MR2654274 (2012b:91001) 91-01 (91A10 91A12 91A18 91A35 91A80)

González-Díaz, Julio; **García-Jurado, Ignacio**; **Fiestras-Janeiro, M. Gloria**

★An introductory course on mathematical game theory.

Graduate Studies in Mathematics, 115.

American Mathematical Society, Providence, RI; Real Sociedad Matemática Española, Madrid, 2010. *xiv*+324 *pp.* *ISBN* 978-0-8218-5151-7

This book presents an introductory and up-to-date course on game theory addressed to mathematicians and economists, and also to other scientists having a basic mathematical background and working in such areas as political science, biology, psychology and computer science. It introduces us to the mostly essential topics of game theory like cooperative and noncooperative game models, the theory of equilibrium and decision theory. The book is self-contained and written very rigorously, but on the other hand, it is also very friendly to the reader, containing a lot of explanations and interpretations of game theory notions, as well as very many examples describing and analyzing various economic and other models with an application to game theory results. This makes it very suitable for graduate and advanced undergraduate courses on game theory.

The book consists of five main chapters. Chapter 1, Introduction to Decision Theory, provides a brief introduction to mathematical decision theory for problems with one *decision maker*, commonly known as *decision theory*. It acquaints the readers with such basic notions as *decision problems*, *preferences* and *utility functions*, and provides some important facts about their properties and mutual relationships.

Chapter 2, Strategic Games, introduces us to different static models that describe interactive situations among several players, where all the players make their decisions simultaneously and independently. It starts from a general notion of *strategic games* (*games in normal form*) and *Nash equilibrium*. Next, a lot of types of such games are widely analyzed in terms of the existence of Nash equilibria and properties of the players' optimal strategies. We can list here *two-person zero-sum games* in a general setting, and *matrix* and *bimatrix games*. The next part of this chapter analyzes possible properties of Nash equilibria, that is, it considers several refinements of this notion and also its generalization. In particular, *perfect*, *proper* and *undominated* Nash equilibria, and *correlated equilibria* are widely discussed and illustrated through many examples and comments, and also their mutual relationships are shown. The chapter ends with the section "Epistemic Foundations of Game Theory", which considers games together with an *information model with common prior*. This provides a convenient tool to model situations in which players may have different information when playing the game. It is worth mentioning here that this chapter also contains a set of basic fixed-point theorems and some other ones related to convex sets.

Chapter 3, Extensive Games, considers some modifications of an n-person finite strategic game where the players can take their decisions in a number of stages (not necessarily constant), and where the information of a player may be different on every stage, and additionally, it may depend on all the payers' decisions made before. For such *extensive games* some subfamily of games with *perfect recall* is analyzed in terms of the existence of Nash equilibria in *behavior strategies*.

Also some essential refinements of Nash equilibrium in extensive games, like *subgame perfect equilibrium*, *sequential equilibrium* and *perfect equilibrium*, are widely discussed in terms of their existence and mutual relationships. *Repeated games* are the next topic considered here. In general, they are extensive games in which the number of stages is infinite. The three types of equilibrium mentioned above are also studied for repeated games. This chapter ends with several so-called *folk theorems*, describing the existence of a Nash equilibrium with a special structure.

Chapter 4, Games with Incomplete Information, considers games in which some important feature of a game is unknown to some of the players in the sense that a player might lack some information about the set of players, the set of actions, or the payoff functions. It starts by defining the notions of *Bayesian game* and *Bayesian Nash equilibrium*, two fundamental concepts for the analysis of games with incomplete information. The problem of the existence of such an equilibrium in Bayesian games and its relationships with sequential equilibria is considered. This theory is applied in an analysis of the *chain store paradox*. Bayesian games are also applied to *auction theory* and to a description of *mechanism design* and *revelation principle*. Also *multistage games with incomplete information* are analyzed including the relationship between perfect Bayesian equilibria and sequential equilibria for these games. This chapter ends with an outline of Harsanyi's approach to extensive-form games with *imperfect information*.

Chapter 5, Cooperative Games, starts with a definition of the classical *nontransferable utility games* and next, two solutions, the *Nash solution* and the *Kalai-Smorodiński solution* for *bargaining problems*, are widely discussed with the help of an axiomatic approach. The second part of this chapter is devoted to *transferable utility games* and different solution concepts for them. We can list here the *core*, the *Shapley value* and the *nucleolus*. All three solutions are discussed here in terms of their existence, and their properties are widely studied. Next, some approaches from noncooperative and cooperative games are discussed which allows the solutions of cooperative games to be adapted to find solutions of noncooperative ones. The theory and the results presented here are applied next to find a solution of such classical games like *airport games*, *bankruptcy games*, *voting games*, *linear production games*, *maximum flow games* and *inventory games*.

To sum up, one can see that this book is very valuable and has a very "fresh" depiction of the topic. It is worthy of recommendation.

Reviewed by *Tadeusz Radzik* (Jelenia Góra)