Spatial Autoregressive and Moving Average Hilbertian Processes: Structural Properties and Functional Inference

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Abstract. *Since the initial work on spatial statistical models developed in Bhattacharyya, Khalil and Richardson (1996), Basu and Reinsel (1993), Güyon (1995), Martin (1979, 1990, 1996), Jain (1981), Tjostheim (1978, 1981, 1983), among others, the spatial series modeling framework has been widely considered in several applied fields such as geology, geophysics, biology, agriculture, spatial econometrics, image processing, etc. High dimensional statistics (see Bosq and Blanke, 2007) has been essentially developed in the context of functional temporal data, i.e., when data are curves or temporal sequences of curves, surfaces, etc. (see, for instance, Bosq, 2000; Ferraty and Vieu, 2006, Ruiz-Medina and Salmerón, 2010; Ruiz-Medina, Salmerón and Angulo, 2007, and Salmerón and Ruiz-Medina, 2009).*

The functional counterpart of the spatial series modeling framework is considered in this work for high-dimensional spatial data processing. Specifically, this paper addresses the introduction and study of structural properties of Hilbert-valued spatial autoregressive processes (SARH(1) processes), and Hilbert-valued spatial moving average processes (SMAH(1) processes), with innovations given by two-parameter (spatial) matingale differences. For inference purposes, conditions for the generation of two-parameter Hilbertian processes (SARH(1) and SMAH(1) processes) from the tensorial product of standard autoregressive Hilbertian (ARH(1)) processes (respectively, from the tensorial product of standard moving average Hilbertian (MAH(1)) processes) are investigated. Recent results in Bosq (2010), on the standard character of the tensorial product of ARH(p) and MAH(q) processes, are also considered in the derivation of functional inference results for the class of spatial functional models introduced. In the context of spatial diffusion processes, some examples, related with the functional observation of such diffusion processes are studied. Simulation results are also displayed to illustrate the modeling and asymptotic inference results derived.

Keywords. *Functional spatial statistics; Spatial Hilbert-valued processes; Tensorial product of Hilbert-valued processes; Two-parameter diffusion processes; Two-parameter martingale differences.*

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