Cores of Convex Games and Pascal's Triangle

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Northwestern University
and
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....

(joint with Estela Sánchez-Rodríguez)

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  (an $(n-1)$-dimensional polytope inside the set of imputations)

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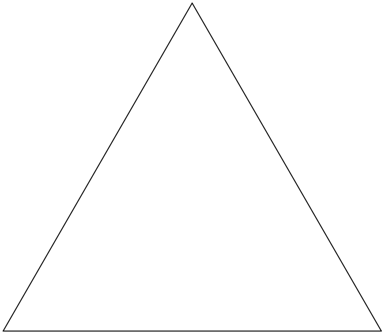

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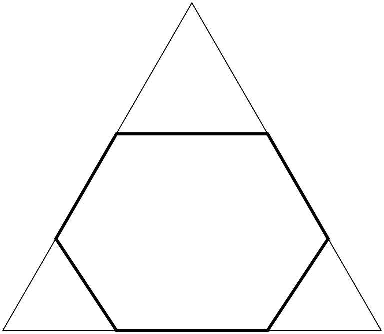

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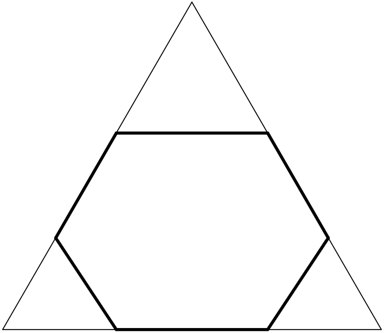

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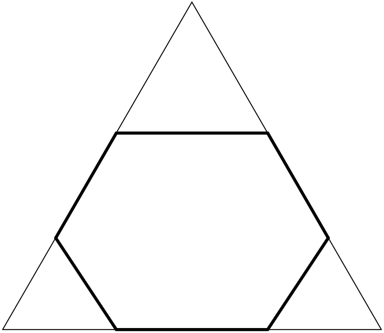

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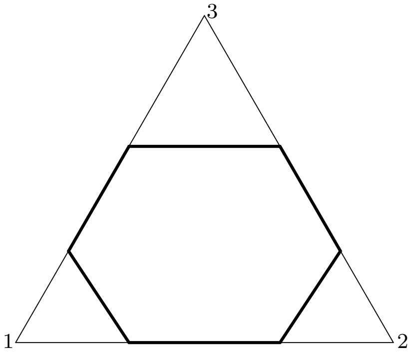

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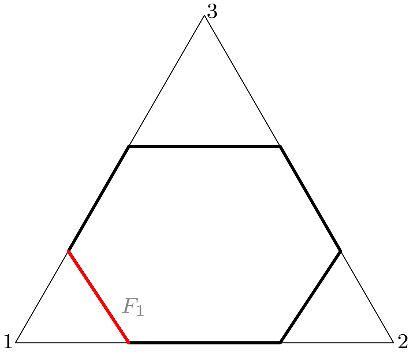

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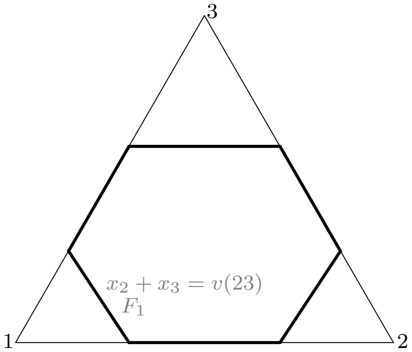

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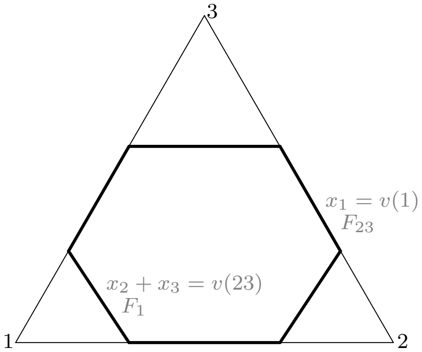

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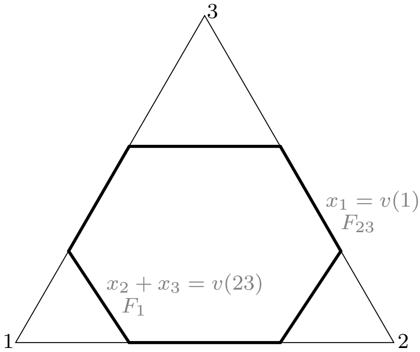

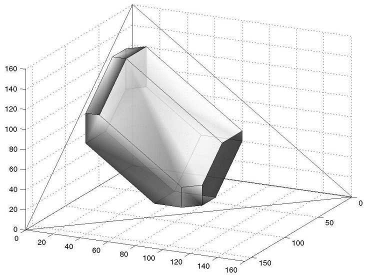

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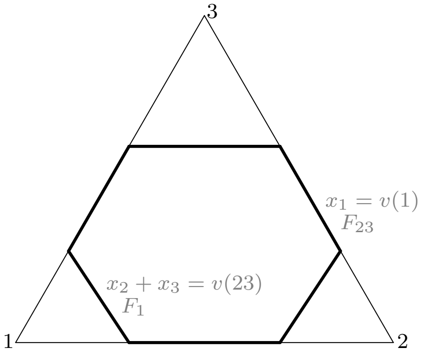

