The Role of Commitment in Repeated Games

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Motivation

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- Commitment

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- Repeated games

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- Unilateral commitments in repeated games

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- Delegation games

Outline

1. Virtually Subgame Perfect Equilibrium
   - Some Examples
   - Formal Definitions
   - Discussion
2. Unilateral Commitments
   - Definitions
   - Delegation Models and Unilateral Commitments
   - Results
3. Conclusions

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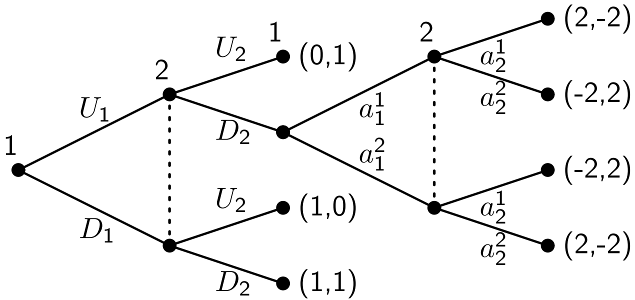
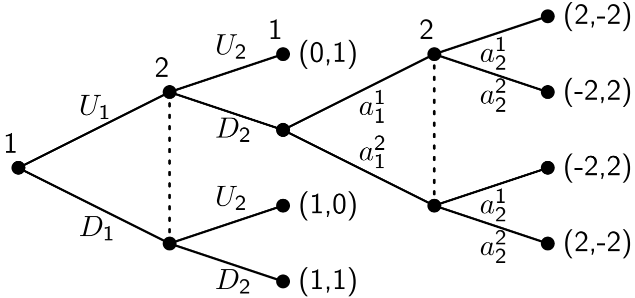
Virtually Subgame Perfect Equilibrium

First Example



Virtually Subgame Perfect Equilibrium
First Example

Pure Strategies !!!



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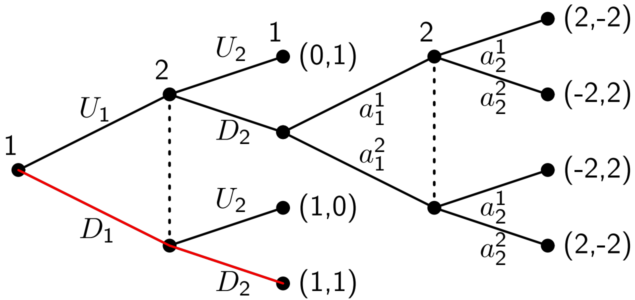

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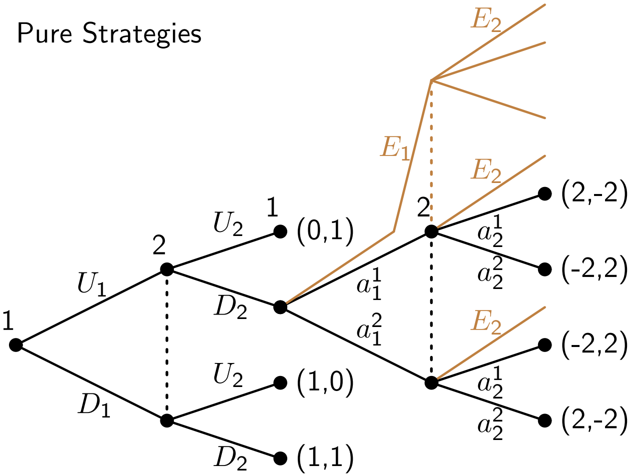

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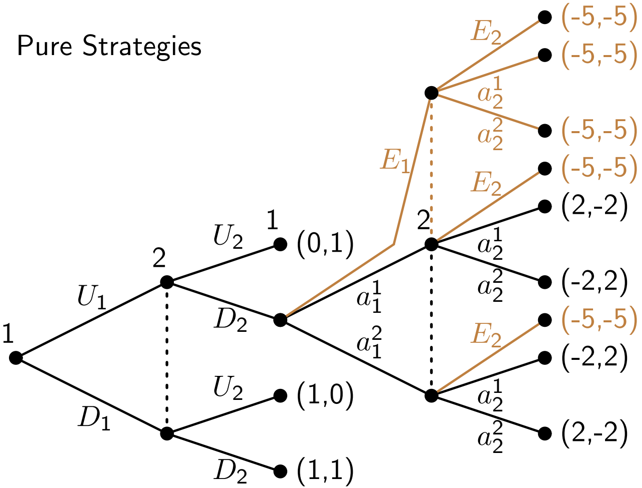

