UNIVERSIDADE DE
SANTIAGO DE COMPOSTELA
DEPARTAMENTO DE
ESTATÍSTICA E INVESTIGACIÓN OPERATIVA

Assignment problems in wildfire suppression: a case study on control of flight resources

J. Rodríguez-Veiga, I. Gómez-Costa, M. J. Ginzo-Villamayor, B. Casas-Méndez

Report 14-03

Reports in Statistics and Operations Research

Assignment problems in wildfire suppression: a case study on control of flight resources

Jorge Rodríguez-Veiga[1] Iván Gómez-Costa[2]
María José Ginzo-Villamayor[3] Balbina Casas-Méndez[4]

Abstract

The phenomenon of wildfires has become one of the biggest problems our forests are suffering due to the high frequency and intensity that has acquired in recent decades. As the budget and fire resources are limited, it is essential to control these catastrophic fires by making efficient decisions. In this paper, we make use of operations research techniques that allow the optimal assignments of aircrafts to extinguishing wheels and to refueling points, which are two important tasks to be performed by the controller of aerial resources in a forest fire.

Keywords: *wildfire management, aerial resource assignment, integer linear programming, extinguishing wheels, refueling points.*

1 Introduction

The design of decision support systems for logistics is an extremely active area of research and applications in modern Operations Research. In the framework of forest fire control, it is essential to make efficient decisions because the budget and fire resources are limited. In this sense, it is worth mentioning that economic theory plays a central role in the management of forest fires. Precursors in the economic study of forest fires were Headley

[1]ITMATI, Technological Institute of Industrial Mathematics, Santiago de Compostela, Spain, jorge.rodriguez.veiga@gmail.com

[2]INDRA, Technology and Consulting, A Coruña, Spain, ivan.gomez.cs@gmail.com

[3]Modestya Research Group, Department of Statistics and Operations Research, University of Santiago de Compostela, Santiago de Compostela, Spain, mariajose.ginzo@usc.es

[4]Modestya Research Group, Department of Statistics and Operations Research, University of Santiago de Compostela, Santiago de Compostela, Spain, balbina.casas.mendez@usc.es

(1916) and Sparhawk (1925), who describe how to establish an optimum program of fire management.

A theoretical framework used to identify the most efficient way to manage the costs of a wildfire has been the Cost Plus Net Value Change (C + NVC) (Gorte and Gorte, 1979). Thus, it is intended to minimize the cost for the use of resources in the fire fighting, plus a cost produced by the hectares of land burned, where not only must take into account material losses in the fire (trees, urban goods, etc.) but also restocking or reconstruction of these areas.

In the case of Spain, in 2010, it emerged the *Prometeo* project with the goal of improving efficiency in fire fighting. *Prometeo* was one of the biggest applied research projects awarded to a business consortium in Spain in the fight forest fire. The project involved more than 16 companies and the active participation of government towards achieving the following objectives: To mitigate environmental damage in case of fire in an efficient way, and to reduce the number and size of large wildfires, ensuring the safety of extinguishing devices.

Following the *Prometeo* project, they arise in 2013 and 2015 the so-called projects *Lumes* and *Enjambre*, respectively, which again involved various companies of public and private sectors. The main objective of these projects is the development of new advanced technologies for comprehensive fight against major forest fires, allowing reduce the number and the surface of these, and generating a security enclosure in operations that significantly reduce the accident rate of the participants (technical, brigade and pilots).

Various activities have been of great interest to perform in projects of this magnitude. These activities range from image processing that provides information about the vegetation structure and the evolution of the fire (according to which resources are selected), the study of the feasibility of unmanned aircraft in this context, the algorithms and strategies to ensure the safety of terrestrial environments, analysis of extinction night operations and coordination of air traffic.

In this paper, we conduct an investigation focusing on the coordination of air traffic. In situations of large wildfires, where different points of loading and discharge of water and the presence of up to 20 aircrafts working at once are located (some of them from other administrations), air traffic increases in quantity. This means that it can produce situations where there is risk of collision between different aircrafts (for example, in cases in which two circuits wheel share the same water intake but have different point of discharge), or situations where some aircrafts are awaiting orders being likely to exceed the maximum working time. Thus, the presence of an air traffic coordination governing each aircraft order, controlling which wheel is assigned, their working time, and the risk of collision between aircrafts, is necessary. Initially, we concentrated on two tasks that are the assignment of aircrafts to wheels and the assignment of aircrafts to refueling points. Moreover, we program and solve the corresponding models with the free software `R` (cf. `http://www.r-project.org/`) and show some of the numerical and computational results obtained.

The goal is to automate these tasks, in order to gain time and save costs. All this is to be realized using models and techniques borrowed from operational research. The idea came from a company in the security sector and the approach is to integrate this tool into a larger application that includes different utilities.

The assignment problem is one of the classical problems in linear programming, having appeared with the work of Votaw and Orden (1952) and becoming more relevant with the publication of the Hungarian method for solving it (cf. Kuhn, 1955). A recent review of this problem and its generalizations is due to Pentico (2007). It is worth mentioning that the first model we will consider is close to three-dimensional assignment problem (cf. Geetha and Vartak, 1994) because we try to assign aircrafts to wheels that are in turn allocated to different fire fronts. However, a slightly more general situation arises here because simultaneously several objectives are considered (cf. Geetha and Nair, 1993) as they are to maximize water discharge on the fronts and minimize distances between aircrafts and wheels, while specific restrictions are also introduced in aircrafts and water points related with capacity, as wheel as preferences in the percentage of water received on the fronts.

