A two-step sequential linear programming algorithm for MINLP problems:

An application to gas transmission networks

Julio González-Díaz
Ángel M. González-Rueda
María P. Fernández de Córdoba

University of Santiago de Compostela
Technological Institute for Industrial Mathematics (ITMATI)

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GANESO$^{\mathrm{TM}}$: Gas Networks Simulation and Optimization

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- Main functionalities of **GANESO**$^{\mathrm{TM}}$:
  - Steady-state and transient simulation
    - Gas loss analysis
    - Gas quality tracking
    - Linepack control
  - Steady-state optimization
    - Network planning and design under uncertainty
  - Computation of tariffs for network access
  - Database management for indexing network scenarios
  - User Interface (daily usage of GANESO$^{\mathrm{TM}}$ by end-user)

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Nonlinear optimization

Stochastic programming

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Nonlinear optimization

Gas transmission networks

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Ingredients of the optimization problem

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Identify feasible gas flows

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Main problem constraints

- Meet demands (security of supply)
- Gas pressure is kept within specified bounds

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Different objective functions

- Minimize gas consumption at compressor stations
- Minimize boil-off gas at regasification plants
- Maximize network linepack
- Maximize/minimize exports of different zones
- Control bottlenecks

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Network flow problem

Network flow problem

Flow conservation constraints

$$\sum_{k \in A_i^{\text{ini}}} q_k - \sum_{k \in A_i^{\text{fin}}} q_k = c_i$$

$\forall i \in N^C$ demand nodes

$$0 \leq \sum_{k \in A_i^{\text{ini}}} q_k - \sum_{k \in A_i^{\text{fin}}} q_k \leq s_i$$

$\forall i \in N^S$ supply nodes

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$$\underline{q}_k \leq q_k \leq \bar{q}_k$$

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$\forall i \in N$ **pressure bounds**

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$\forall i \in N$ **pressure bounds**

Variables of the optimization problem

- Flow through each pipe
- **Pressure** at each node

Gass loss equations

Given a pipe between two nodes i and j, we have

$$p_i{}^2 - p_j{}^2 = \frac{16 L_k \lambda_k}{\pi^2 D_k^5} Z(p_m, T_m) R T_m |q_{ij}| q_{ij} + \frac{2g}{R T_m} \frac{p_i{}^2 + p_j{}^2}{2 Z(p_m, T_m)} (h_j - h_i)$$

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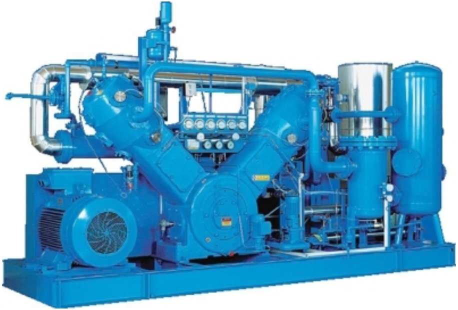
As many nonlinear constraints as pipes

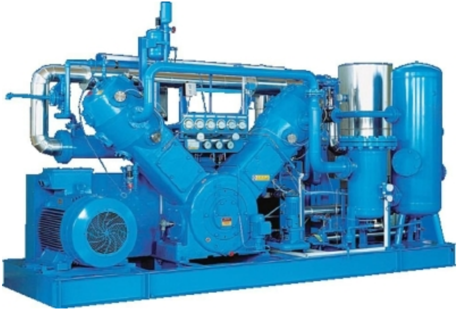
Gas consumption at compressors



Given input pressure p_i and output pressure p_j, we have

$$g_{ij} = \frac{1}{e_h H^c} \frac{\gamma}{\gamma - 1} Z(p_m, T_{in}) R T_{in} \left(\left(\frac{p_j}{p_i}\right)^{\frac{\gamma-1}{\gamma}} - 1 \right) q_{ij}$$

Gas consumption at compressors



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As many nonlinear constraints as compressors in the network

Nonlinear nonconvex optimization problem

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Obj. Function: $\min \sum_{k \in A^c} g_k$

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Flow conservation constraints

$$\sum_{k \in A_i^{\text{ini}}} q_k - \sum_{k \in A_i^{\text{fin}}} q_k = c_i \qquad \forall i \in N^C \text{ flow conservation at demand nodes}$$

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Nonlinear nonconvex optimization problem (continuous)

Obj. Function: $\min \sum_{k \in A^c} g_k$

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Size and complexity of real instances

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Spanish primary gas network

- ≈ 1000 variables (≈ 500 pipes and ≈ 500 nodes)
- ≈ 1000 constraints (and ≈ 2000 box constraints)
- ≈ 500 constraints are **nonlinear**

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To be solved routinely by the company

(A twist on) Sequential Linear Programming Algorithms

Approaches to solve the problem

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- **Global optimization algorithms** on approximations of the problem

  nonlinearities $\Longrightarrow$ piecewise linear functions $+$ integer variables

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How to solve this problem?

- **Global optimization algorithms** on approximations of the problem (cannot handle real-size problems)
  nonlinearities $\Longrightarrow$ piecewise linear functions $+$ integer variables
- **Local optimization algorithms** such as sequential linear programming, **SLP**, or sequential quadratic programming, **SQP**

Our initial approach

Classic SLP

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- We get a solution using **Classic SLP**

Our initial approach

Classic SLP + Control Theory

- We get a solution using **Classic SLP**
- We refine it using **control theory** by including some second order elements

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Additional network elements

Elements that require the use of binary variables

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- Different types of **control valves**
- **Operational ranges** of each compressor station
- Boil-off gas at regasification plants

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Mixed-integer nonlinear nonconvex programming problem

- $\approx$ 1000 continuous variables and 1000 constraints
- No more than 100-200 binary variables

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- **Step 1.** Study a **simplified version** of the problem to fix all binary choices

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How are these problems normally tackled?

