another commonly assumption is that the log-returns of an asset are iid as normal with mean μ and variance σ^2. Then, the simple returns are then iid lognormal with mean and variance:

$$E(R_t) = \exp\left(\mu + \frac{\sigma^2}{2}\right) - 1 \qquad Var(R_t) = \exp\left(2\mu + \sigma^2\right)\left[\exp\left(\sigma^2\right) - 1\right].$$

  However, the lognormal assumption is not consistent with all the properties of stock returns. In particular, many stock returns exhibit a positive excess kurtosis.

- Stable distributions, such as the Cauchy distribution, are a generalization of the normal capable of capturing excess kurtosis. However, nonnormal stable distributions do not have a finite variance, which is in conflict with most finance theories.

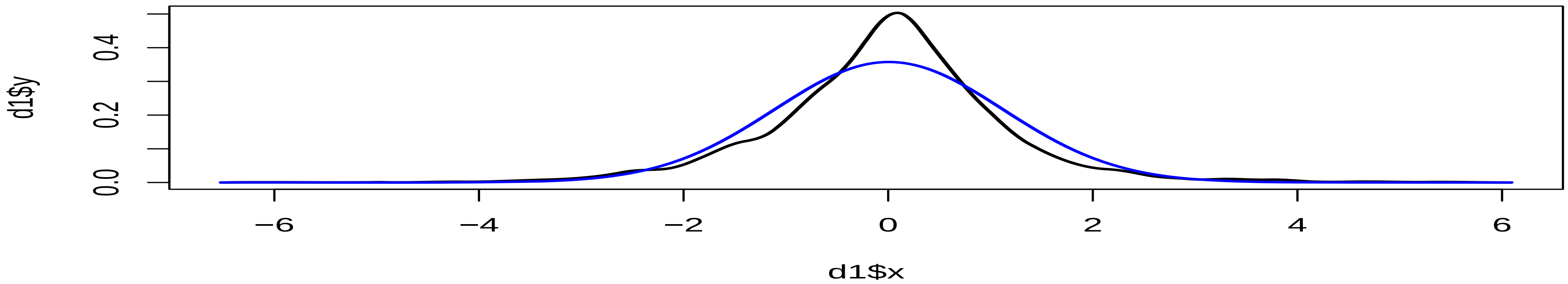
Empirical characteristics of financial returns

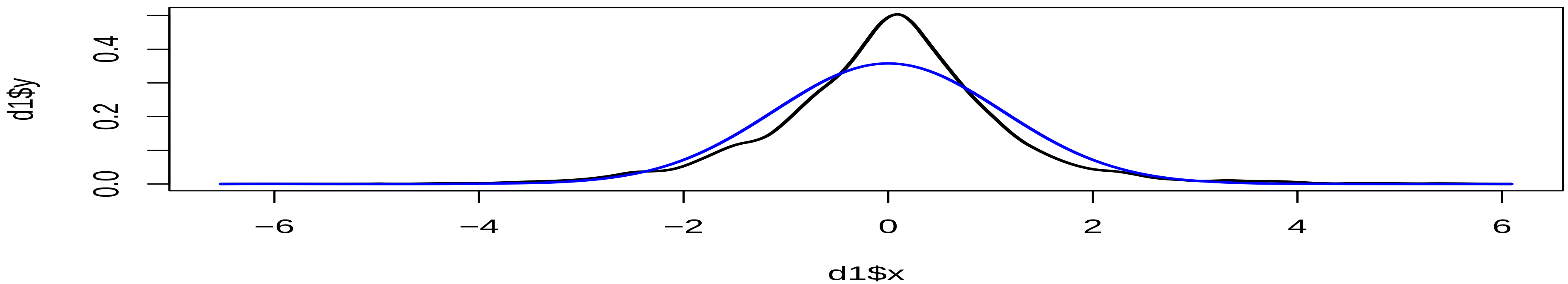
- Recent studies tend to use scale mixtures or finite mixture of normal distributions. Under this assumption, the log-return r_t is normally distributed with mean μ and variance σ^2, where σ^2 is a random variable that follows a positive distribution. For instance,
$$r_t \sim \alpha \times N\left(\mu_1, \sigma_1^2\right) + (1-\alpha) \times N\left(\mu_2, \sigma_2^2\right)$$
where $\alpha \in (0,1)$.

- Then, if α is large, σ_1^2 small and σ_2^2 large, most of the returns are distributed with a $N\left(\mu_1, \sigma_1^2\right)$ while a few of them are generated with a $N\left(\mu_2, \sigma_2^2\right)$ enabling to put more mass at the tails of the distribution. Advantages of the mixtures includes that they maintain the tractability of the normal distribution, have finite higher order moments, and can capture the excess kurtosis.

Empirical characteristics of financial returns

The figure shows the empirical density functions of monthly simple and log returns of SP index from 2000 to 2007. Also, the normal pdf using the sample means and variances.