On the other hand, we should mention that different references can be found in the literature related to systems of decision support in fighting fires, or reviews on employment of operational research techniques in firefighting in particular and disaster situations in general. For example, Keramitsoglou et al. (2004) use Dijkstra algorithm in order to optimize operations that involve routing of response units dispatched to attack fire front from various sides. Other papers are Dimopoulou and Giannikos (2004), Galindo and Batta (2013), Mavsar et al. (2013), and Minas et al. (2012).

2 The models

At present, the number of airline resources that can coincide in time and space in a forest fire has increased considerably compared to what happened in the relatively recent past, when the number of aircrafts was not very large, they had little autonomy and few bases of recovery. In general, the organization of the air strike did not pose complications and was developed between aircraft pilots themselves.

Today, not having proper air coordination function would substantially increase risk of flight safety and a serious reduction in the effectiveness.

2.1 Assignment of aircrafts to wheels

It is very common in a wildfire that aircraft work is organized in training wheel type, so that a set of aircrafts is flying over the fire forming a circuit from which each aircraft has access to a point of water intake and later it discharge thereof. Naturally, if these operations are done disorganized then there is a high-risk operation, because if it is not clear about the status of all aircrafts continuously, the possibility of collision is high. The advantages of this type of work through wheels are mainly: a shorter occupation of airspace in the fire, which means, besides not waste the time of other aircraft, do maximum damage to the fire in the shortest possible time, increased strength in areas of high flames, and the possibility of defense lines longer and more accurate.

If we suppose that it has been determined the number and type of aircrafts to be sent to the fire, a problem for the coordinator is to distribute optimally the resources at its disposal. We build a function in `R` language that provides optimal allocation of resources and serve as a support for decision-making by the coordinator during the fire. Thus, we will propose an integer linear programming model whose purpose is to maximize the output per hour of aircrafts that are acting on the fire, that is, the objective is maximize the amount of water discharged into the fire. All this is done through an appropriate allocation of aircrafts among the wheels subject to the restriction that no left unattended fronts of the fire, that the maximum number of aircrafts is respected by wheel (this number is previously determined), and that the percentage of water for each front, chosen for the coordinator, is also respected.

2.1.1 Notation and decision variables

The main elements of our formulation are the following.

Sets

$\mathcal{A} = \{1, \ldots, nA\}$ is the set of aircrafts (resources).
$\mathcal{W} = \{1, \ldots, nW\}$ is the set of wheel circuits.
$\mathcal{P} = \{1, \ldots, nP\}$ is the set of water recharge points.
$\mathcal{F} = \{1, \ldots, nF\}$ is the set of fire fronts.

For each $k \in \mathcal{F}, \mathcal{W}_k$ is the subset of $\mathcal{W}$ formed by the wheel circuits assigned to front k.[1]

For each $l \in \mathcal{P}, \mathcal{W}_l$ is the subset of $\mathcal{W}$ formed by the wheel circuits assigned to water recharge point l.

Parameters

For each $i \in \mathcal{A}, CAP_i$ is the capacity in liters of water of airplane i.

[1] Once the fire is declared, we assume, for simplicity, that the number of wheels is determined from the available charging water points and the number of fire fronts. In addition, each wheel is initially assigned automatically to a front and to a water point.

For each $j \in \mathcal{W}, NUA_j$ is the maximum number of aircrafts entering wheel j.

For each $j \in \mathcal{W}, DOA_j$ is the number of downloads per hour performing an aircraft in wheel j.

For each $i \in \mathcal{A}, j \in \mathcal{W}, DIS_{ij}$ is the distance from the base of aircraft i to wheel j.

For each $l \in \mathcal{P}, NUW_l$ is the number of wheels can share water point l.

For each $k \in \mathcal{F}, PER_k$ is the percentage of water (expressed as the relative frequency) intended for front k.

Decision variables

We use four sets of decision variables in our formulation.

For each $i \in \mathcal{A}, j \in \mathcal{W}, a_{ij}$ is a binary variable that takes value 1 if aircraft i is assigned to wheel j, and 0 otherwise.

For each $k \in \mathcal{F}, m_k^+, m_k^-$ are real variables that measure the difference (in absolute value) between the amount of water used in front k and the initially assigned (although this assignment can not be satisfied in full).

For each $k \in \mathcal{F}, f_k$ is a binary variable that takes value 1 if front k left unattended , and 0 otherwise.

For each $j \in \mathcal{W}, w_j$ is a binary variable that takes value 1 if wheel j is assigned to an aircraft, and 0 otherwise.

2.1.2 Objective function

$$\max \sum_{i \in \mathcal{A}} \sum_{j \in \mathcal{W}} \left(DOA_j CAP_i - \frac{DIS_{ij}}{\max\limits_{i \in \mathcal{A}, j \in \mathcal{W}} DIS_{ij}} \right) a_{ij} - \sum_{k \in \mathcal{F}} M \, \left(m_k^+ + m_k^- + f_k \right).$$

The goal to make the assignment of aircrafts to the wheels is to maximize the performance of the same, in terms of quantity of water discharged. This explains the first term in the above expression in which we also take into account the fact that we prefer to choose among aircrafts with equal performance those that are closest to the wheel.

