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Spatial pattern classification for children with attention deficit

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Abstract. *The eye trackers (eye loggers) are based on the technique of infrared corneal reflection, and constitutes the eye tracker system most used today in psychological studies that analyze eye movements. This technique sends a small infrared beam in the pupil that due to the refraction of the vitreous and aqueous humor of the eye Purkinje images (i.e. two small points of light visible within the pupil) are pictured. By using the first Purkinje image and pupil center a photosensitive camera captures light and sends signals to the central unit of the eye tracker obtaining the relative position between the corneal reflection point and the center center. Thus this device provides a database with coordinates of the location of the eyes in a particular time. This can be considered a space-time point pattern. Psychologists define a fixation or unit of concentration such situation in which the eye does not move apart more than 35 pixels within 100 milliseconds. Patients with an attention deficit can not define fixations in a right way. We aim to detect space-time clusters of fixations to classify and detect particular problems related to attention deficit.*

Keywords. *Eye trackers; Feature detection; LISA function; SEM algorithm; Spatial point processes.*

1 Introduction: eye trackers

After recording spatial and temporal patterns, we can easily get data on linguistic processing instructions. Considering the fact that people tend to look for the words you are hearing, Tanenhaus and his students established the paradigm of eye movement measurement for viewing scenes while listening. These authors enhanced the techniques of eye tracking processing to better understand the oral language, highlighting the major advantages of analyzing oral language than written. Many studies using eye tracking systems have explored different aspects of language comprehension. Some authors have adapted this experimental paradigm to children between 4 and 6 years while listening to spoken sentences with the aim of detecting forms of autism. In these studies real time eye movements were measured while asked different things about objects in the visual display in response to oral instructions.

In this context, we analyze a study conducted with children on real-time understanding of sentences with good reading comprehension items in two situations. They used eye tracker systems. The eye trackers (eye loggers) are based on the technique of infrared corneal reflection, an eye tracker system mostly used today in psychological studies that analyze eye movements. This technique sends a small infrared beam in the pupil that due to the refraction of the vitreous and aqueous humor of the eye Purkinje images (i.e. two small points of light visible within the pupil) are pictured. By using the first Purkinje image and pupil center a photosensitive camera captures light and sends signals to the central unit of the eye tracker obtaining the relative position between the corneal reflection point and the center center. Thus this device provides a database with coordinates of the location of the eyes in a particular time. A fixation or unit of concentration in a cluster in space and time, as it defines such situation in which the eye does not move apart more than 35 pixels within 100 milliseconds. Analyzing fixations give hints to detect attention problems. An example of such patterns is given in Figure 1.

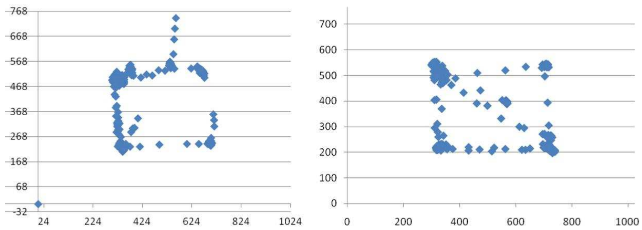

Figure 1: *Left*: patient with attention deficit; *Right*: normal patient

2 The problem and spatial point processes

Consider the problem of detecting clusters within an image of 1024×678 pixels. The clusters may correspond to fixations (fasteners) or not. This scientific problem can also be thought of as searching for regions of higher density of the point process that is the result of the data collection.

2.1 Product densities and LISA functions

A planar point process N is a stochastic model governing the locations of events $\{\mathbf{s}_i\}$ in some set A, a bounded region in $\mathbb{R}^2$. If the events are located at random and independently, the resulting point process is called a *Poisson process* and constitutes the reference process in every type spatial point analysis. When estimated from point process data, the empirical product density function provides a description of the density of inter-event distances in an observed point pattern. For instance, high values for small distances are indicative of an overabundance of short inter-event distances (this is a typical situation for cluster processes, where data tend to form groups). Conversely, if short inter-event distances are rare, this will indicate that an inhibitory structure is present, and points tend to separate from each other. In the homogeneous and isotropic case ([5, 6]), the product density depends only on the distance $r = \|\mathbf{s} - \mathbf{t}\|$ between the points $\mathbf{s}$ and $\mathbf{t}$, and thus we write, for the sake of simplicity, $l(r)$. For a Poisson process $l(r) = \lambda^2$, the square of the intensity of the process.