Two-step algorithms

- **Step 1.** Study a **simplified version** of the problem to fix all binary choices
- **Step 2.** Apply **SLP**, **SQP**,... to the resulting **continuous problem**

Our two-step approach for MINLP problems

Classic SLP

- We get a solution using **Classic SLP**

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- **Step 2. Classic SLP.** Binary variables already **fixed**
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2SLP: **SLP-NTR** + **Classic SLP**

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Step 1 runs on the full model. No simplification needed

SLP-NTR (No Trust Region)

Nonlinear programming problem: **NLP**

$$\begin{aligned} \textbf{minimize} \quad & f(\boldsymbol{x}) \\ \textbf{subject to} \quad & \\ \textbf{inequality contraints} \quad & g_i(\boldsymbol{x}) \leq 0, \quad i = 1, \cdots, m \\ \textbf{equality constrains} \quad & h_j(\boldsymbol{x}) = 0, \quad j = 1, \cdots, l \\ \textbf{linear constraints} \quad & \boldsymbol{x} \in X = \{\boldsymbol{x} \in \mathbb{R}^n : A\boldsymbol{x} \leq b\} \end{aligned}$$

where f, g_i and h_j are **nonlinear** functions.

SLP-NTR (**N**o **T**rust **R**egion)

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SLP-NTR (No Trust Region)

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Hard to accommodate binary variables with the trust region

SLP-NTR (No Trust Region)

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\textbf{inequality constraints} & g_i(\boldsymbol{x}^k) + \nabla g_i(\boldsymbol{x}^k)^t(\boldsymbol{x} - \boldsymbol{x}^k) \leq 0 & i = 1, \cdots, m \\
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\textbf{linear constraints} & \boldsymbol{x} \in X = \{\boldsymbol{x} \in \mathbb{R}^n : A\boldsymbol{x} \leq b\} & \\
\textbf{trust region} & \cancel{-d_k \leq \boldsymbol{x} - \boldsymbol{x}^k \leq d_k} &
\end{array}
$$

- We **remove** the constraints that define the **trust region**

SLP-NTR (No Trust Region)

SLP-NTR

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- We solve the linearization of **NLP** about $\boldsymbol{x}^k$, LP($\boldsymbol{x}^k$):

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Theoretical justification for the removal of the trust region?

SLP-NTR vs classic SLP (in the continuous case, NLP problems)

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Classic SLP

- ++ Accumulation points of the sequence are **KKT** points of **NLP**
- ++ In practice it normally converges

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Our contribution

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NLP problems

Theoretical foundation for the **SLP-NTR algorithm**

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Heuristic approach based on the **SLP-NTR algorithm**

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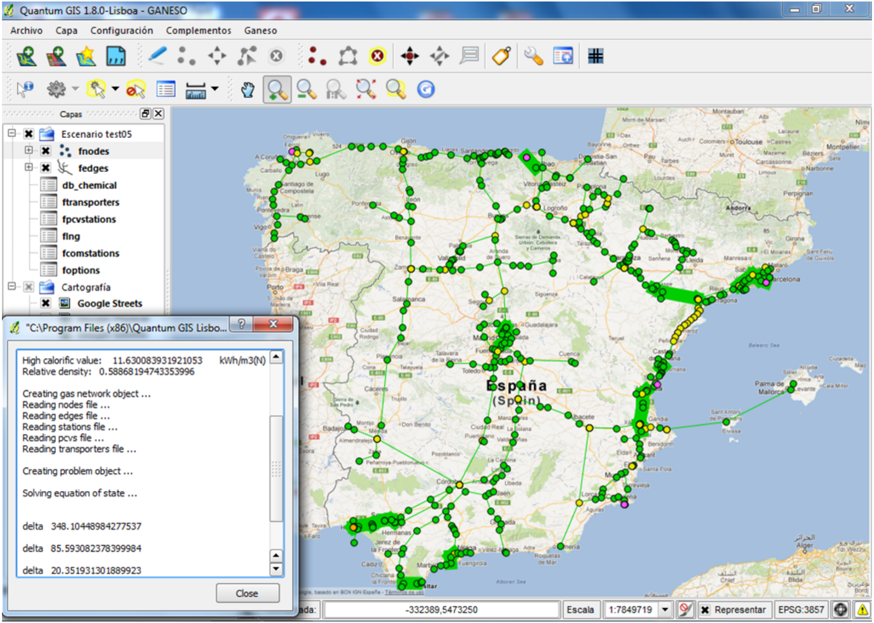
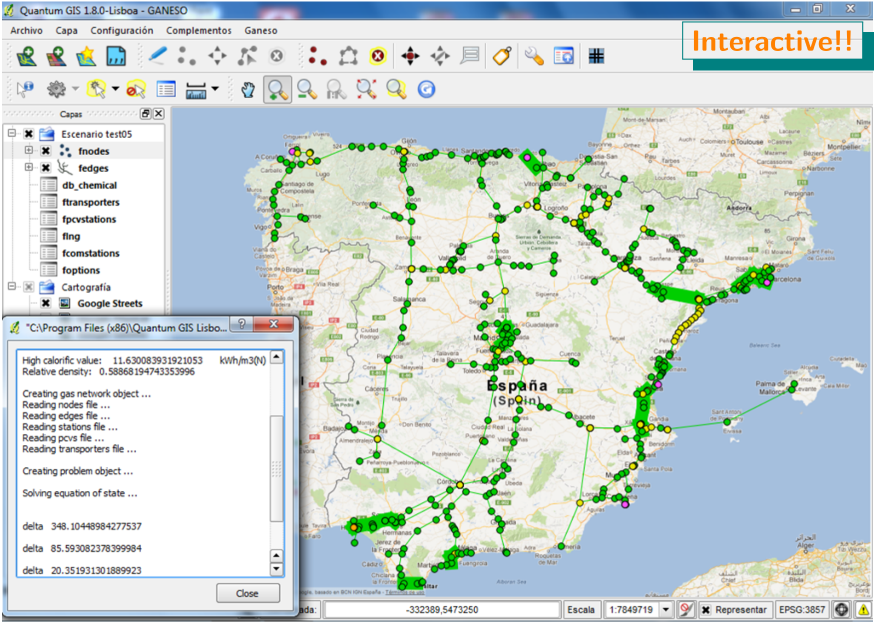
MINLP stochastic problems