The second term appears because we want to respect as much as possible the distribution of water among fronts chosen by the coordinator and also serve the greatest number of fronts.

M is a constant with a sufficiently large value. In tests, it was taken equal to how many liters of water is discharged per hour if all aircrafts are on the wheel with higher performance, this is $nA \max_{j \in \mathcal{W}} DOA_j$.

2.1.3 Constraints

The relationships that describe the real-world model are translated in our formulation via mathematical constraints.

$$\forall i \in \mathcal{A}, \sum_{j \in \mathcal{W}} a_{ij} = 1.$$

Each aircraft is assigned to a unique wheel.

$$\forall j \in \mathcal{W}, \quad \sum_{i \in \mathcal{A}} a_{ij} \leq NUA_j.$$

The number of aircrafts assigned to a wheel is limited.

$$\forall k \in \mathcal{F}, \quad \sum_{i \in \mathcal{A}} \sum_{j \in \mathcal{W}_k} a_{ij} \geq 1 - f_k.$$

All fronts must be served unless the number of aircrafts is not sufficient.

$$\forall i \in \mathcal{A}, \ \forall j \in \mathcal{W}, \quad w_j \geq a_{ij}.$$

If aircrafts are not assigned to a certain wheel then this wheel is not considered active.

$$\forall l \in \mathcal{P}, \sum_{j \in \mathcal{W}_l} w_j \leq NUW_l.$$

The capacities at points of water must not be exceeded.

$$\forall k \in \mathcal{F}, \quad \sum_{i \in \mathcal{A}} \sum_{j \in \mathcal{W}_k} CAP_i a_{ij} = PER_k \sum_{i \in \mathcal{A}} CAP_i + m_k^+ - m_k^-.$$

The quantity of water assigned to each front is respected as much as possible.

2.2 Allocation of aircrafts to refueling points

When aircrafts assigned to a fire start the rest period, they have to make fuel refueling. The task of assigning these aircrafts to refueling bases is not simple, because we have to take into account various factors, such as time of arrival at refueling points, the amount of fuel available at each point, etc. The following integer linear programming model provides the optimal allocation of aircrafts to the refueling points, in such a way that the total time taken for all aircrafts in the operation is minimum. The model should take into account the following aspects. First, the number of aircrafts that can refuel at a time on the same basis. A refueling point can be constituted by a tanker in the middle of an open area, being a single tanker with a single hose, and then the simultaneous supply of various aircrafts becomes impossible. The fuel liters of each base and the fuel capacity of each aircraft are also relevant. It could also happen that a refueling base was available very next to the fire, but the fuel of which there is less available than required for any of the aircraft. This would cause that not all aircrafts could be assigned to the base, despite the closeness of it. Moreover, aircrafts may prefer to wait for an aircraft completes its refueling in a base and then supplies it, than to have to go to a farther base (thereby losing more time). In addition, it is intended that, once found the optimal allocation there is a warning of the new capacity of refueling points, so that, if necessary, fuel is reloaded in some basis.

2.2.1 Notation and decision variables

In this case, the elements of the formulation are the following.

Sets

$\mathcal{A} = \{1, \ldots, nA\}$ is the set of aircrafts (resources).
$\mathcal{B} = \{1, \ldots, nB\}$ is the set of refueling bases.

$\mathcal{T} = \{1, \ldots, nT\}$ is the set of periods of time (we take each period equal to five minutes).

Parameters

For each $i \in \mathcal{A}, LOA_i$ is the fuel load of aircraft i.

For each $i \in \mathcal{A}, REF_i$ is the time refueling of aircraft i.

For each $b \in \mathcal{B}, FUE_b$ is the quantity of fuel available in base b.

For each $b \in \mathcal{B}, NUM_b$ is the number of aircrafts that can refuel simultaneously on basis b.

For each $i \in \mathcal{A}, b \in \mathcal{B}, TIM_{ib}$ is the time it takes to move aircraft i to base b.

For each $t \in \mathcal{T}, PER_t$ is the accumulated time in period t.

Decision variables

We use here three sets of decision variables in our formulation.

For each $i \in \mathcal{A}, b \in \mathcal{B}, a_{ib}$ is a binary variable that takes the value 1 if aircraft i is assigned to base b, and 0 otherwise.

For each $i \in \mathcal{A}, b \in \mathcal{B}, t \in \mathcal{T}, s_{ibt}$ is a binary variable that takes the value 1 when aircraft i starts refueling in base b in period t, and 0 otherwise.

For each $i \in \mathcal{A}, b \in \mathcal{B}, t \in \mathcal{T}, e_{ibt}$ is a binary variable that takes the value 1 when aircraft i ends refueling in base b in period t, and 0 otherwise.