Empirical characteristics of financial returns

- Assume that the return series r_t is weakly stationary. Therefore, $E(r_t) = \mu$ is a constant, and $Cov(r_t, r_{t+k}) = \gamma_k$, only depends on k. In particular, $\gamma_0 = Var(r_t)$ and $\gamma_k = \gamma_{-k}$.

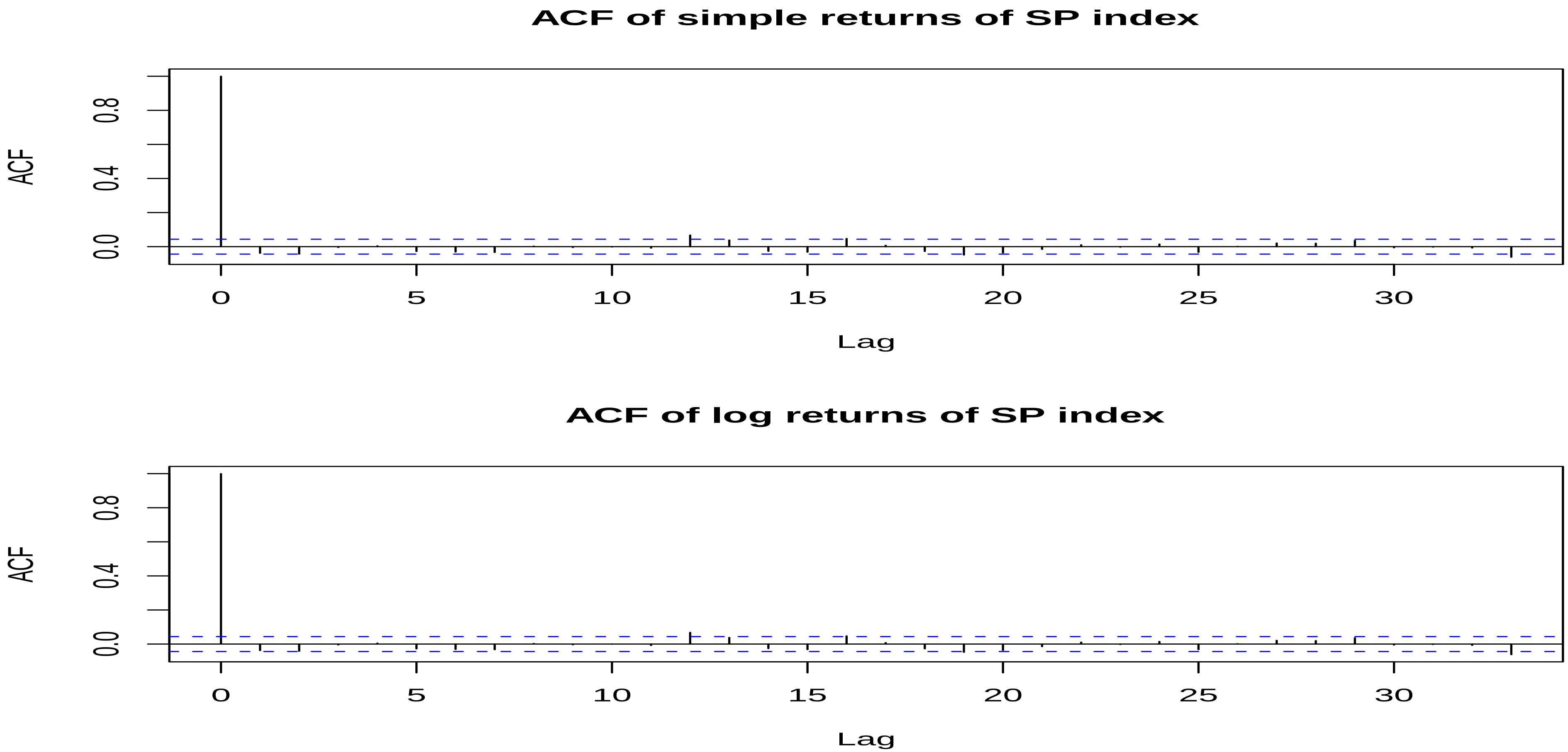
- Under weakly stationarity, the autocorrelation function (ACF) of a return series measures the correlation between values of the return series at time t and $t+k$, for $k \geq 1$. This is given by $\rho_k = \gamma_k / \gamma_0$. Note that it is only required to compute ρ_k for k positive becuase $\rho_k = \rho_{-k}$.

- Assuming a return series $r_1, \ldots, r_T$, the sample ACF is given by $\widehat{\rho}_k = \widehat{\gamma}_k / \widehat{\gamma}_0$ where $\widehat{\gamma}_0$ and $\widehat{\gamma}_k$ are the estimates of γ_0 and γ_k, respectively.

- If r_t is an iid sequence $E\left(r_t^2\right) < \infty$, then $\widehat{\rho}_k$ is asymptotically normal with mean zero and variance $1/T$ for any fixed positive integer k.