- Long-term **infrastructure planning under uncertainty** (prices and demands)
- Implementation of a **lagrangian decomposition algorithm** (progressive hedging) that uses **SLP-NTR algorithm** to solve the MINLP subproblems

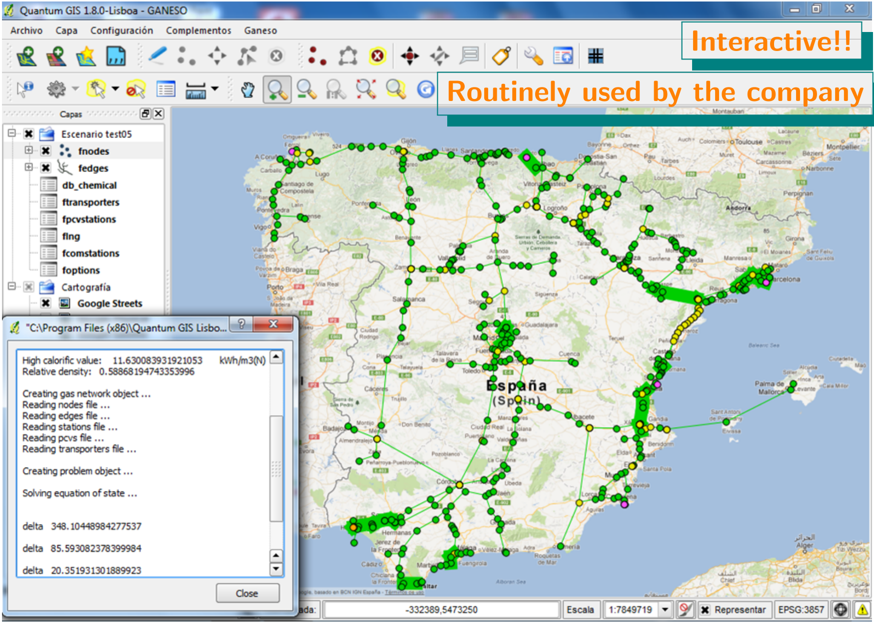
GANESO^{TM} user interface



GANESO$^{\mathrm{TM}}$ user interface



GANESO™ user interface



Interactive!!

Routinely used by the company

Numerical results

1. Comparisons on the Spanish gas transmission network
2. Comparisons on related gas transmission problems
3. Comparisons on multicommodity flow problems

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Work in progress

Tests on the Spanish Gas Transmission Network (NLP)

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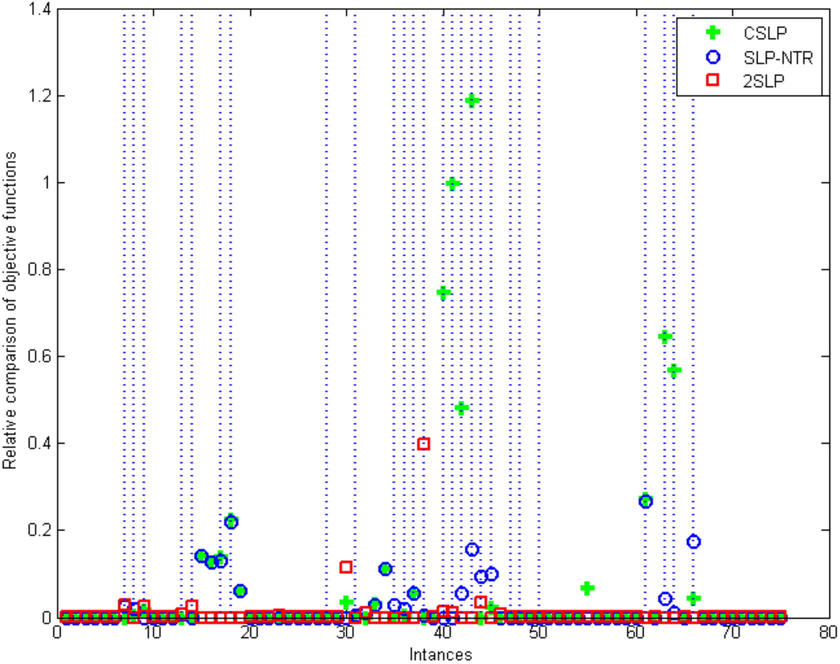

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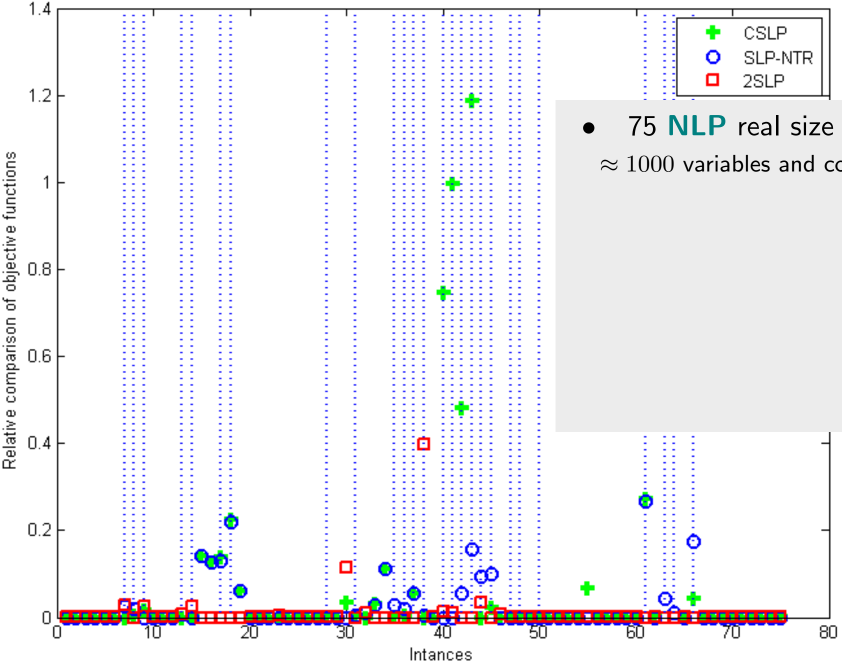