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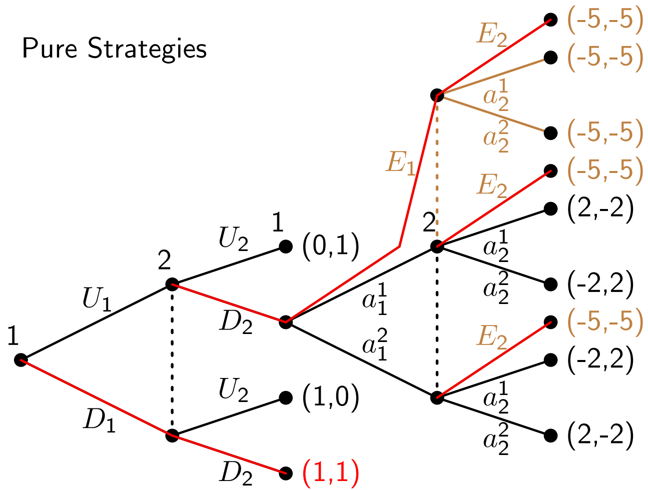

Virtually Subgame Perfect Equilibrium

Second Example

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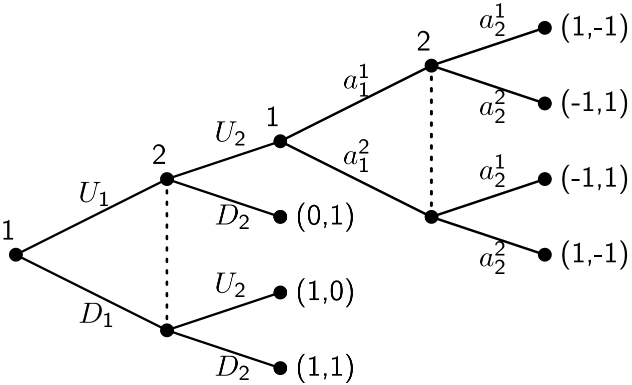

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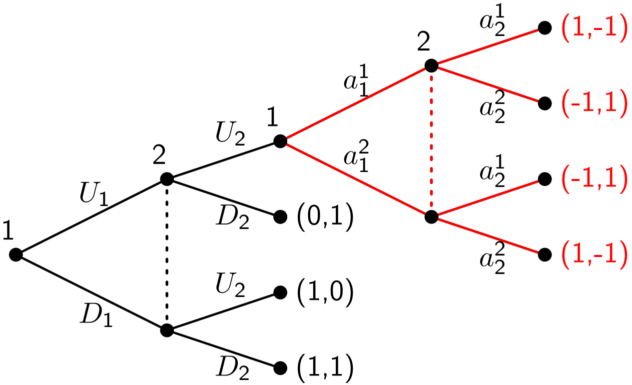

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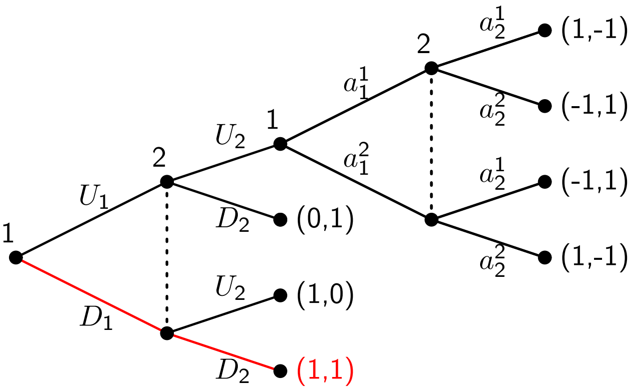

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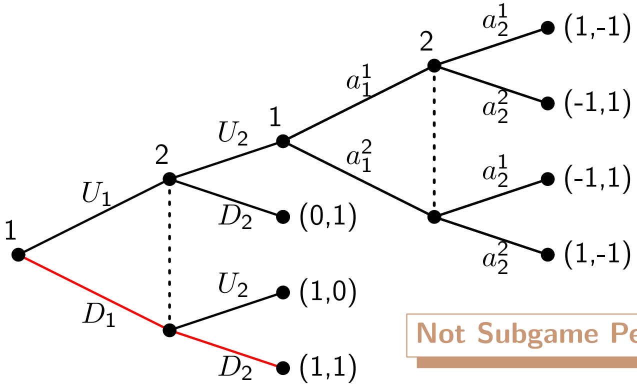

