A Non-cooperative Approach to Bankruptcy Problems*

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Abstract

We propose an elementary game form that allows to obtain the allocations proposed by any acceptable bankruptcy rule as the unique payoff vector of the corresponding Nash equilibria.

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1 Introduction

A bankruptcy problem describes a case in which a planner has to allocate a given amount of a divisible good E among a set N of agents, when their claims $(d_i)_{i \in N}$ exceed the available amount (that is, $\sum_{i \in N} d_i > E$). Most rationing situations can be given this form. Relevant examples are the execution of a will with insufficient assets, the allocation of a commodity with excess demand in a fixed price setting, the collection of a given amount of taxes, and, of course, the liquidation of a bankrupt firm among its creditors. Rationing problems encompass a wide range of distributive situations and are analytically very simple (indeed, a bankruptcy problem can be summarized by a triple (N, E, d)).

The literature on bankruptcy problems is large and keeps growing. The main contributions refer to the analysis of different solutions following an axiomatic approach or translating to this context some of the standard solutions to coalitional games. The reader is referred to the works of Moulin (2002) and Thomson (2003) for comprehensive surveys of this literature and for a deeper discussion of the examples mentioned above.

There is also a number of contributions that study different strategic aspects of the problem (see Thomson (2003, # 7)). The first one already appears in O'Neill's seminal paper (see O'Neill (1982, # 5.2)), where the minimal overlap rule is analyzed as the Nash equilibrium of a non-cooperative game. Chun (1989) introduces a different strategic consideration in the bankruptcy problem by allowing the agents involved to propose solution concepts, rather than allocations. He devises a procedure that converges to the outcome associated with the constrained equal awards rule. A dual formulation is presented in Herrero (2003); in this case the procedure converges to the constrained equal losses rule. Sonn (1992) obtains the constrained equal awards rule as the limit of a process in which each player makes a proposal for someone else, who either accepts and leaves or rejects and takes the place of the proposer. Dagan et al. (1997) define a sequential game whose unique subgame perfect equilibrium outcome corresponds to the allocation of a given resource monotonic and consistent rule (an extension of the result in Serrano (1995)). The sequential game, which depends on the consistency assumption, can be summarized as follows. The player with the highest claim proposes a distribution of the amount available. The other agents can either accept and leave, or else reject and leave, obtaining what the two-person rule recommends for the problem made of him and the proposer, with a budget made of the amounts allotted initially for them by the proposer. See also Dagan et al. (1999) for a further extension.

Along these lines, Corchón and Herrero (2004) discuss the implementation of bankruptcy rules when the proposals made by the agents are bounded by their claims. Herrero et al. (2003) provide an experimental analysis of the strategic behaviour in bankruptcy problems.

Somehow between the axiomatic and the strategic approaches there are those contributions on the manipulation of the rules via merging or splitting claims (see de Frutos (1999), Ju (2003) and Moreno-Ternero (2004)).

In this paper we provide a non-cooperative support to the resolution of bankruptcy problems. The basic idea is the following. Each agent is asked to declare what reward she is ready to admit, given that there is not enough to satisfy all the demands. Of course this is a strategic situation and each player will declare the amount that maximizes her expected payoff. We propose here an elementary game form in which those who "demand less" are given priority in the distribution, so that in a Nash equilibrium all players demand "the same". Interestingly enough the equilibrium payoff vector is unique and so is, in many cases, the strategy profile. Moreover, the Nash equilibria of this game are all strong. By specifying properly what we mean by "demanding less" and "getting the same" through the rules of each particular game of this type, we obtain all the different solutions to bankruptcy problems as Nash equilibria.

We implicitly assume, as it is usual in this literature, that both the amount to divide and the claims are known by all the agents. There are, however, two features that make this game form much simpler than those in other contributions: (1) No sequential procedure is involved (which, incidentally, makes the result independent on consistency). (2) Each agent's message only refers to her decision variable, in contrast with most of the results in which each agent proposes a whole allocation. Moreover, our results apply to virtually all acceptable bankruptcy rules (where "unacceptable" rules are those that allow some agent to get her full claim and, at the same time, give zero to some else).