2.2.2 Objective function

$$\min \sum_{i \in \mathcal{A}} \sum_{b \in \mathcal{B}} \sum_{t \in \mathcal{T}} PER_t s_{ibt} + \sum_{i \in \mathcal{A}} \sum_{b \in \mathcal{B}} \sum_{t \in \mathcal{T}} PER_t e_{ibt}.$$

The objective is to minimize the total time spent refueling. Thus, the first term in the above formula represents the time from the beginning of the rest of the aircrafts until the beginning of their refueling in a base. The second term computes the refueling time plus time back to the fire.

2.2.3 Constraints

The relationships that describe the real-world model of allocation of aircrafts to refueling points are formulated by means of the following restrictions.

$$\forall i \in \mathcal{A}, \quad \sum_{b \in \mathcal{B}} a_{ib} = 1.$$

Each aircraft must be assigned to a single base.

$$\forall i \in \mathcal{A}, \ \forall b \in \mathcal{B}, \quad a_{ib} \geq \sum_{t \in \mathcal{T}} s_{ibt}.$$

Each aircraft starts refueling at the assigned base.

$$\forall i \in \mathcal{A}, \quad \sum_{b \in \mathcal{B}} \sum_{t \in \mathcal{T}} s_{ibt} = 1.$$

Every aircraft must have a beginning of its refueling.

$$\forall i \in \mathcal{A}, \quad \sum_{b \in \mathcal{B}} \sum_{t \in \mathcal{T}} e_{ibt} = 1.$$

Every aircraft must have an end of its refueling.

$$\forall b \in \mathcal{B}, \ \sum_{i \in \mathcal{A}} LOA_i a_{ib} \leq FUE_b.$$

Aircrafts assigned to each base may not need more fuel than the fuel available at that refueling point.

$$\forall i \in \mathcal{A}, \ \forall b \in \mathcal{B}, \quad TIM_{ib} a_{ib} \leq \sum_{t \in \mathcal{T}} PER_t s_{ibt}.$$

Aircrafts can not begin refueling before reaching supply point. Also, the equal condition is not imposed to allow aircrafts wait if another aircraft is refueled (if it is more favorable waiting to go to a farther base).

$$\forall i \in \mathcal{A}, \ \forall b \in \mathcal{B}, \quad (TIM_{ib} + REF_i) a_{ib} \leq \sum_{t \in \mathcal{T}} PER_t e_{ibt}.$$

Completion of refueling has to be once it has been performed the flight period until supply point and once the filling time of the aircraft ended.

$$\forall b \in \mathcal{B},\ \forall t \in \mathcal{T},\ \ \sum_{i\in\mathcal{A}}\sum_{k=1}^{t} s_{ibk} - \sum_{i\in\mathcal{A}}\sum_{k=1}^{t} e_{ibk} \leq NUM_b.$$

It must respect the maximum number of aircrafts that can be refueled in the same base simultaneously.

$$\forall i \in \mathcal{A},\ \ \sum_{b\in\mathcal{B}}\sum_{t\in\mathcal{T}} PER_t e_{ibt} - \sum_{b\in\mathcal{B}}\sum_{t\in\mathcal{T}} PER_t s_{ibt} \geq REF_i.$$

The time between the beginning and the end of refueling of an aircraft can not be less than the estimated duration of the refueling of such aircraft.

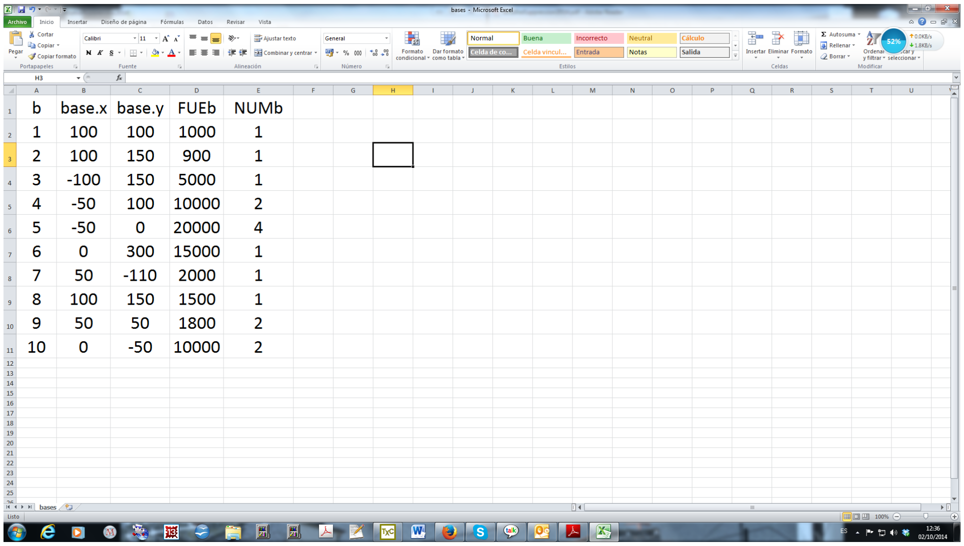
3 Examples and numerical results

To solve the above models, two functions have been created in the free software `R`.[2] They work with four databases containing information, respectively, on the fronts of the fire, the water charge points, the available aircrafts, and the refueling bases.

Figure 1 shows an example of database, specifically for refueling points. It contains the coordinates of different bases, with the corresponding fuel capacities and the number of aircrafts that can reload simultaneously in each base.