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Virtually Subgame Perfect Equilibrium

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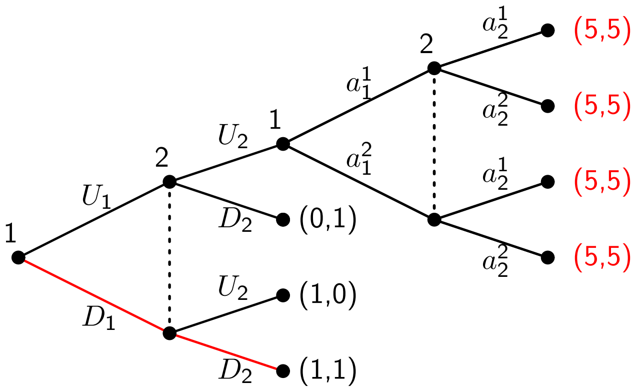

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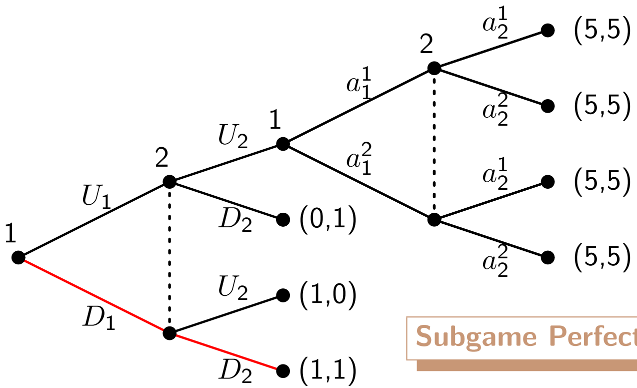

Subgame Perfect!!!

Virtually Subgame Perfect Equilibrium

Formal Definitions

Virtually Subgame Perfect Equilibrium
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- Let Γ be an extensive-form game and let x and σ be a *single-node* information set and a strategy profile, respectively. Then, Γ_x denotes the *subgame* of Γ that begins at node x and σ_x the restriction of σ to Γ_x.

▸ Example

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- Now, let Γ be an extensive-form game, σ a strategy profile of Γ, and x a single-node information set. Then, the subgame Γ_x is *σ-relevant* if either (i) $\Gamma_x = \Gamma$, or (ii) there are a player i, a strategy σ'_i, and a single-node information set y such that Γ_y is σ-relevant and node x is reached by $(\sigma_{-i}, \sigma'_i)_y$. Example

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- Let Γ be an extensive-form game. The strategy profile σ is a *virtually subgame perfect equilibrium* of Γ if for each σ-relevant subgame Γ_x, then σ_x is a Nash equilibrium of Γ_x.

Virtually Subgame Perfect Equilibrium

Discussion

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Subgame Perfect Vs Virtually Subgame Perfect

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Why do we need VSPE?

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Subgame Perfect Vs Virtually Subgame Perfect

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We cannot use the classic results for the existence of SPE

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Complete Information

Delegation Models and Unilateral Commitments

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Fershtman et al (1991)

Delegation Models and Unilateral Commitments

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Players: 2 principals, 2 agents

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$$v_i = \min_{a_{-i}\in A-i} \max_{a_i\in A_i} \varphi_i(a_i, a_{-i})$$

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Nash Folk Theorem (without UC)

G must have a Nash equilibrium in which some player gets more than his minmax payoff

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No assumption is needed for the Nash folk theorem with UC.

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Subgame Perfect Folk Theorem (without UC)

G must have a pair of Nash equilibra in which some player gets different payoffs

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Let $\bar{a} \in A$ be a Nash equilibrium of G. Then, the game $U(G)$ has a VSPE with payoff $\varphi(\bar{a})$.

The Folk Theorems
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No assumption is needed for the VSPE folk theorem when we have two stages of commitments.

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Theorem 1 No assumptions for the Nash folk theorem with UC.