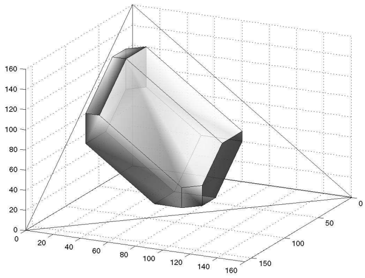

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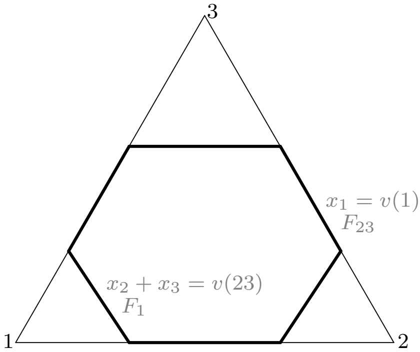

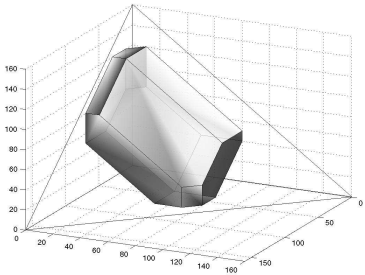

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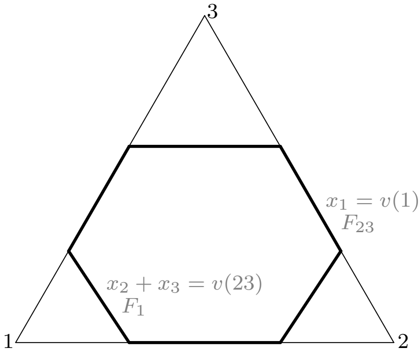

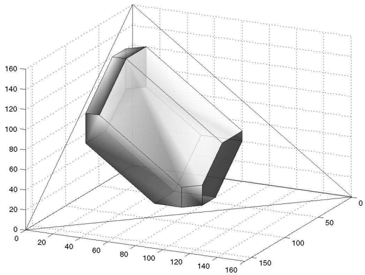

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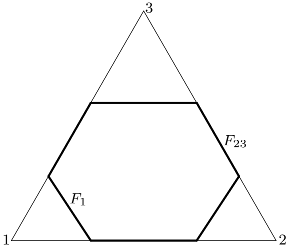
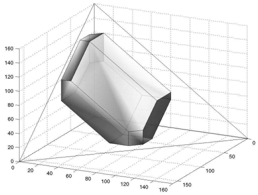
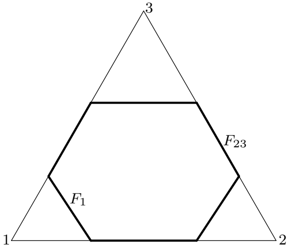
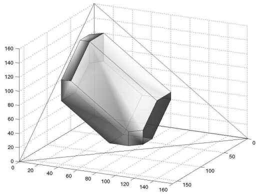

3
F_{23}
F_1
1
2







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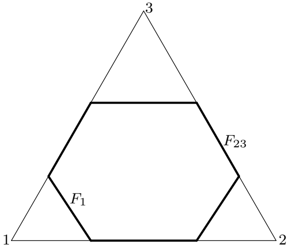

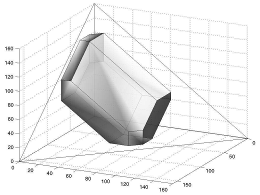

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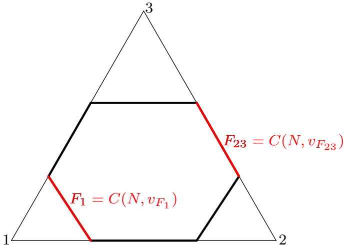

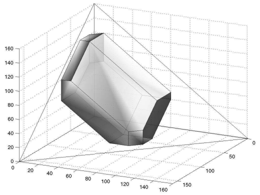

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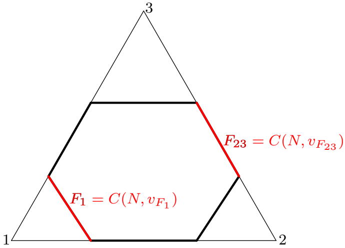

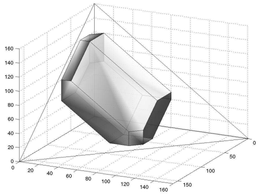

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Therefore, $C(N, v) = \text{co}\{C(N, v_{F_T}) : \emptyset \neq T \subsetneq N\}$

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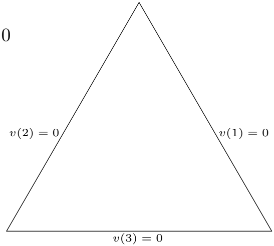