These programming problems can be solved with the library lpSolveAPI or alternatively the Gurobi solver. The lpSolveAPI uses simplex method and branch and bound algorithms. However, Gurobi also employs different cutting plane methods. We can get further information from these solvers in `http://cran.r-project.org/web/packages/lpSolveAPI/index.html` and `http://www.gurobi.com`, respectively.

[2] The `R` code is available from the authors.



b	base.x	base.y	FUEb	NUMb
1	100	100	1000	1
2	100	150	900	1
3	-100	150	5000	1
4	-50	100	10000	2
5	-50	0	20000	4
6	0	300	15000	1
7	50	-110	2000	1
8	100	150	1500	1
9	50	50	1800	2
10	0	-50	10000	2

Figure 1: Example of database for refueling points.

3.1 Assignment of aircrafts to wheels

Before executing the function, we have information about the maximum number of aircrafts per wheel and the discharges per hour of every aircraft.

To show the use of the first function through an example, we assumed to have the aircrafts BellB412-2, Ka32-1, and BellB407-3. It is further assumed that there are two fronts in the fire and from each of them, one can access to three water loading points. In this case, the model has 4 continuous variables and 22 binary variables. Tables 1 and 2 summarize the main results, concerning the assignment of aircrafts to fronts and to water points, and the water quantity assigned to each front (in liters and percent), respectively.

In Table 1, with 1 we indicate that the aircraft (in a row) is assigned to the wheel that can be formed by joining the front i and the point j of water (in a column). In other case, we put 0.

In Table 2, we see that it spends 73 % of capacity at front 1 and 27 % at front 2, which is very close to the percentages specified in the database corresponding to the fronts, which are 75 % and 25 %, respectively.

$Tab.Ass

	F1-P1	F1-P2	F1-P3	F2-P1	F2-P2	F2-P3
BellB412-2	0	0	0	1	0	0
Ka32-1	0	0	1	0	0	0
BellB407-3	0	1	0	0	0	0

Table 1: Allocation of aircrafts to wheel (front and point of water).

$Tab.Amo

	Amount.Water	Percentage
Front 1	2400	72.72727
Front 2	900	27.27273

Table 2: Allocation of water to different fronts.

3.2 Allocation of aircrafts to refueling points

To demonstrate the use of the second function by an example, we assumed to have nine aircrafts and ten refueling points. In this case, where we assume that all aircrafts have to rest, the model has 7963 binary variables.

Among the results, we have Table 3 of zeros and ones, where a one indicates that the corresponding aircraft is assigned to the corresponding base. We note that an aircraft is assigned to the bases 5, 6 and 7, two on the base 4 and four to the base 10.

$AiB

	B1	B2	B3	B4	B5	B6	B7	B8	B9	B10
BellB412-1	0	0	0	0	1	0	0	0	0	0
BellB412-2	0	0	0	1	0	0	0	0	0	0
BellB212-1	0	0	0	0	0	0	0	0	0	1
BellB212-2	0	0	0	0	0	0	0	0	0	1
Ka32-1	0	0	0	0	0	1	0	0	0	0
Ka32-2	0	0	0	0	0	0	1	0	0	0
BellB407-1	0	0	0	0	0	0	0	0	0	1
BellB407-2	0	0	0	1	0	0	0	0	0	0
ABellB407-3	0	0	0	0	0	0	0	0	0	1

Table 3: Allocation of aircrafts to bases.

In addition, we also obtain Table 4 indicating the number of aircrafts per base in each time period. Only two planes coincide in station 10, which is permitted because this base accommodates up to two aircrafts refueling simultaneously.

$BbT

	0	5	10	15	20	25	30	35	40	45	...
B1	0	0	0	0	0	0	0	0	0	0	
B2	0	0	0	0	0	0	0	0	0	0	
B3	0	0	0	0	0	0	0	0	0	0	
B4	1	1	1	0	0	0	0	0	0	1	
B5	0	0	0	0	1	1	1	0	0	0	
B6	0	0	0	0	1	1	1	1	1	1	
B7	0	0	0	0	0	0	1	1	1	1	
B8	0	0	0	0	0	0	0	0	0	0	
B9	0	0	0	0	0	0	0	0	0	0	
B10	0	0	1	1	1	1	2	2	2	1	

Table 4: Number of aircrafts per base in each time period.

Similarly, Table 5 indicates the periods in which aircrafts made refueling to minimize the time employee.

$AiT

	0	5	10	15	20	25	30	35	40	45	...
BellB412-1	0	0	0	0	1	1	1	0	0	0	
BellB412-2	1	1	1	0	0	0	0	0	0	0	
BellB212-1	0	0	0	0	0	0	0	0	0	0	
BellB212-2	0	0	1	1	1	1	1	1	1	0	
Ka32-1	0	0	0	0	1	1	1	1	1	1	
Ka32-2	0	0	0	0	0	0	1	1	1	1	
BellB407-1	0	0	0	0	0	0	1	1	1	1	
BellB407-2	0	0	0	0	0	0	0	0	0	1	
BellB407-3	0	0	0	0	0	0	0	0	0	0	

Table 5: Periods in which aircrafts made refueling.