The paper is organized as follows. Section 2 presents the formal model and the main results. Section 3 applies those results to the bankruptcy problem. It is shown that the allocation proposed by any acceptable bankruptcy rule can be obtained as the Nash equilibrium of a specific game within the family presented in Section 2. We conclude with a few final comments in Section 4.

2 The model and the main results

A **bankruptcy problem** is a triple (N, E, d), where $N = \{1, 2, ..., n\}$ is a collection of agents, $E > 0$ is the amount to divide, and $d \in \mathbb{R}^n_{++}$ is the vector of claims. The essence of the problem under consideration implies that $\sum_{i \in N} d_i > E > 0$. A **bankruptcy rule** R is defined as a function mapping each bankruptcy problem (N, E, d) into $\mathbb{R}^n_+$, so that $R_i(N, E, d) \in [0, d_i]$ for all $i \in N$, and $\sum_{i \in N} R_i(N, E, d) = E$. The rule R represents a sensible way of distributing the available amount E, with two natural restrictions. One is that no agent gets more than she claims or less than zero. The other that the total amount E is divided among the agents.

Our **non-cooperative bankruptcy game** for the bankruptcy problem (N, E, d) is a strategic game with N as the set of players, who are endowed with strategy spaces $(D_i)_{i \in N}$, and payoff functions $(\pi_i)_{i \in N}$ that describe what each agent gets as a function of the joint strategy vector.

In our game, player i's strategy space is a closed interval $D_i = [0, m_i]$, for some scalar $m_i > 0$. A strategy for agent i will be denoted by α_i, whereas $\alpha \in D = \prod_{i \in N} D_i$ is a strategy combination, and α_{-i} is an $(n-1)$-vector consisting of the strategies of all agents other than agent i. We interpret α_i as a message monotonically related to the total payoff that she declares admissible, given the rationing situation. For instance, α_i may describe the share of d_i that agent i would be ready to accept. Or it may correspond to the loss she is ready to admit. This is described through a function $f_i : D_i \to [0, d_i]$ that specifies the relationship between the agent's message and her intended reward. For instance $f_i(\alpha_i) = \alpha_i d_i$, when α_i corresponds to agent i's admissible share. A strategy profile α is **feasible** if $\sum_{i \in N} f_i(\alpha_i) \leq E$.

Concerning those f_i functions we assume:

Axiom 1. *For all $i \in N$, f_i is a monotone function that defines a bijection from $[0, m_i]$ to $[0, d_i]$.*

Axiom 2. *All f_i's are simultaneously increasing or decreasing.*

Note that axiom 1 implies that every f_i is also a continuous and strictly monotone function. Because of this, functions f_i will be referred to as increasing or decreasing meaning strictly increasing or strictly decreasing, respectively. For simplicity we assume in axiom 2 that the orientation of the messages of the agents is uniform, which means that functions f_i are either all increasing or all decreasing. It follows from axioms 1 and 2 that $f_i(0) = 0$

and $f_i(m_i) = d_i$, when functions f_i are increasing, and $f_i(0) = d_i$, $f_i(m_i) = 0$ when they are decreasing.

If the f_i's are increasing (decreasing) functions, we denote by $[i]$ the agent whose message occupies the ith position in the increasing (decreasing) ordering of messages. Although it will not be important for the results of this paper, a tie-breaking rule must be considered in order to have a well defined reordering of the agents. Here we consider the following: if there is a tie between two or more players, the ranking will be made in increasing ordering or their indices within N. So:

$\alpha_{[1]} \leq \alpha_{[2]} \leq ... \leq \alpha_{[n]}$ if the f_i functions are increasing.