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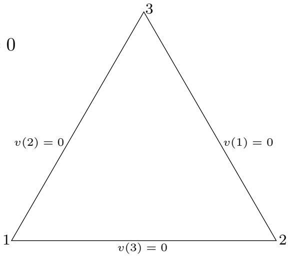

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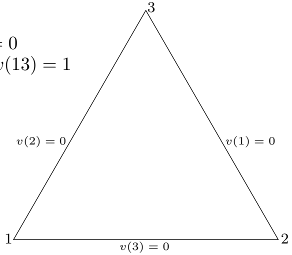

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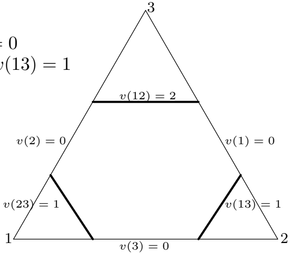

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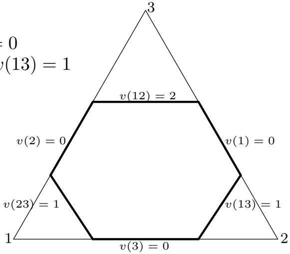

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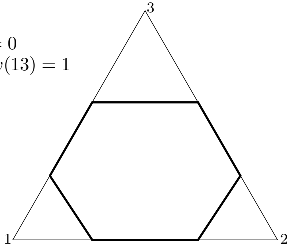

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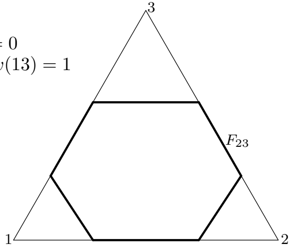

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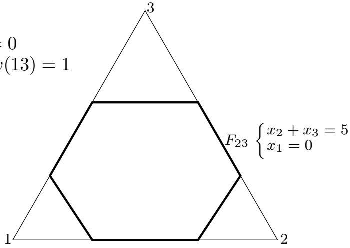

$v(N) = 5$
$v(1) = v(2) = v(3) = 0$
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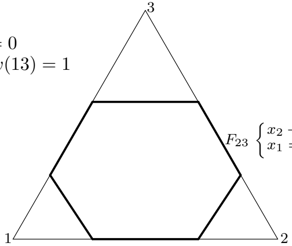

$F_{23} \begin{cases} x_2 + x_3 = 5 \\ x_1 = 0 \end{cases}$ $\quad F_{23} =$

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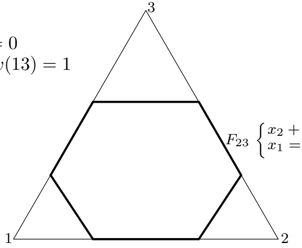

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Decomposable Games

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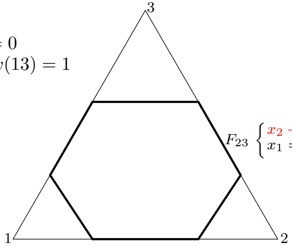

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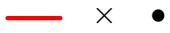
$F_{23} =$ 

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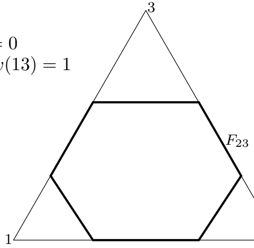

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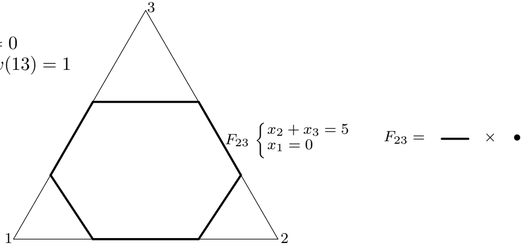

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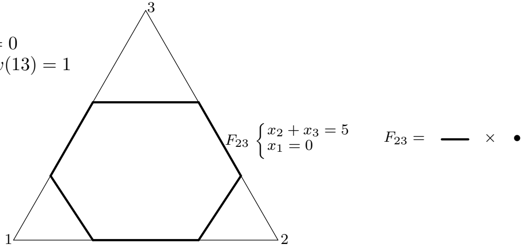

Decomposable Games

- In F_T the "negotiations" between T and $N\backslash T$ have been decided in favor of T
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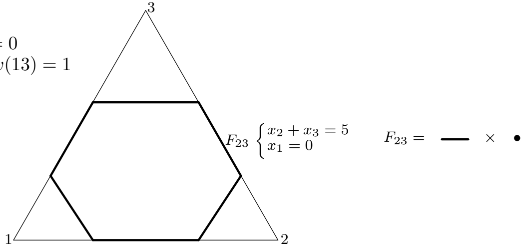

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- If $|T| > 1$, the players in T still have to "negotiate" (similarly in $(N\backslash T, v^{N\backslash T})$)

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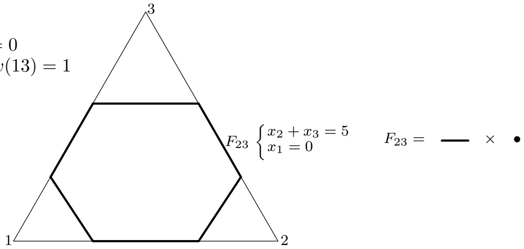