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$\Longrightarrow$ $U(U(G(\delta, T)))$ has a VSPE

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Proposition 2 Let $\bar{a} \in A$ be a Nash equilibrium of G. Then, the game $U(G)$ has a VSPE with payoff $\varphi(\bar{a})$.

Theorem 2

No assumption is needed for the VSPE folk theorem when we have two stages of commitments.

Proof

1. Apply Theorem 1 to $G(\delta, T)$ $\Longrightarrow$ $U(G(\delta, T))$ has a Nash
2. Apply Proposition 2 to $U(G(\delta, T))$
$\Longrightarrow$ $U(U(G(\delta, T)))$ has a VSPE

- Moreover, the VSPE can be chosen such that

The Folk Theorems
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- Moreover, the VSPE can be chosen such that the subgame that begins after the first stage of commitments has a unique Nash payoff

The Folk Theorems
Discussion

Theorem 2

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Remarks

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Remarks

- Are two stages of commitments natural??
- We "allow for" commitments on commitments
- President $\longrightarrow$ Manager $\longrightarrow$ Director

	Without UC	**1 stage of UC**	**2 stages of UC**
Nash Theorem Infinite Horizon	None (Fudenberg and Maskin, 1986)		
(Virtual) Perfect Th. Infinite Horizon	Non-Equivalent Utilities (Abreu et al., 1994)		
Nash Theorem Finite Horizon	Minimax-Bettering Ladder (González-Díaz, 2003)		
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The State of Art

Necessary and Sufficient Conditions for the Folk Theorems

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Outline

1. Virtually Subgame Perfect Equilibrium
    - Some Examples
    - Formal Definitions
    - Discussion
2. Unilateral Commitments
    - Definitions
    - Delegation Models and Unilateral Commitments
    - Results
3. Conclusions

Conclusions

Our contribution

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- UC lead to weaker assumptions for the folk theorems.

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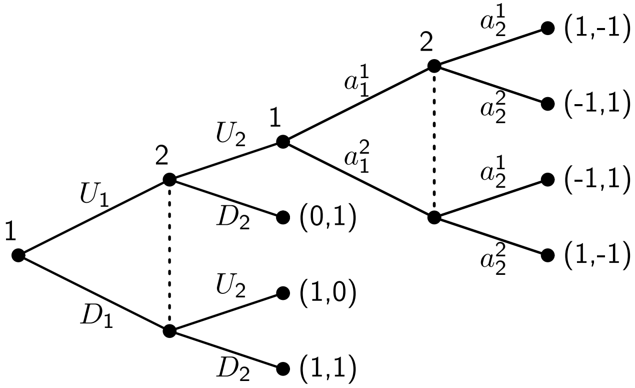
- UC lead to weaker assumptions for the folk theorems.
- Nonetheless, some assumptions are still needed for some VSPE folk theorems.

Thanks



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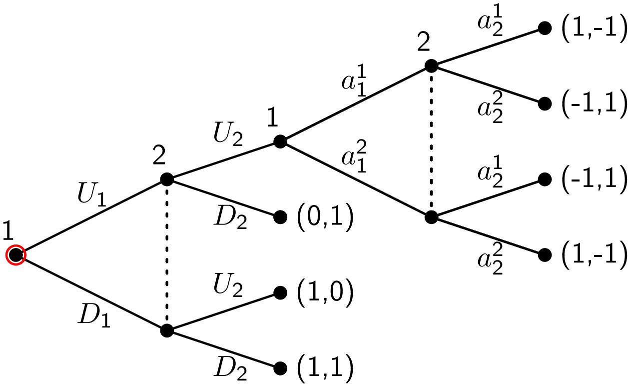

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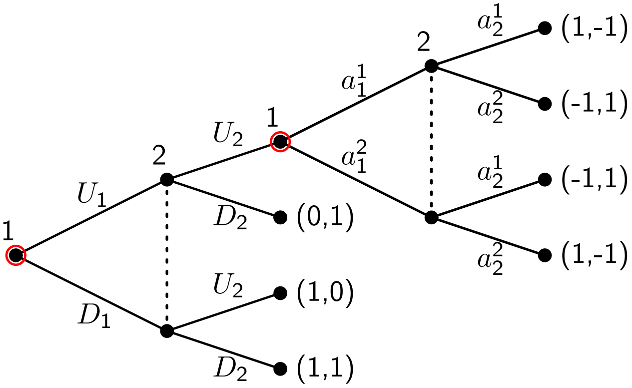