$\alpha_{[1]} \geq \alpha_{[2]} \geq ... \geq \alpha_{[n]}$ if the f_i functions are decreasing.

Consider now the following procedure. Each agent j chooses her message α_j. If the corresponding profile α is feasible, then each agent j gets $f_j(\alpha_j)$. If α is not feasible, then E is allocated among the players with lowest ranking. That is, let $[h]$ denote the smallest index for which $\sum_{[i]\leq[h]} f_{[i]}(\alpha_{[i]}) > E$. Then:

$$\pi_{[j]}(\alpha) = \begin{cases} f_{[j]}(\alpha_{[j]}) & [j] < [h] \\ E - \sum_{[i]\leq[h-1]} f_{[i]}(\alpha_{[i]}) & [j] = [h] \\ 0 & [j] > [h] \end{cases}$$

This non-cooperative bankruptcy game will be denoted by $\langle N, D, \pi\rangle$. Note that, under axioms 1 and 2, the strict monotonicity of the f_i functions is translated to the functions $\pi_i(\alpha_{-i}, \cdot)$ for every α_{-i}, provided $\sum_{i\in N} f_i(\alpha_i) < E$. By construction, $\sum_{i\in N} \pi_i(\alpha) \leq E$, for all $\alpha \in D$.

We define a **Nash equilibrium** of the game $\langle N, D, \pi\rangle$, associated with a bankruptcy problem (N, E, d), as a strategy profile $\alpha^* \in D$ such that $\pi_i(\alpha^*) \geq \pi_i(\alpha^*_{-i}, \alpha_i)$, for all $\alpha_i \in D_i$, and all $i \in N$.

A strategy profile $\alpha^* \in D$ is a **strong equilibrium** (Aumann (1959)) if there do not exist $T \subset N$ and $\alpha_T \in \prod_{i\in T} D_i$ such that $\pi_i(\alpha^*) < \pi_i(\alpha^*_{N\setminus T}, \alpha_T)$, for all $i \in T$.

Lemma 1. *Under axioms 1 and 2, if $\alpha^* \in D$ is a Nash equilibrium of the bankruptcy game $\langle N, D, \pi\rangle$, then $\sum_{i\in N} \pi_i(\alpha^*) = E$, i.e., all the Nash equilibria are efficient.*

The proof is immediate and so it is omitted. The lemma is used in the following proposition which shows two features of each non-cooperative bankruptcy game. First, that it has, at least, one Nash equilibrium. Second, that it can have multiple Nash equilibra but, in this case, the equilibrium payoff is unique.

Proposition 1. *Given a bankruptcy problem (N, E, d) and an associated non-cooperative bankruptcy game $\langle N, D, \pi\rangle$, the following statements are true under axioms 1 and 2:*

(i) There exists a constant ρ such that the game has a Nash equilibrium in which $\alpha^*_i = \min\{\rho, m_i\}$, i.e., every player selects the strategy in $[0, m_i]$ which is closer to ρ.

(ii) The equilibrium payoff for this game is unique.

Proof. **Case i)** Functions f_i are increasing. Let $F : \mathbb{R}_+ \longrightarrow \mathbb{R}_+$ be such that for each $x \in \mathbb{R}_+$, $F(x) = \sum_{i\in N} f_i(x_i)$, where $x_i = \min\{x, m_i\}$. The continuity of all f_i's implies the continuity of F. Hence, since i) $F(0) = 0$, ii) $F(\max_{i\in N}\{m_i\}) = \sum_{i\in N} d_i > E$, and (iii) F is strictly increasing in the interval $[0, \max_{i\in N}\{m_i\}]$, there is a unique $\rho \in (0, \max_{i\in N}\{m_i\})$ such that $F(\rho) = E$.