Results

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The *combinatorial complexity* of the core of a game is the number of "different" kinds of polytopes there are among its facets

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For each $t \in \{1, \ldots, n-1\}$, $C(N, v)$ has $2\binom{n}{t}$ "equal" facets (decomposable as the product of the cores of two strictly convex games with t and $n-t$ players, respectively)

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$$\binom{0}{0}$$

$$\binom{1}{0} \quad \binom{1}{1}$$

$$\binom{2}{0} \quad \boldsymbol{\binom{2}{1}} \quad \binom{2}{2}$$

$$\binom{3}{0} \quad \boldsymbol{\binom{3}{1}} \quad \boldsymbol{\binom{3}{2}} \quad \binom{3}{3}$$

$$\binom{4}{0} \quad \boldsymbol{\binom{4}{1}} \quad \boldsymbol{\binom{4}{2}} \quad \boldsymbol{\binom{4}{3}} \quad \binom{4}{4}$$

$$\binom{5}{0} \quad \boldsymbol{\binom{5}{1}} \quad \boldsymbol{\binom{5}{2}} \quad \boldsymbol{\binom{5}{3}} \quad \boldsymbol{\binom{5}{4}} \quad \binom{5}{5}$$

$$\binom{6}{0} \quad \boldsymbol{\binom{6}{1}} \quad \boldsymbol{\binom{6}{2}} \quad \boldsymbol{\binom{6}{3}} \quad \boldsymbol{\binom{6}{4}} \quad \boldsymbol{\binom{6}{5}} \quad \binom{6}{6}$$

$$\vdots$$

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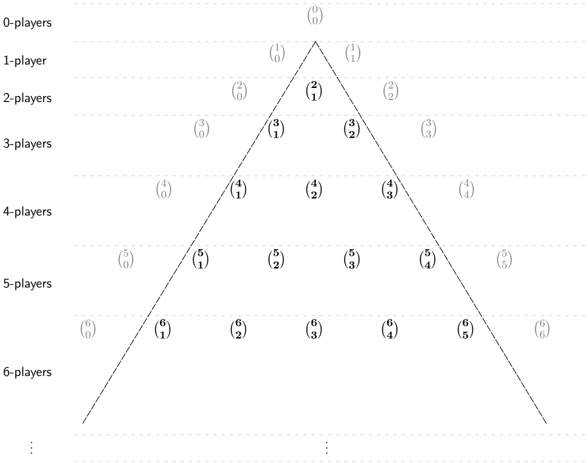

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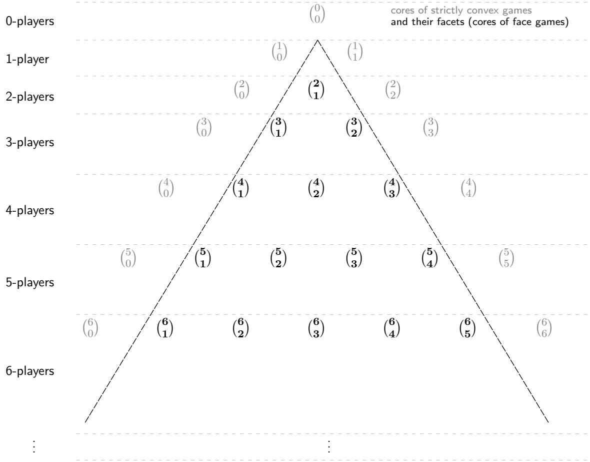

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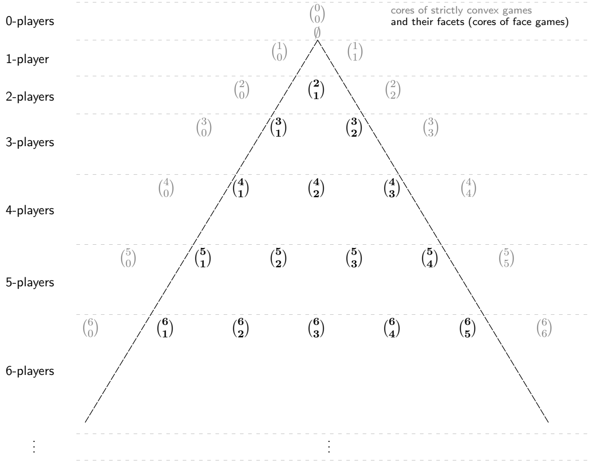

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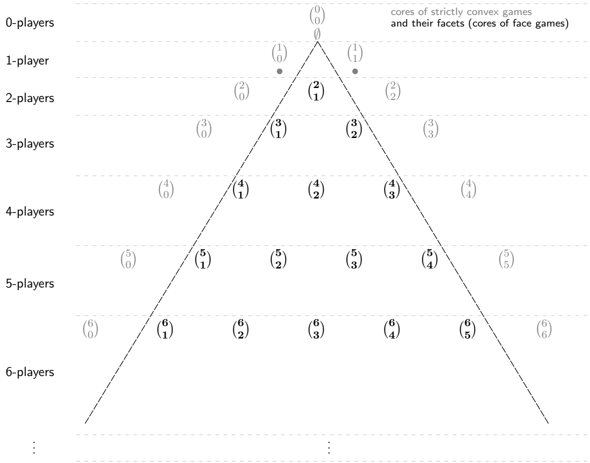

