1 Hipóteses de traballo

- Supoñamos que definimos a incertidumbre dunha variable Z como a medida adimensional da súa variabilidade dada por
$$U = \frac{\sigma}{x},$$
onde x e σ son a media e a desviación típica de Z, respectivamente.
- Supoñamos que dicimos que dúas variables Z_1 e Z_2 son interdependientes cando
$$Cov(Z_1, Z_2) = \sqrt{Var(Z_1)Var(Z_2)}.$$
- Supoñamos que temos n observacións $Z_1, \ldots, Z_n$ con medias $x_1, \ldots, x_n$ e desviacións típicas $\sigma_1, \ldots, \sigma_n$, respectivamente (evidentemente, as súas incertidumbres serán $U_i = \frac{\sigma_i}{x_i}$ con $i = 1, \ldots, n$).
- Utilizaremos o método delta multidimensional:
$$\sqrt{n}\left((Z_1, \ldots, Z_n)^t - (x_1, \ldots, x_n)^t\right) \to N\left((0, \ldots, 0)^t, \Sigma\right)$$
$$\Rightarrow \sqrt{n}\left(g((Z_1, \ldots, Z_n)^t) - g((x_1, \ldots, x_n)^t)\right) \to N\left((0, \ldots, 0)^t, \nabla g((x_1, \ldots, x_n)^t)^t \Sigma \nabla g((x_1, \ldots, x_n)^t)\right).$$

2 Incertidumbre da suma

2.1 Cando as variables son independentes

Neste caso,

- $\mathbb{E}\left(\sum_{i=1}^n Z_i\right) = \sum_{i=1}^n \mathbb{E}(Z_i) = \sum_{i=1}^n x_i$, e
- $Var\left(\sum_{i=1}^n Z_i\right) = \sum_{i=1}^n Var(Z_i) = \sum_{i=1}^n \sigma_i^2$.

Entón a incertidumbre da suma será
$$U_{total} = \frac{\sqrt{\sum_{i=1}^n \sigma_i^2}}{\sum_{i=1}^n x_i} = \frac{\sqrt{\sum_{i=1}^n x_i^2 U_i^2}}{\sum_{i=1}^n x_i}.$$

2.2 Cando as variables son interdependientes

Neste caso,

- $\mathbb{E}\left(\sum_{i=1}^n Z_i\right) = \sum_{i=1}^n \mathbb{E}(Z_i) = \sum_{i=1}^n x_i$, e
- $Var\left(\sum_{i=1}^n Z_i\right) = \sum_{i=1}^n Var(Z_i) + 2\sum_{i<j} Cov(Z_i, Z_j)$
$$= \sum_{i=1}^n \sigma_i^2 + 2\sum_{i<j} \sigma_i \sigma_j = \left(\sum_{i=1}^n \sigma_i\right)^2.$$

Entón a incertidumbre da suma será
$$U_{total} = \frac{\sum_{i=1}^n \sigma_i}{\sum_{i=1}^n x_i} = \frac{\sum_{i=1}^n x_i U_i}{\sum_{i=1}^n x_i}.$$

3 Incertidumbre do producto

Neste caso, utilizamos o método delta con $g((Z_1, \ldots, Z_n)^t) = \prod_{i=1}^n Z_i$. Entón

- $g((x_1, \ldots, x_n)^t) = \prod_{i=1}^n x_i$ e
- $\nabla g((x_1, \ldots, x_n)^t) = (\prod_{i\neq}^n x_i, \ldots, \prod_{i\neq n}^n x_i)$

3.1 Cando as variables son independentes

Neste caso $\Sigma = diag(\sigma_1^2, ..., \sigma_n^2) = diag(x_1^2 U_1^2, ..., x_n^2 U_n^2)$.

- $\mathbb{E}\left(\prod_{i=1}^n Z_i\right) = g((x_1, \ldots, x_n)^t) = \prod_{i=1}^n x_i$, e
- $Var\left(\prod_{i=1}^n Z_i\right) = \nabla g((x_1, \ldots, x_n)^t)^t \Sigma \nabla g((x_1, \ldots, x_n)^t$

$$= (\prod_{i\neq 1}^n x_i, \ldots, \prod_{i\neq n}^n x_i)^t \Sigma (\prod_{i\neq 1}^n x_i, \ldots, \prod_{i\neq n}^n x_i)$$

$$= (\prod_{i\neq 1}^n x_i, \ldots, \prod_{i\neq n}^n x_i)^t (x_1^2 U_1^2 \prod_{i\neq 1}^n x_i, \ldots, x_n^2 U_n^2 \prod_{i\neq n}^n x_i)$$

$$= \left(\prod_{i=1}^n x_i^2\right) \sum_{i=1}^n U_i^2.$$

Entón a incertidumbre será

$$U_{total} = \sqrt{\sum_{i=1}^n U_i^2}.$$

3.2 Cando as variables son interdependentes

Neste caso $\Sigma = (\sigma_i \sigma_j)_{i,j} = (x_i U_i x_j U_j)_{i,j}$. Facendo as contas

- $\mathbb{E}\left(\prod_{i=1}^n Z_i\right) = g((x_1, \ldots, x_n)^t) = \prod_{i=1}^n x_i$, e
- $Var\left(\prod_{i=1}^n Z_i\right) = \left(\prod_{i=1}^n x_i^2\right) \left(\sum_{i=1}^n U_i\right)^2$.

Entón a incertidumbre será

$$U_{total} = \sum_{i=1}^n U_i.$$