- 75 **NLP** real size instances

  ≈ 1000 variables and constraints

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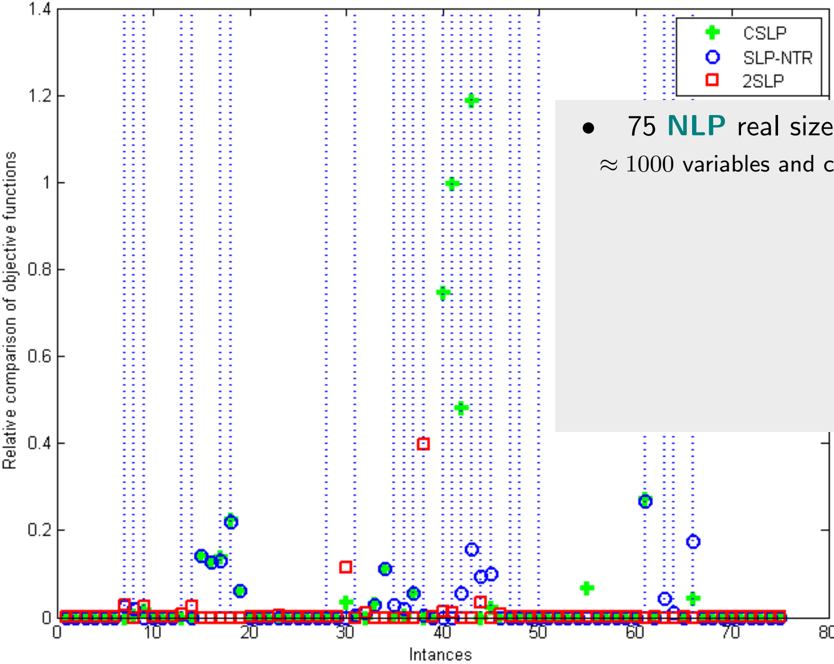

CSLP

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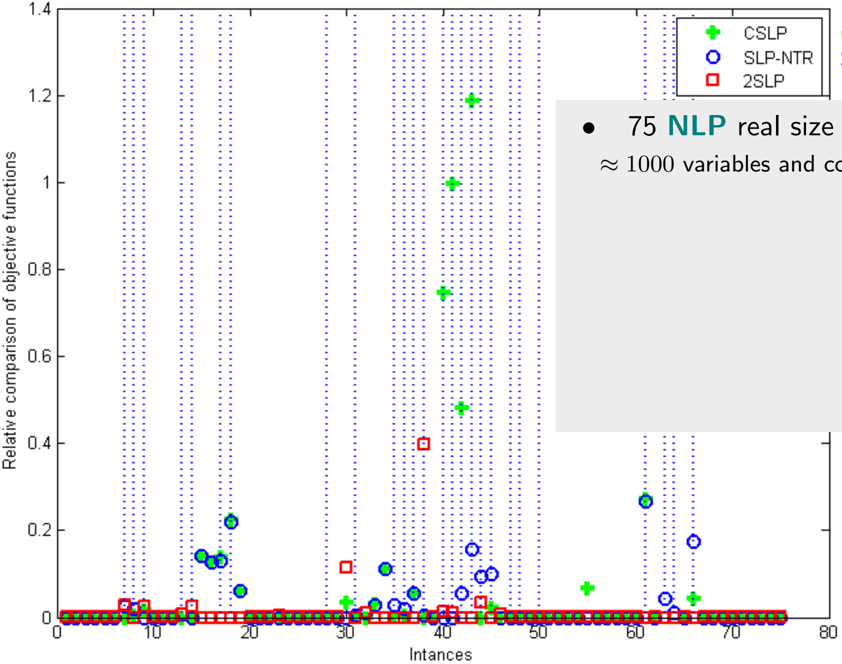

CSLP
SLP-NTR

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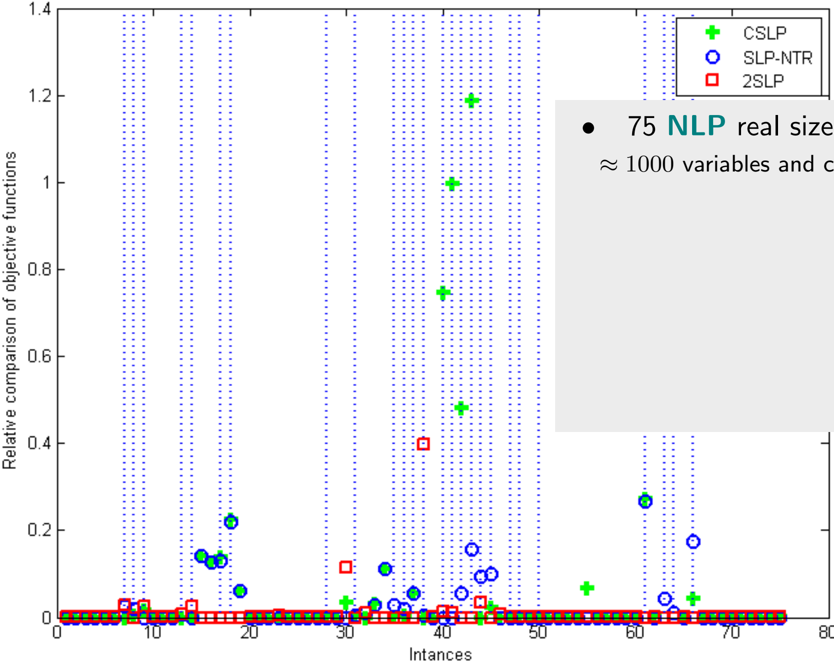