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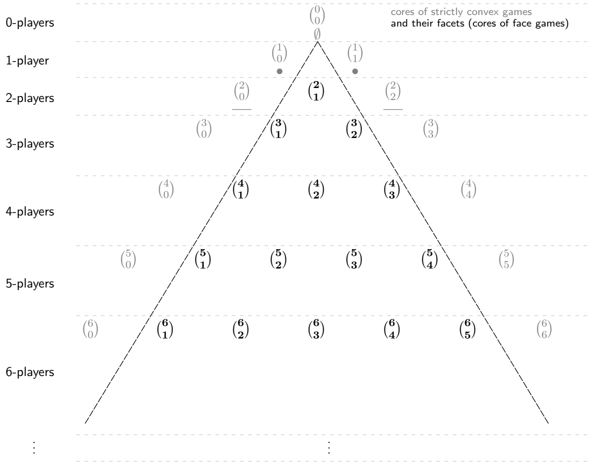

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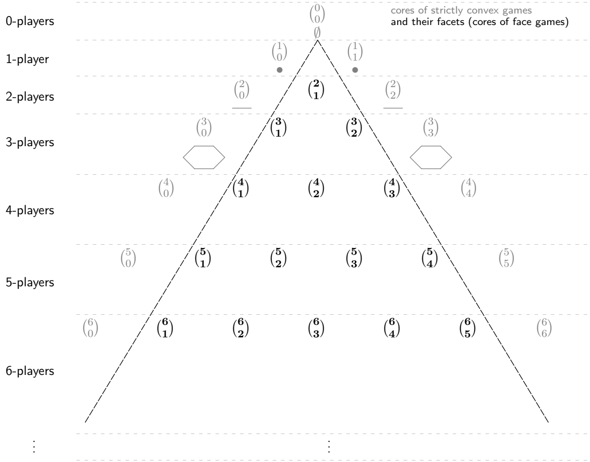

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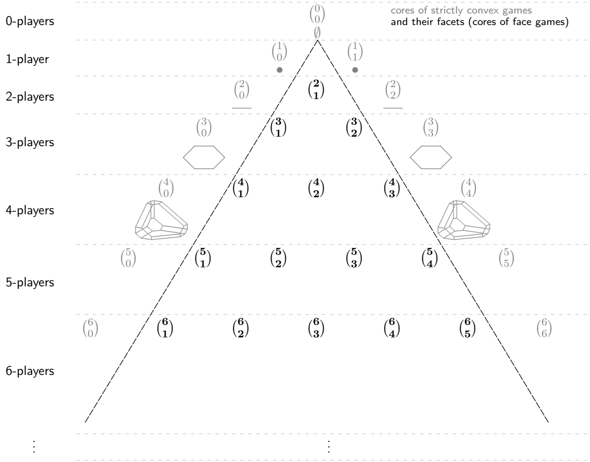

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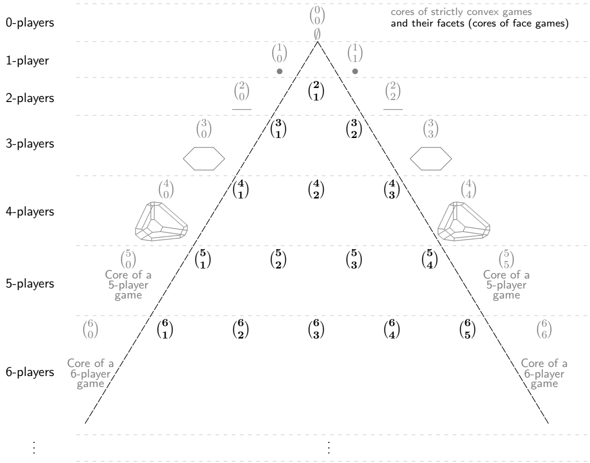

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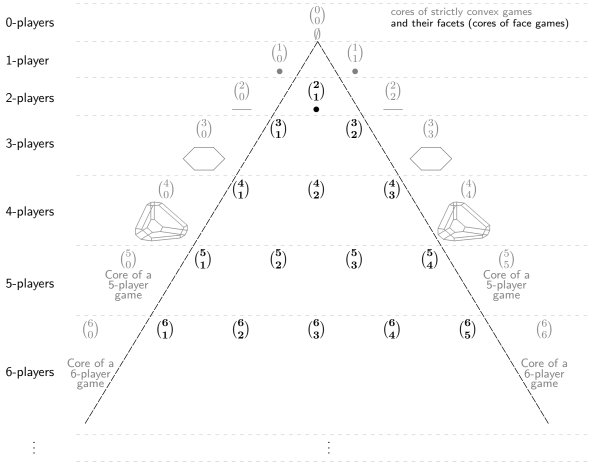