Suppose that the profile α^* is a Nash equilibrium and that, for some fixed i, j, we have $\alpha^*_j > \alpha^*_i$ and $m_i > \alpha^*_i$. If $\pi_j(\alpha^*) = 0$, then player j can ensure for herself a positive payoff with the strategy ε, for ε small enough. Hence, $\pi_j(\alpha^*)$ must be positive. Now, for $\delta > 0$

small enough, agent i can switch to a strategy $\alpha_i^* + \delta$ such that player i has still a lower index than player j and ensuring himself a payoff $\pi_i(\alpha_i^* + \delta) = f(\alpha_i^* + \delta) > f(\alpha_i^*) = \pi_i(\alpha_i^*)$. Hence, if α^* is a Nash equilibrium and $\alpha_j^* > \alpha_i^*$ for some $i, j \in N$, we have $\alpha_i^* = m_i$. Combining this with the fact that ρ is the unique positive real number for which $F(\rho) = E$, we get that the strategies $\alpha_i^* = \min\{\rho, m_i\}$ define a Nash equilibrium.

Let $\bar{\alpha}$ be a Nash equilibrium of $\langle N, D, \pi \rangle$. Suppose that $\pi(\bar{\alpha}) \neq \pi(\alpha^*)$. Then, by Lemma 1, there are $i, j \in N$ such that $\pi_i(\bar{\alpha}) > \pi_i(\alpha^*)$ and $\pi_j(\bar{\alpha}) < \pi_j(\alpha^*)$. Since for each $k \in N$, $\pi_k(\alpha^*) = f_k(\alpha^*)$, then $\bar{\alpha}_i > \alpha_i^*$ and $\bar{\alpha}_j < \alpha_j^*$. Then, in $\bar{\alpha}$, j can deviate to strategy α_j^* and get a higher profit.

Case ii) Functions f_i are decreasing. Take again the function F, now we have i) $F(0) = \sum_{i \in N} d_i > E$ and ii) $F(\max_{i \in N}\{m_i\}) = 0$. Define again ρ as the unique real number in $(0, \max_{i \in N}\{m_i\})$ such that $F(\rho) = E$. The situation is similar to the case with increasing functions: the profile α^* with $\alpha_i^* = \min\{\rho, m_i\}$ is again a Nash equilibrium. The part for the uniqueness of the equilibrium payoff is analogous. □

Moreover, all the Nash equilibria in the Proposition above are in fact strong equilibria, as the following Proposition shows.

Proposition 2. *Given a bankruptcy problem (N, E, d), an associated non-cooperative bankruptcy game $\langle N, D, \pi \rangle$ that satisfies axioms 1 and 2, and a strategy profile α^*, then α^* is a Nash equilibrium if and only if α^* is a strong equilibrium.*

Proof. Since a strong equilibrium is a Nash equilibrium only one implication has to be proved. Assume that the functions f_i are increasing. Let α^* be the Nash equilibrium defined in Proposition 1, *i.e.*, there exists ρ such that $\alpha_i^* = \min\{\rho, m_i\}$ for all $i \in N$. Suppose that $\bar{\alpha}$ is a Nash equilibrium which is not strong. Then, there are a coalition $T \subset N$, with at least two players, and $\alpha_T \in \prod_{j \in T} D_j$ such that, for each $j \in T$, $\pi_j(\bar{\alpha}) < \pi_j(\bar{\alpha}_{N \setminus T}, \alpha_T)$. Since $\bar{\alpha}$ is a Nash equilibrium, for each $i \in N$, $\pi_i(\bar{\alpha}) = \pi_i(\alpha^*) = f_i(\alpha_i^*)$. Now, for each $j \in T$,

$$f_j(\alpha_j) \geq \pi_j(\bar{\alpha}_{N \setminus T}, \alpha_T) > \pi_j(\bar{\alpha}) = f_j(\alpha_j^*).$$

Hence, $\alpha_j > \alpha_j^*$. This means that $\alpha_j^* < m_j$. Thus, for each $j \in T$, $\alpha_j^* = \rho$ and $\alpha_j > \rho$. Moreover, since for each $j \in T$, $f_j(\bar{\alpha}_j) \geq \pi_j(\bar{\alpha}) = f_j(\rho)$, then $\bar{\alpha}_j \geq \rho$.