CSLP
SLP-NTR
2SLP

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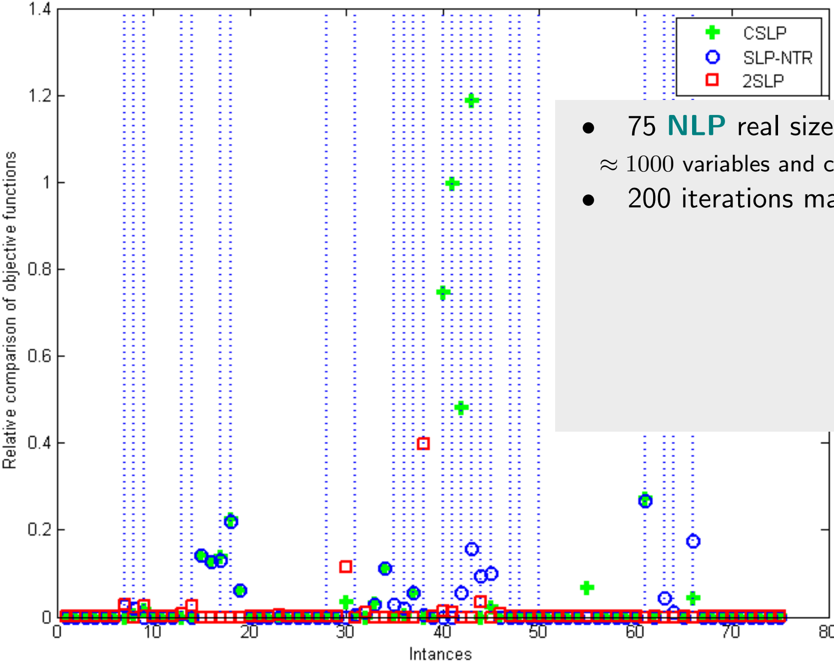

CSLP
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- 200 iterations maximum

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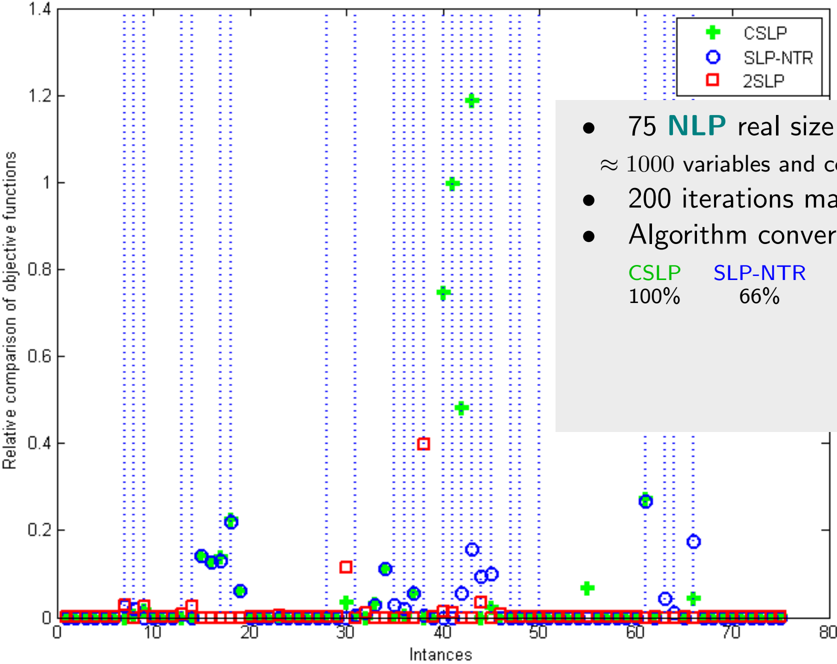

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CSLP	SLP-NTR	2SLP
100%	66%	100%

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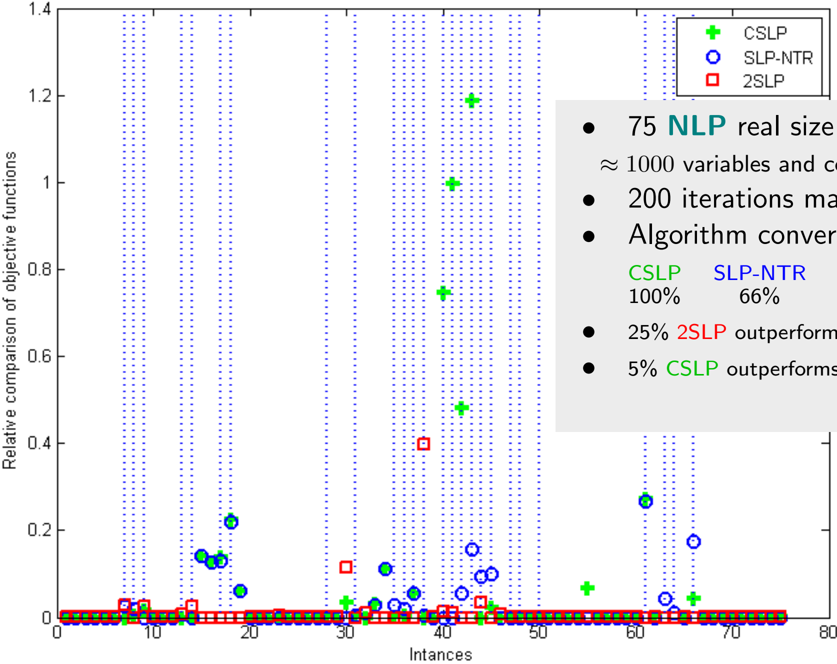