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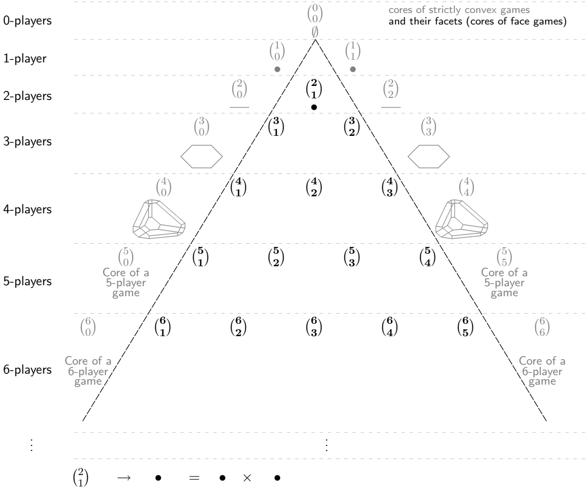

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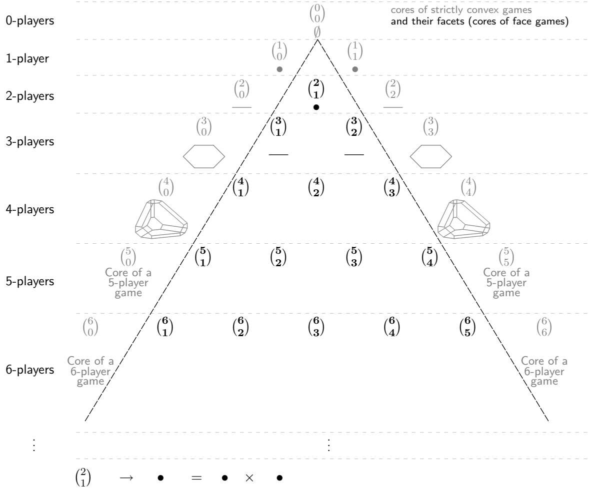

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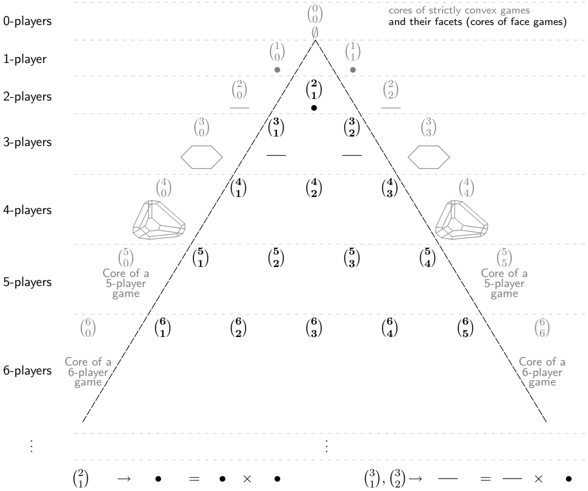

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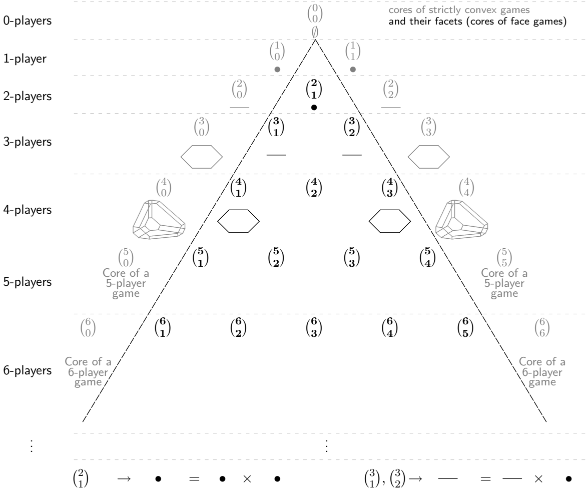

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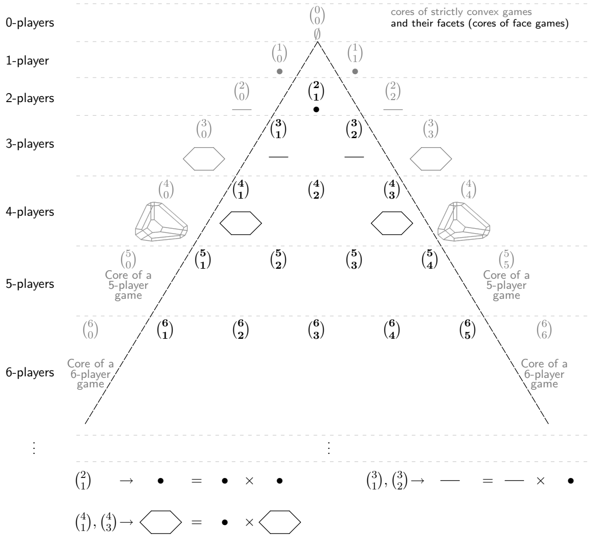