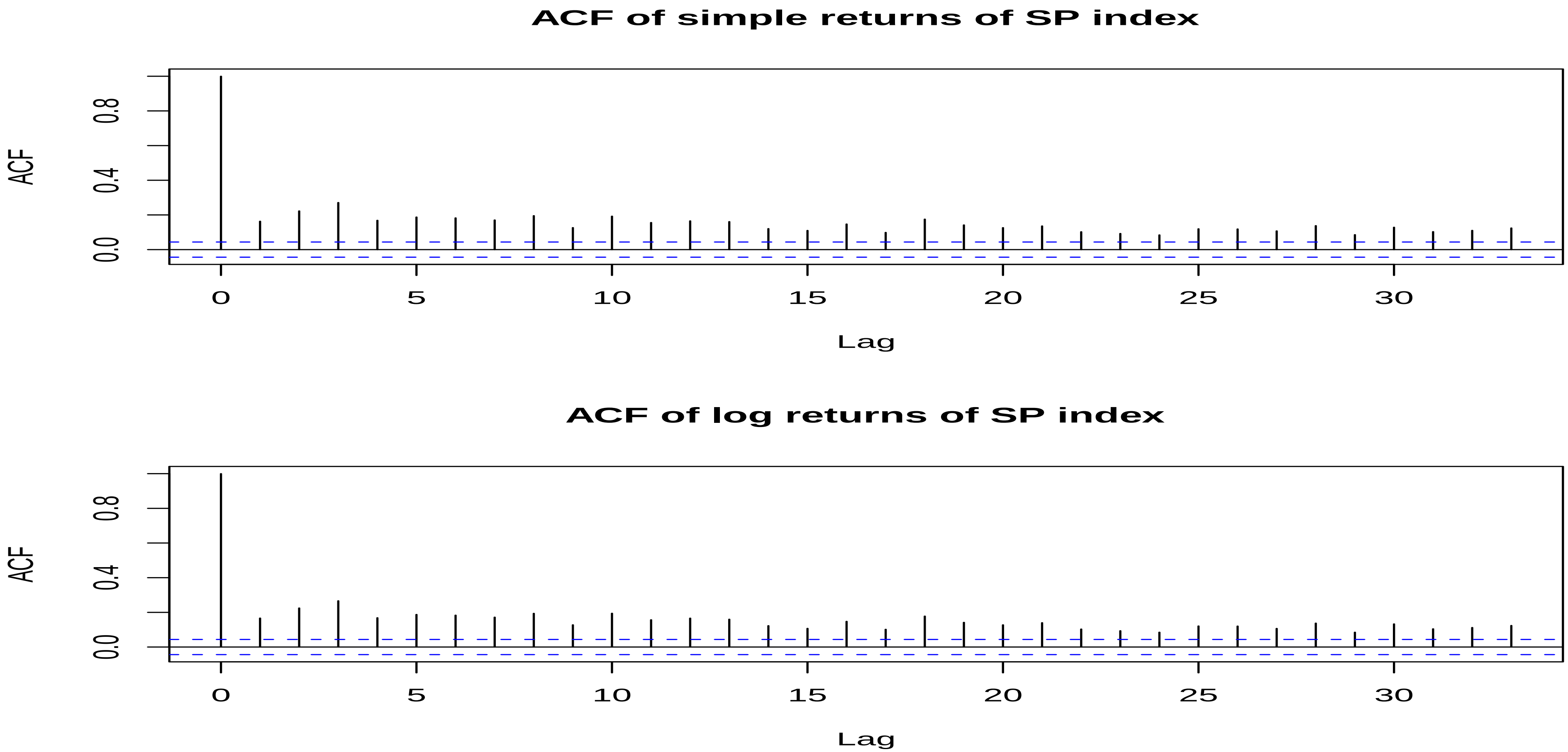
Empirical characteristics of financial returns

The figure shows the sample ACFs of monthly simple and log returns of IBM stock from 1926 to 2008.



Empirical characteristics of financial returns

The figure shows the sample ACFs of the squared monthly simple and log returns of IBM stock from 1926 to 2008. These ACFs illustrate that returns are time dependent.



Empirical characteristics of financial returns

- Besides the return series, we also consider the volatility process and the behavior of extreme returns of an asset.

- The volatility process is concerned with the evolution of conditional variance of the return over time. This is a topic of interest because the variabilities of returns vary over time and appear in clusters. Volatility plays an important role in pricing options and risk management. Models for volatility are in Chapters 2 and 3.

- On the other hand, an extreme of a return series is a large positive or negative return. The negative extreme returns are important in risk management, whereas positive extreme returns are critical to holding a short position (selling an asset one does not own). The impact of extremes will be covered in Chapter 4.

- Also, moments of and the relationship between multiple asset returns will be seen in Chapter 5.

Chapter outline

1. Financial returns and their statistical properties.

2. Empirical characteristics of financial returns.

We are ready now for **Chapter 2: Conditional heteroscedastic models**.