Table 6 details the liters of fuel remaining in each base after making the assigned refueling.

```
$NewFUEb
 B1    B2     B3     B4     B5     B6    B7    B8    B9   B10
1000   900   5000   5000  17000  14100  1100  1500  1800  2000
```

Table 6: Liters of fuel remaining in each base.

By graphical output, it is easily seen the bases that must be filled fuel again. As we can see in Figure 2, base 10 decreases by more than 75 % their initial capacity, therefore, it should be indicated to the operators to call a tanker as soon as possible to fill the fuel tank of said base; this alarm is shown in red. Another case is that of base 4, where the capacity is decreased by more than 50 % but less than 75 %; latter alarm is displayed orange. This figure also shows the aircrafts assigned to each base and the period in which start charging.

Table 7 details the time of each aircraft is expected to perform, if necessary, before begin refueling. We see that no aircraft has to perform a wait until the end of the other aircraft refueling.

```
$TExp
BellB412-1  BellB412-2  BellB212-1  BellB212-2  Ka32-1  Ka32-2
         0           0           0           0       0       0
BellB407-1  BellB407-2  BellB407-3
         0           0           0
```

Table 7: Time performed by aircrafts begin refueling.

The optimal value of the objective function is shown in Table 8. This value represents the sum of the refueling time that the aircrafts take to perform that operation.

```
$obj
[1] 12.5
```

Table 8: Optimal value of the objective function.

It can be seen as the results shown in the figure and in the tables of this subsection are consistent.

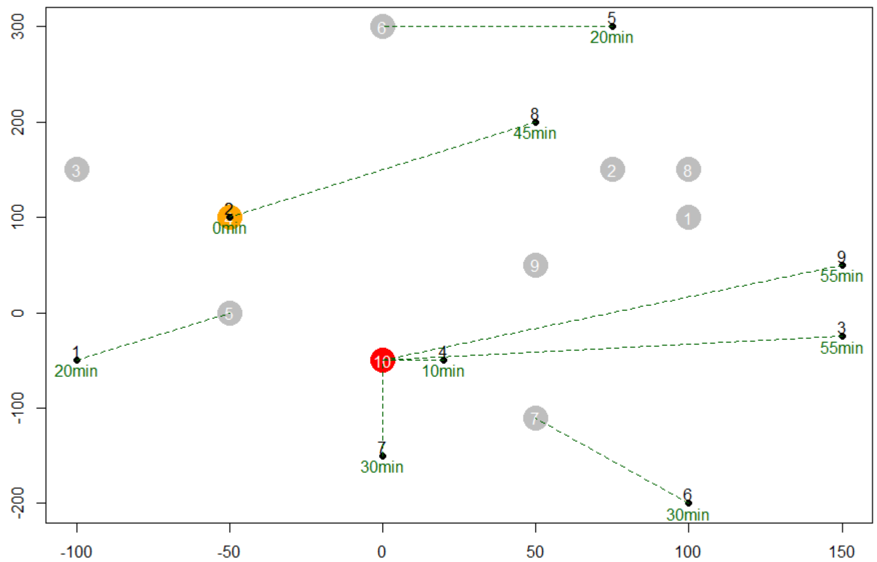


Figure 2: Alerts caused by the level of refueling points.

3.3 The algorithms on bigger instances

In this section, we explore the feasibility of solving real-size instances in reasonable execution times. To do this, we apply the two algorithms over instances ranging from 3 to 12 aircrafts, from 3 to 6 water recharge points and from 3 to 6 fire fronts. The algorithms have been solved in a notebook Intel(R) Core(TM) i5-4200U CPU @ 1.60GHz 2.30 GHz with 8,00 GB of RAM memory.

Table 9 lists the execution times (in seconds) solving with lpSolveAPI for different instances. First column indicates the number of aircrafts, the second one the number of water charge points, and the third one the number of fronts. Column 4 indicates the mean time and column 5 the standard deviation. Finally, column 6 shows the maximum value obtained for the time of execution.

Airplanes	Water points	Fronts	Mean Time	S.D. Time	Max. Time
3	3	3	0.0288	0.0437	0.2974
6	3	3	1.6305	3.9132	25.8624
9	3	3	148.5566	349.8206	2055.2332
12	3	3	3600.0000	NA	NA
3	6	6	96.8911	515.3076	3600.0000
6	6	6	3600.0000	NA	NA
9	6	6	3600.0000	NA	NA
12	6	6	3600.0000	NA	NA

Table 9: Executing times with lpSolveAPI.

Table 10 lists the execution times when using the solver Gurobi.

Airplanes	Water points	Fronts	Mean Time	S.D. Time	Max. Time
3	3	3	0.0528	0.2523	2.5276
6	3	3	0.0592	0.1665	0.8688
9	3	3	0.1689	0.3803	2.4638
12	3	3	9.1129	47.4721	330.9605
3	6	6	0.0508	0.0361	0.3339
6	6	6	0.0966	0.0936	0.8482
9	6	6	0.1889	0.1819	1.0124
12	6	6	71.4598	499.9814	3600.0000

Table 10: Executing times with Gurobi.

In Tables 9 and 10, we see that for some high values of the problem parameters that define the scenario, the execution time can be high, especially with lpSolveAPI, but also in some cases with Gurobi.