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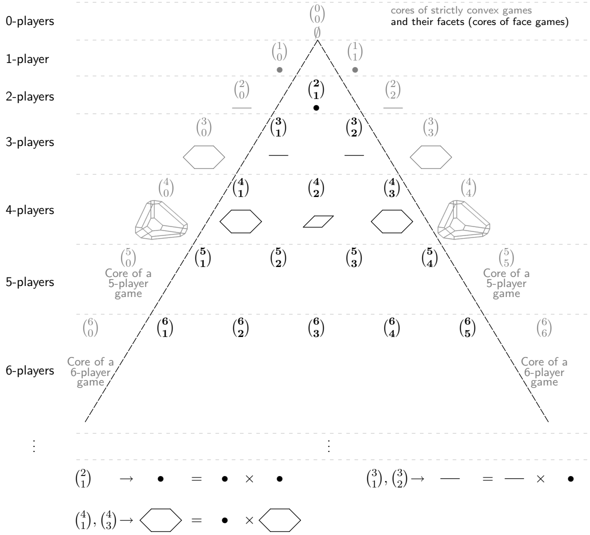

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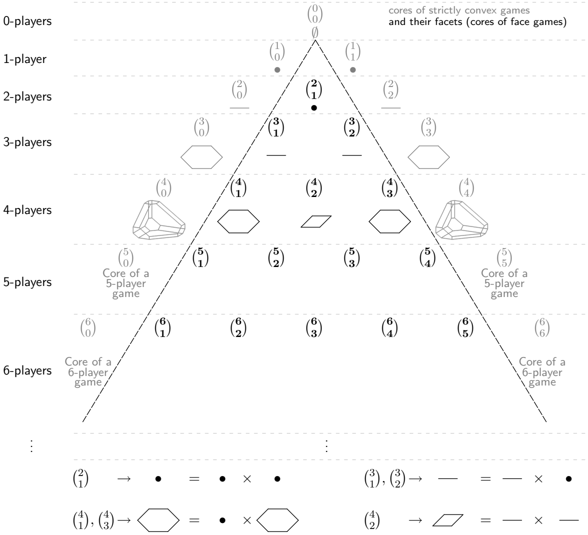

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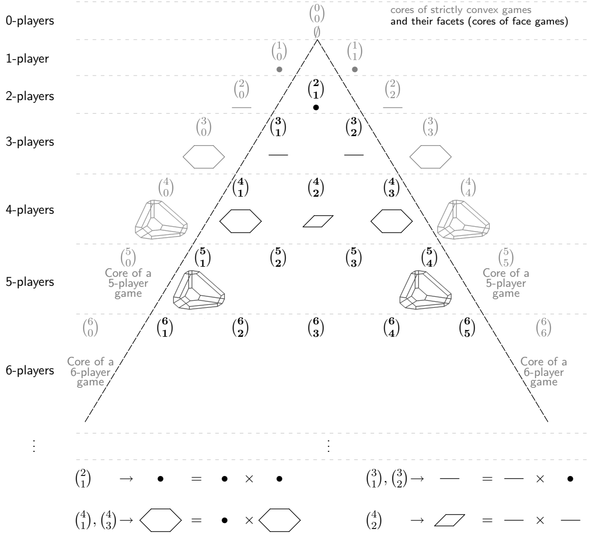

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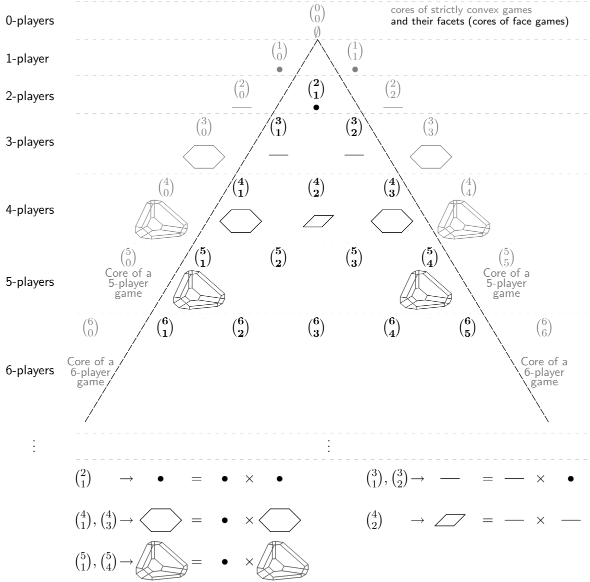

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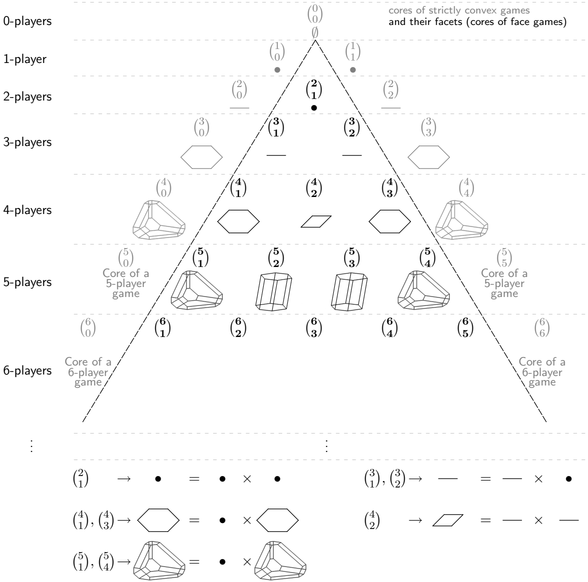