Now, we distinguish two cases:

Case i) For each $i \in T$, $\bar{\alpha}_i = \rho$. Take now the player $k \in T$ with the lowest priority for $(\bar{\alpha}_{N \setminus T}, \alpha_T)$. Then, since k's priority does not get worse when moving from $(\bar{\alpha}_{N \setminus T}, \alpha_T)$ to $(\bar{\alpha}_{-k}, \alpha_k)$ and for each $i \in N$, $f_i((\bar{\alpha}_{N \setminus T}, \alpha_T)_i) \geq f_i((\bar{\alpha}_{-k}, \alpha_k)_i)$, we have that $\pi_k(\bar{\alpha}_{-k}, \alpha_k) \geq \pi_k(\bar{\alpha}_{N \setminus T}, \alpha_T) > \pi_k(\bar{\alpha})$. Hence, $\bar{\alpha}$ is not a Nash equilibrium.

Case ii) $\max_{j \in T} \bar{\alpha}_j > \rho$. Let $i \in \text{argmax}_{j \in T} \bar{\alpha}_j$. Let $k \neq i, k \in T$. Now, $\bar{\alpha}_k < \bar{\alpha}_i$, since otherwise we would have $f_i(\bar{\alpha}_i) > \pi_i(\bar{\alpha}) = f_i(\rho)$ and $f_k(\bar{\alpha}_k) > \pi_k(\bar{\alpha}) = f_k(\rho)$, and this is not possible by the construction of payoff functions. Now, there is $\alpha_k' \in (\bar{\alpha}_k, \bar{\alpha}_i)$ such that k can profitably deviate to α_k' in the profile $\bar{\alpha}$. Hence, $\bar{\alpha}$ is not a Nash equilibrium.

In the decreasing case, a similar argument can be formulated. □

3 Bankruptcy games and bankruptcy rules

Let us now illustrate how the results in Section 2 apply to some standard bankruptcy rules.

The *proportional rule*, P, which is probably the best known and most widely used solution concept, distributes awards proportionally to claims. It is defined as follows: for all (N, E, d), $P(N, E, d) = \lambda d$, with $\lambda = \frac{E}{\sum_{i \in N} d_i}$. It is easy to see that, if we take $D_i = [0, 1]$

and $f_i(\alpha_i) = \alpha_i d_i$, for all $i \in N$, the unique Nash equilibrium of the game will produce the proportional solution to the bankruptcy problem.

The *constrained equal-awards rule, A*, applies an egalitarian principle on the awards received, provided no agent gets more than she claims. It is defined as follows: for all (N, E, d) and all $i \in N$, $A_i(N, E, d) = \min\{d_i, \lambda\}$, where λ solves $\sum_{i \in N} \min\{d_i, \lambda\} = E$. By letting $D_i = [0, d_i]$ and $f_i(\alpha_i) = \alpha_i$, we get the constrained equal awards solution as the unique Nash equilibrium payoff of the associated bankruptcy game.

The *constrained equal-loss rule*, L, is the dual of the former. It distributes equally the difference between the amount available and the aggregate claims, with one proviso: no agent ends up with a negative transfer. Namely, $L_i(N, E, d) = \max\{0, d_i - \lambda\}$, where λ solves $\sum_{i \in N} \max\{0, d_i - \lambda\} = E$. Taking $D_i = [0, d_i]$ and defining $f_i(\alpha_i) = d_i - \alpha_i$, we obtain the constrained equal-losses solution as the Nash equilibrium payoff of the game.