To try to reduce these runtimes, we have scheduled with `R` the model so that instead of grouping the different objectives into a single function, we have considered them separately, and we have solved the resulting multi-objective model through a lexicographic optimization method. The `R` code for the proposed model and its resolution by the lexicographic optimization method can also be asked to authors. Tables 11 and 12 show the execution times corresponding to lpSolveAPI and Gurobi when using the lexicographic optimization method.

Airplanes	Water points	Fronts	Mean Time	S.D. Time	Max. Time
3	3	3	0.0226	0.0091	0.0670
6	3	3	0.3392	0.3534	1.9191
9	3	3	30.3146	52.3201	299.1597
12	3	3	3600.0000	NA	NA
3	6	6	0.3190	0.3590	2.9261
6	6	6	432.4068	931.9435	8518.4585
9	6	6	3600.0000	NA	NA
12	6	6	3600.0000	NA	NA

Table 11: Executing times with lpSolveAPI and lexicographic method.

Airplanes	Water points	Fronts	Mean Time	S.D. Time	Max. Time
3	3	3	0.0398	0.1071	0.4220
6	3	3	0.0537	0.0826	0.3340
9	3	3	0.0866	0.1083	0.3980
12	3	3	0.1500	0.1754	0.7020
3	6	6	0.0821	0.1477	0.3140
6	6	6	0.1988	0.1211	0.5250
9	6	6	0.3797	0.3213	1.9640
12	6	6	0.9930	1.3365	9.8915

Table 12: Executing times with Gurobi and lexicographic method.

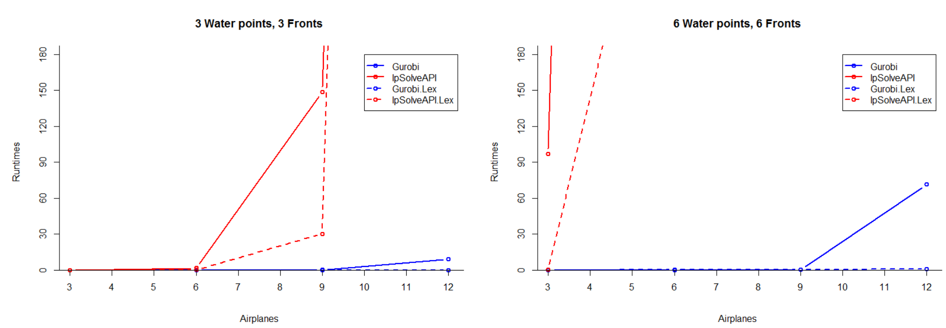


Figure 3: Execution time with different scenarios, solvers and methods.

We should mention that in the second model which is introduced in this paper, the execution times do not suffer in any way significant increases because we have to keep in mind that it is rare to find more than 10 aircrafts refueling at a time. However, in the first model if one considers more than 10 aircrafts, the execution of the algorithms may be slower.

4 Conclusions

In this work, we present two integer linear programming models that solve two real problems that can be found by the traffic control coordinator in a forest fire service. To solve these real problems, we follow a novel way that constitutes an innovative approach taken from the Operations Research, as far as we can know. These models are programmed and solved exactly by using a free software tool such as `R`. In several examples totally inspired by real situations we see that the model resolution is fast. These features indicate, in our opinion, that this material may be useful as a tool to support decision making conducted by the coordinators of traffic control in this type of fire. In fact, along with other tools that our work team has created, as collision avoidance algorithms, this material will be properly integrated into a more complex, complete and user-friendly system for decision support, which also incorporates modern methods of image processing and presentation.

In the models we have defined, resource allocation to wheels and to refueling points is performed. It is important to note that once made this assignment according to certain objectives and restrictions, aircrafts are to integrate effectively the task of extinction and now we will have the necessity of take into consideration a temporal assignment that includes time schedules of pilot work, rest periods, time of flight from the ground, etc. We are currently working on an efficient allocation of this time contemplating the various restrictions and computing costs associated with extinction working according to the C + NVC philosophy mentioned in the introduction to this work.

5 Acknowledgements

The authors wish to thank the interesting proposals, suggestions, comments, and computing support made by W. González-Manteiga, B. Pateiro-López, A. Riera-Álvarez, and J. L. Saiz-Díaz. This research received financial support from *Ministerio de Economía y Competitividad* of Spain through grants MTM2013-41383-P and MTM2014-53395-C3-2-P, from Belgian government through network IAP 97/06 StUdyS, and from ITMATI, Technological Institute of Industrial Mathematics, Santiago de Compostela, Spain, through *Lumes* and *Enjambre* projects, that are gratefully acknowledged.

References

[1] Dimopoulou, M. and Giannikos, I. (2004) Towards an integrated framework for forest fire control. *European Journal of Operational Research* **152**, 476-486.

[2] Galindo, G. and Batta, R. (2013) Review of recent developments in OR/MS research in disaster operations management. *European Journal of Operational Research* **230**, 201-211.

[3] Geetha, S. and Nair, K. P. K. (1993) A variation of the assignment problem. *European Journal of Operational Research* **68**, 422-426.

[4] Geetha, S. and Vartak, M. N. (1994) The three-dimensional bottleneck assignment problem with capacity constraints. *European Journal of Operational Research* **73**, 562-568.