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- 25% 2SLP outperforms CSLP
- 5% CSLP outperforms 2SLP

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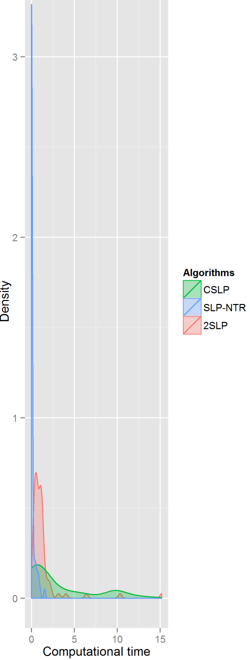

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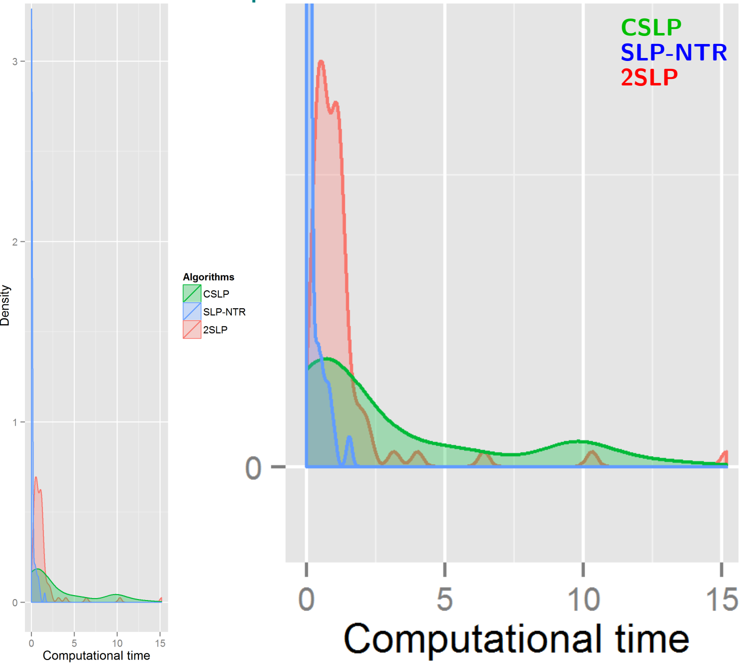

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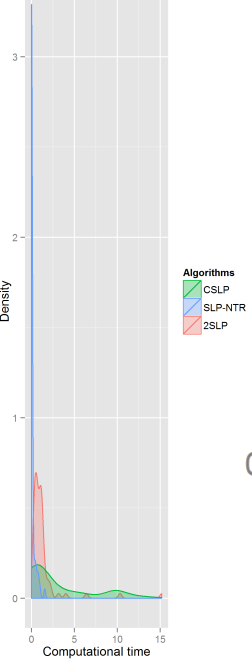

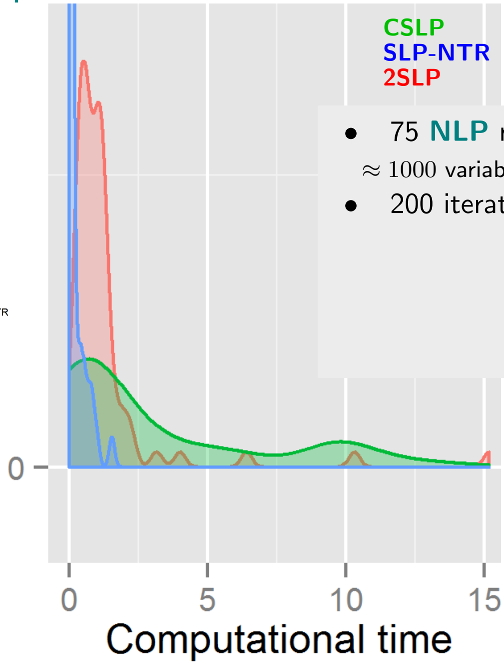

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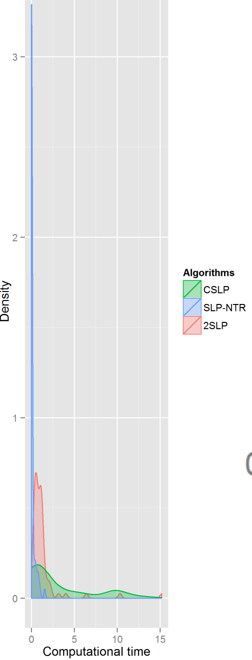

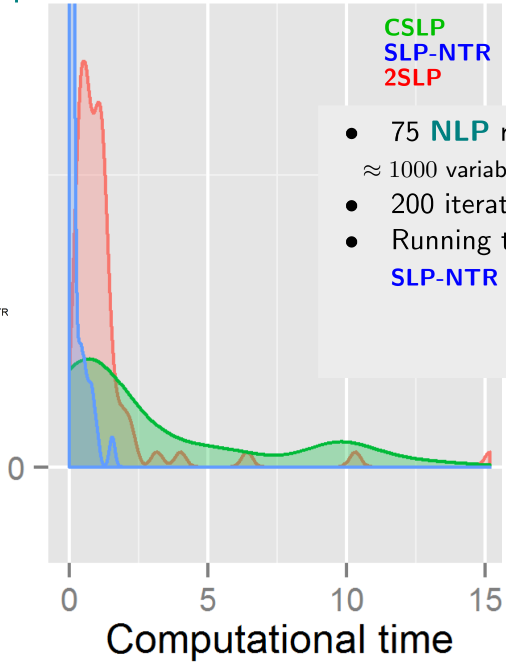

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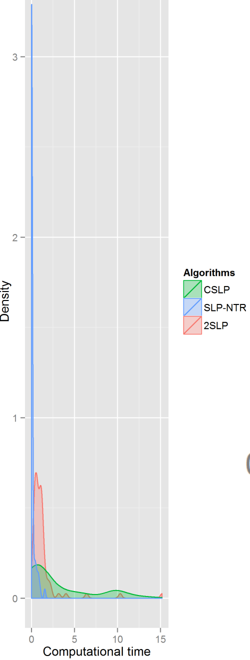