These results illustrate on the applicability of this procedure to provide a non-cooperative support to some well-known bankruptcy rules. But these results can actually be extended to virtually any meaningful rule. Consider now the following definition which introduces an extremely mild requirement on bankruptcy rules:

Definition 1. *A bankruptcy rule R is called* ***acceptable*** *if there are no bankruptcy problem (N, E, d) and agents $i, j \in N$ such that $R_i(N, E, d) = 0$ and $R_j(N, E, d) = d_j$.*

Acceptable rules are those which never concede an agent her claim in full whereas some other agent gets nothing. Most of the rules which have been studied in the literature are acceptable.

The following proposition provides a general result for acceptable bankruptcy rules.

Proposition 3. *Given an acceptable bankruptcy rule R and a bankruptcy problem (N, E, d), there exists a non-cooperative bankruptcy game $\langle N, D, \pi \rangle$, satisfying axioms 1 and 2, whose unique equilibrium payoff coincides with $R(N, E, d)$.*

Proof. The proof consists of showing that we can define sets of strategies D_i and functions f_i in such a way that the result is a consequence of Proposition 1.

Let (N, E, d). To simplify notation we write R_i instead of $R_i(N, E, d)$. Since the rule is acceptable, either $R_i > 0$ for all $i \in N$ or $R_i < d_i$ for all $i \in N$. Next, we define the sets of strategies and the functions f_i.

Case i) $R_i > 0$ for all $i \in N$. Let

$$m_i = \frac{R_1}{R_i} d_i \quad \text{and} \quad f_i(\alpha_i) = \frac{R_i}{R_1} \alpha_i.$$

It is clear that functions f_i are monotone (in fact, increasing) and that, for all $i \in N$, f_i is a bijection mapping $[0, m_i]$ into $[0, d_i]$.

From Proposition 1 we have that the non-cooperative bankruptcy game has a unique equilibrium payoff. Clearly, in this case, $\rho = R_1$. Hence, $\alpha^* = (R_1, \ldots, R_1)$ is a Nash equilibrium (which is, moreover, strong according to Proposition 2); its associated payoff is $R(N, E, d)$.

Case ii) $R_i < d_i$ for all $i \in N$. The reasoning is the same as before, except in that, now, we define:

$$m_i = \frac{d_1 - R_1}{d_i - R_i} d_i \quad \text{and} \quad f_i(\alpha_i) = d_i - \frac{d_i - R_i}{d_1 - R_1} \alpha_i.$$

Proposition 1 gives again the desired result, taking into account that $(d_1 - R_1, ..., d_1 - R_1)$ is a Nash equilibrium of this game, and that its associated payoff vector is $R(N, E, d)$. □

4 Final remarks

We have presented in this paper a simple and intuitive game form which supports virtually all bankruptcy rules. The allocation proposed by each rule is obtained as the unique payoff vector corresponding to the Nash equilibria of a specific game. In this respect, choosing the rules of the game (and most particularly the strategy space of the agents) determines the bankruptcy rule that will emerge.

Interestingly enough, the game form that allows to implement those bankruptcy rules is a one-shot game in which every agent sends a message concerning her own awards exclusively. Those messages refer to the cuts in their claims they might be ready to accept, given their claims and the existing shortage. The game form induces an equilibrium in which all agents choose "the same" message. Selecting the nature of those messages (e.g. awards, shares, losses) amounts to deciding on the bankruptcy rule whose allocation will result (the equal awards-rule, the proportional rule, the equal-losses rule).

The game form proposed here implicitly assumes that all the data of the problem are public knowledge. In particular that the planner may know both the agents' claims and the amount to divide. This is a natural assumption in most of the bankruptcy situations, where claims have to be eventually credited. The case of taxation problems may be an exception in this respect. Dagan et al. (1999) show that those problems are implementable when all agents other than the planner know all the data of the problem. Even though this is an arguable assumption in this context, they also show an impossibility result when this is not the case (see also Corchón and Herrero (2004) on this point).

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