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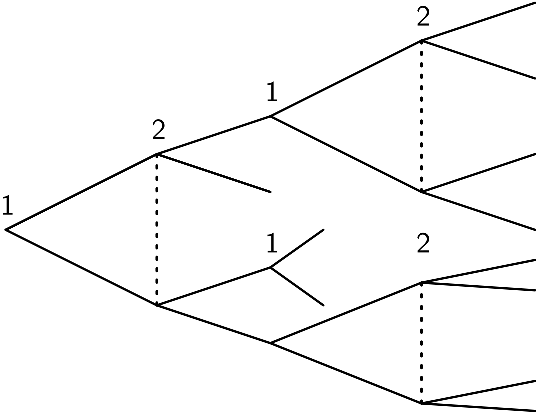

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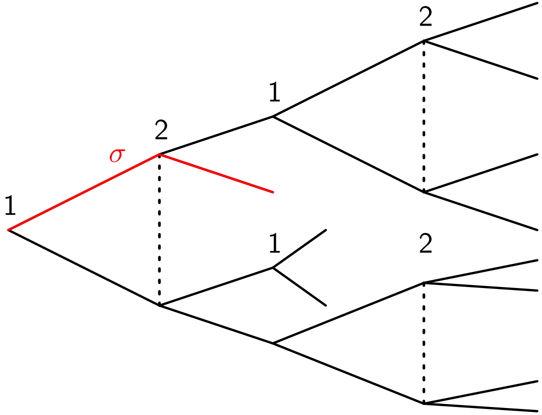

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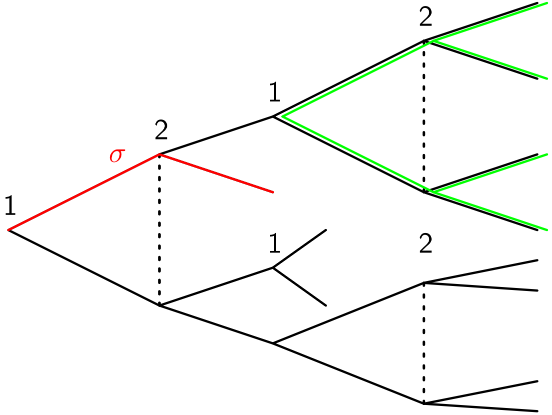

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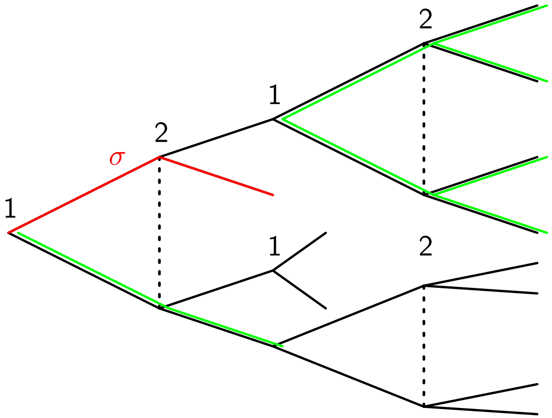

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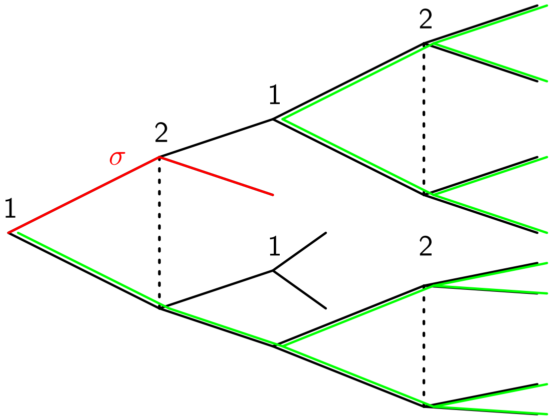

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Proof

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Strategy for a player i

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1. Commitment: $\bar{S}_i^c :=$ "If $\bar{a}$ is played in the first stage, then I play $\bar{a}_i$ forever"

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2. Strategy:
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    - If someone has deviated from the commitment i punishes him forever

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