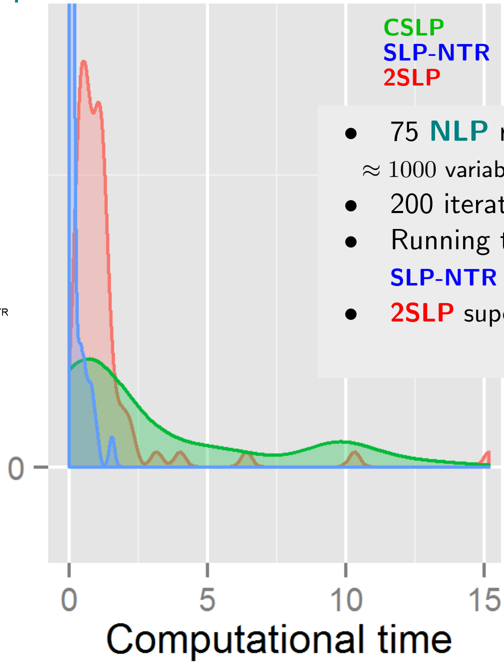

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- Running times:
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- **2SLP** superior performance

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- Real size instance with 10 binary variables
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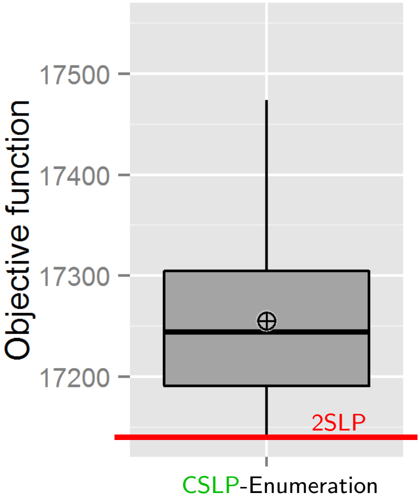

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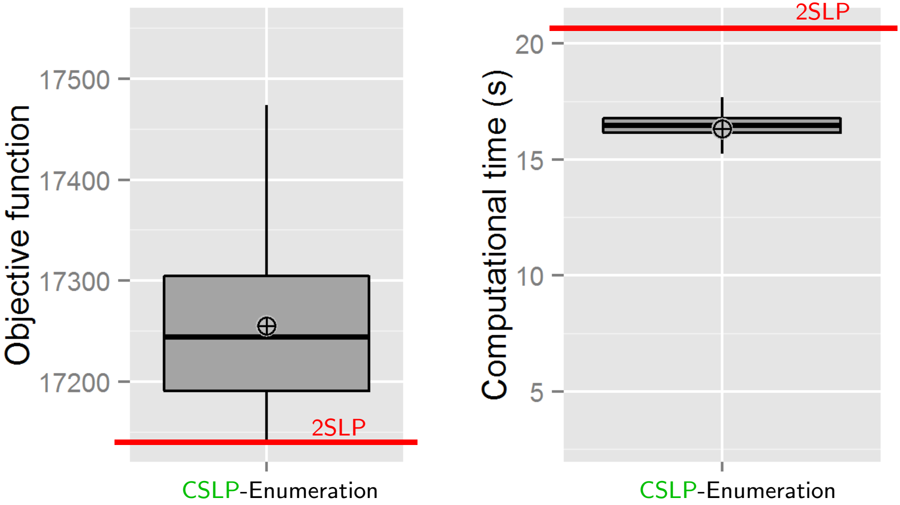

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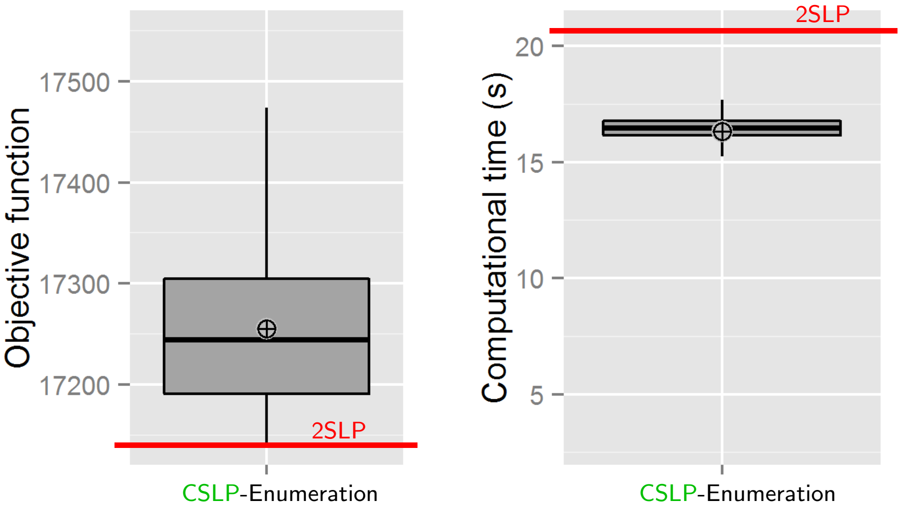

- **Next task.** Designing a full set of test instances

Tests on the Belgian Gas Transmission Network

(de Wolfe and Smeers, 2000)

- Slightly different model of the gas transmission problem
- Small example: ≈ 50 variables and constraints

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NLP problem	**CSLP**	**SLP-NTR**	**2SLP**	BARON	Knitro
Objective function	91.0562	91.0562	91.0562	91.0562	91.0562
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Tests on multicommodity flow problems (NLP)