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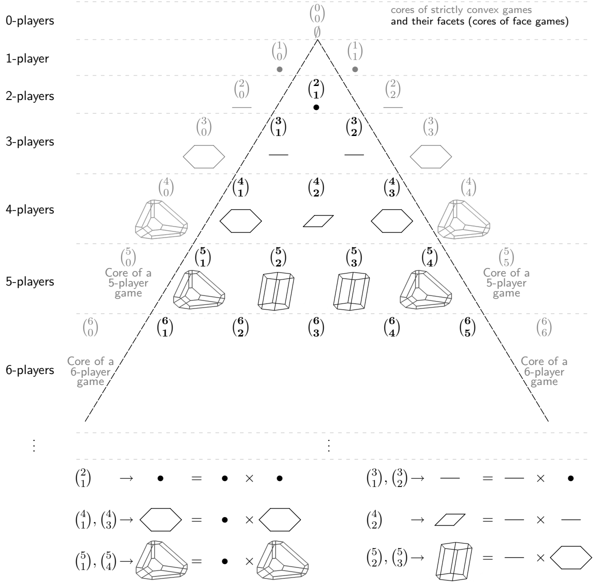

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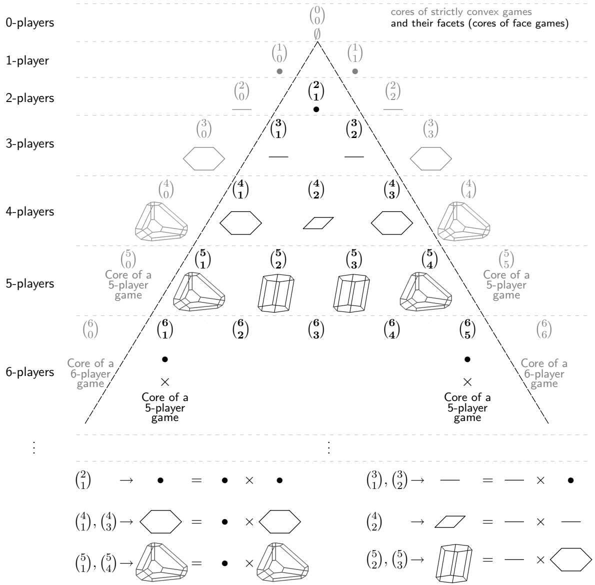

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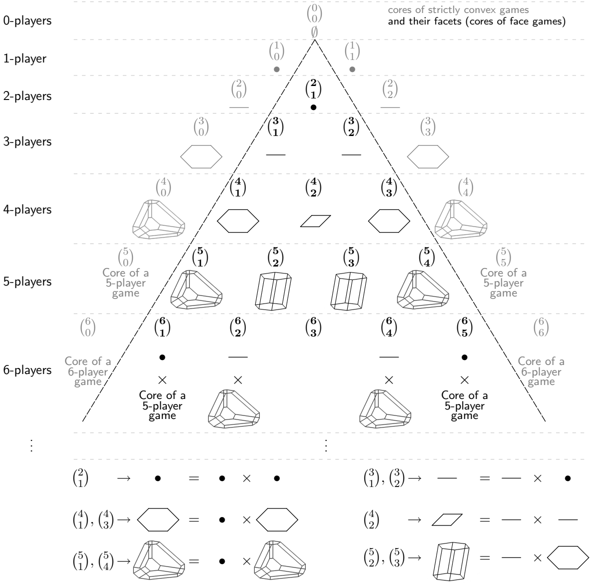

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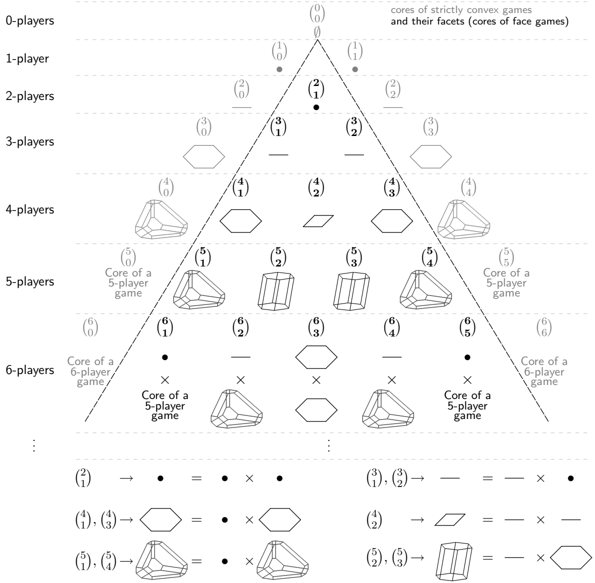

Cores of Convex Games and Pascal's Triangle

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Universidad de Vigo
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(joint with Estela Sánchez-Rodríguez)

July 4th, 2007