Consider now that the planar point process N is observed in a region $A \subseteq R^2$ of area $|A| = a$. For stationary and isotropic point processes in the plane with intensity λ, the K-function ([3, 5, 6]) is defined by $\lambda K(r) \equiv E[N(b(\mathbf{s},r)) \setminus \{\mathbf{s}\} : \mathbf{s} \in N]$, where $E[.]$ denotes the expected number of events, $b(\mathbf{s},r)$ is a disc of radius $r > 0$ centered at an arbitrary event $\mathbf{s}$, and the notation above refers to the element $\mathbf{s}$ being removed from the disc. $K(r)$ provides an interpretable measure of the spatial-dependence structure of the point process. In particular, $\lambda^2 aK(r)$ is the expected number of ordered pairs of events in the observation region A with pairwise distance less than or equal to r.

Both the K-function and the product density function provide a global measure of the covariance structure by summing over the contributions from each event observed in the process. Now we consider individual contributions to the estimated function that are analogous to the local statistics described by [1] and called *local indicators of spatial association* (LISA). An individual LISA product density function $l^{(i)}(.)$ should reveal the extent of the contribution of the event $\mathbf{s}_i$ to the global estimate of $l(.)$, and may provide a further description of structure in the data (e.g., determining events with similar local structure through dissimilarity measures of the individual LISA functions).

A product density LISA function can be constructed in the same manner as the global estimate but considering local features. Define now

$$\{\lambda K(r)\}^{(i)} \equiv E[N(b(\mathbf{s}_i,r)) \setminus \{\mathbf{s}_i\} : \mathbf{s}_i \in N],$$

as the expected number of extra events within distance r from $\mathbf{s}_i$. Here, and for a homogeneous Poisson point process, $(n-1)/a$ provides an unbiased estimator for λ. By analogy with the formation of the global product density estimate, a localised version of the empirical product density function is given by

$$\hat{l}_{\varepsilon}^{(i)}(r) = \frac{n-1}{2\pi r a} \sum_{j \neq i} \frac{2\pi r_{ij}}{\left|\partial b(\mathbf{s}_i, r_{ij}) \bigcap A\right|} t_{\varepsilon}(r_{ij} - r), \tag{1}$$

for $r > \varepsilon > 0$.

2.2 Cluster classification through LISA functions and the SEM algorithm

We aim to detect features present in an image. Following [2, 4], the method we adopt here does not assume any particular shape nor number of features. Thus, we estimate features in quite general situations. We assume that the clutter is randomly distributed throughout the region, through a homogeneous Poisson point process. The features are also supposed to be a Poisson process restricted to a certain part of the region. Now, noting that we know some distributional properties of the LISA functions, for example the expected value of a LISA function under a homogeneous Poisson process, we make use of these properties to develop particular distances, as an alternative to the k^{th} nearest-neighbour distances. Thus, given the whole set of spatial locations of the points in the treated image, we build a marked point process as $N = \{x_i, y_i, d_i, \delta_i\}_{i=1}^n$, with (x_i, y_i) the original spatial locations, d_i the distances between the edge-corrected LISA functions, and the (known) expected value, and $\delta_i \in \{0,1\}$. Moreover, $\delta_i = 1$ if the i^{th} observation belongs to a feature, and $\delta_i = 0$ if it belongs to the clutter.

Thus, our method follows the steps and philosophy of the methodology proposed by [2], but using distances between product density-based LISA functions rather than k^{th} nearest-neighbour distances. By reasoning this way, we develop exploratory data analytic tools to examine individual points in the point pattern in terms of how they relate to their neighbouring points. As features are clearly clustered, if we analyse them from the perspective of the whole image, we need to rely on those statistical tools that can measure clustering. In fact, a product density is a rate of change of the K-function, which is a measure of clustering in a point process.

The individual distances, $\{d_i\}_{i=1}^n$, between an edge-corrected LISA function and the (constant and known) expected value can be postulated to be distributed as a mixture of two distributions. With a strong distinction between feature and clutter, this distribution becomes highly bimodal. Thus, given the marked point process $N = \{x_i, y_i, d_i, \delta_i\}_{i=1}^n$, built upon a binary parameter, we can use the EM algorithm to estimate the classification parameter, $\delta_i \in \{0, 1\}$. However, the E-step needs the evaluation of the density function associated to the distances between LISA functions, and this is unknown. Thus, the EM method is not applicable because the expectation step cannot be performed in closed form. We then follow a stochastic approximation of the EM algorithm (SEM), which replaces the expectation step of the EM algorithm by iterations of a stochastic approximation procedure following [4]. The stochastic version of the EM algorithm we use here introduces an additional step, the S-step, between the usual E and M steps, in which the (unknown) density function $f_{D_{LISA}}(d_i; .)$ is estimated through simulation.

3 Applications

The data we analyze belongs to a group of adults, children without specific language disorder, and a third group of children with this symptom. All of them were passed the same image and the same recording, and their eye movements were recorded. We then apply the SEM algorithm to detect space-time clusters and particular fixations to provide the interested scientists with more objective information.

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