- Linear constraints and **nonlinear** objective function

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Problem	$\|N\|$	$\|E\|$	$\|T\|$	Constr.	Variab.	z_{opt}	Relative error		
Planar problems							**CSLP**	**SLP-NTR**	**2SLP**
P30	30	150	92	2760	13800	4.445×10^7	0.0074	0.0085	0.0074
P50	50	250	267	13350	66750	1.212×10^8	0.0202	0.0212	0.0202
P80	80	440	543	43440	238920	1.819×10^8	0.0174	0.0188	0.0174
P100	100	532	1085	108500	577220	2.291×10^8	0.0212	0.0219	0.0212
Grid problems									
G1	25	80	50	1250	4000	8.336×10^5	0.0003	0.0054	0.0004
G2	25	80	100	2500	8000	1.727×10^6	0.0006	0.0089	0.0005
G3	100	360	50	5000	18000	1.532×10^6	0.0000	0.0065	0.0002
G4	100	360	100	10000	36000	3.055×10^6	0.0000	0.0066	0.0000
G5	225	840	100	22500	84000	5.079×10^6	0.0000	0.0069	0.0000
G6	225	840	200	45000	168000	1.051×10^7	0.0001	0.0108	0.0002
G7	400	1520	400	160000	608000	2.607×10^7	0.0000	0.0031	0.0000
Telecommunication-like problems									
N22	14	22	23	322	506	1.871×10^3	0.0131	0.0131	0.0131
N148	58	148	122	7076	18056	1.402×10^5	0.0000	0.0002	0.0000
Transportation problems									
S-F	24	76	528	12672	40128	3.202×10^5	0.0050	0.0051	0.0050

Tests on multicommodity flow problems (NLP)

- Linear constraints and **nonlinear** objective function (feasibility ✓)
- Benchmark test sets available (Babonneau et al. 2004)

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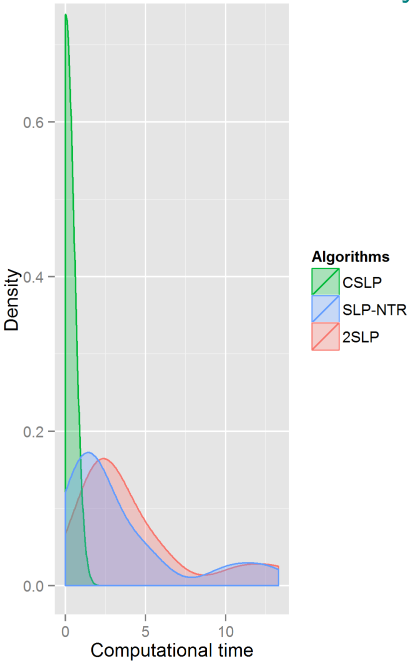
- All approaches very competitive in terms of objective function

Tests on multicommodity flow problems (NLP)



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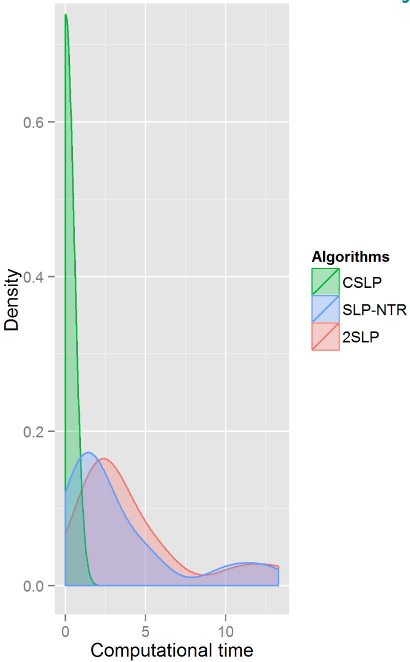

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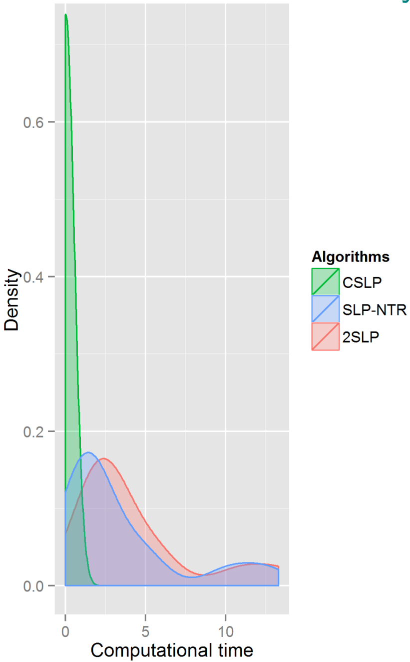

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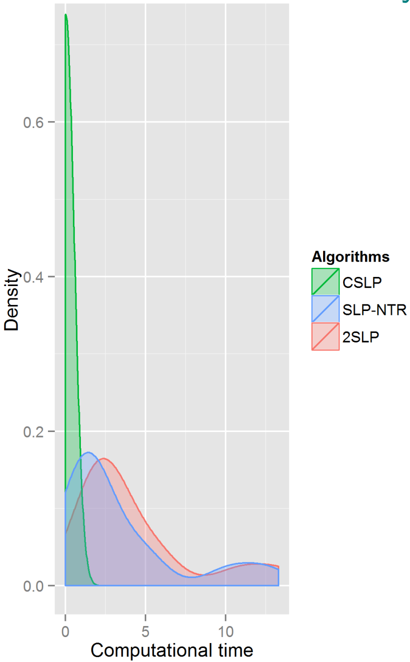

- Now **CSLP** is the fastest one   Why??
- Apparently, the trust region helps to solve faster **very large** linearized **subproblems**

A two-step sequential linear programming algorithm for MINLP problems:

An application to gas transmission networks

Julio González-Díaz
Ángel M. González-Rueda
María P. Fernández de Córdoba

University of Santiago de Compostela
Technological Institute for Industrial Mathematics (ITMATI)

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