[5] Gorte, J. K. and Gorte, R. W. (1979) Application of economic techniques to fire management - a status review and evaluation. *General Technical Report INT-53*. Ogden, UT, USDA Forest Service, Intermountain Forest and Range Experiment Station, 26 p.

[6] Headley, R. (1916) Fire Suppression District 5. USDA Forest Service, Washington, 58 p.

[7] Keramitsoglou, I., Kiranoudis, C. T., Sarimveis, H., and Sifakis, N. (2004) A multidisciplinary decision support system for forest fire crisis management. *Environmental Management* **33**, 212-225.

[8] Kuhn, H. W. (1955) The Hungarian method for the assignment problem. *Naval Research Logistics Quarterly* **2**, 83-97. Republished in 2005.

[9] Mavsar, R., González-Cabán, A., and Varela, E. (2013) The state of development of fire management decision support systems in America and Europe. *Forest Policy and Economics* **29**, 45-55.

[10] Minas, J. P., Hearne, J. W., and Handmer, J. W. (2012) A review of operations research methods applicable to wildfire management. *International Journal of Wildland Fire* **21**, 189-196.

[11] Pentico, D. W. (2007) Assignment problems: a golden anniversary survey. *European Journal of Operational Research* **176**, 774-793.

[12] Sparhawk, W. N. (1925) The use of liability ratings in planning forest fire protection. *Journal of Agricultural Research* **30**, 693-792.

[13] Votaw, D. F. and Orden, A. (1952) The personnel assignment problem. *Symposium on Linear Inequalities and Programming.* SCOOP (Scientific Computation of Optimum Programs Project) 10 US Air Force, 155-163.

List of Figures

List of Tables

Reports in Statistics and Operations Research

2005

05-01 SiZer Map for Evaluating a Bootstrap Local Bandwidth Selector in Nonparametric Additive Models. M. D. Martínez-Miranda, R. Raya-Miranda, W. González-Manteiga and A. González-Carmona.

05-02 The Role of Commitment in Repeated Games. I. García Jurado, Julio González Díaz.

05-03 Project Games. A. Estévez Fernández, P. Borm, H. Hamers.

05-04 Semiparametric Inference in Generalized Mixed Effects Models. M. J. Lombardía, S. Sperlich.

2006

06-01 A unifying model for contests: effort-prize games. J. González Díaz.

06-02 The Harsanyi paradox and the "right to talk" in bargaining among coalitions. J. J. Vidal Puga.

06-03 A functional analysis of NOx levels: location and scale estimation and outlier detection. M. Febrero, P. Galeano, W. González-Manteiga.

06-04 Comparing spatial dependence structures. R. M. Crujeiras, R. Fernández-Casal, W. González-Manteiga.

06-05 On the spectral simulation of spatial dependence structures. R. M. Crujeiras, R. Fernández-Casal.

06-06 An L_2-test for comparing spatial spectral densities. R. M. Crujeiras, R. Fernández-Casal, W. González-Manteiga.

2007

07-01 Goodness-of-fit tests for the spatial spectral density. R. M. Crujeiras, R. Fernández-Casal, W. González-Manteiga.

07-02 Presmothed estimation with left truncated and right censores data. M. A. Jácome, M. C. Iglesias-Pérez.

07-03 Robust nonparametric estimation with missing data. G. Boente, W. González-Manteiga, A. Pérez-González.

07-04 k-Sample test based on the common area of kernel density estimators. P. Martínez-Camblor, J. de Uña Álvarez, N. Corral-Blanco.

07-05 A bootstrap based model checking for selection-biased data. J. L. Ojeda, W. González-Manteiga, J . A. Cristobal.

07-06 The Gaussian mixture dynamic conditional correlation model: Bayesian estimation, value at risk calculation and portfolio selection. P. Galeano, M. C. Ausín.

2008

08-01 ROC curves in nonparametric location-scale regression models. W. González-Manteiga, J. C. Pardo Fernández, I. Van Keilegom.

08-02 On the estimation of α-convex sets. B. Pateiro-López, A. Rodríguez-Casal.

2009

09-01 Lasso Logistic Regression, GSoft and the Cyclyc Coordinate Descent Algorithm. Application to Gene Expression Data. M. García-Magariños, A. Antoniadis, R. Cao, W. González-Manteiga.

2010

10-01 Asymptotic behaviour of robust estimators in partially linear models with missing responses: The effect of estimating the missing probability on simplified marginal estimators. A. Bianco, G. Boente, W. González-Manteiga, A. Pérez-González.

10-02 First-Price Winner-Takes-All Contents. J. González-Díaz.

10-03 Goodness of Fit Test for Interest Rate Models: an approach based on Empirical Process. A. E. Monsalve-Cobis, W. González-Manteiga, M. Febrero-Bande.

2011

11-01 Exploring wind direction and SO_2 concentration by circular–linear density estimation. E. García–Portugués, R.M. Crujeiras, W. González–Manteiga.

11-02 Utilities for Statistical Computing in Functional Data Analysis: The R Package fda.usc. M. Oviedo de la Fuente, M. Febrero-Bande.

11-03 Multivariate uniformity tests: the distance to boundary method for the case of unknown support. J. R. Berrendero, A. Cuevas, B. Pateiro-López.

11-04 On the meaning, properties and computing of a coalitional Shapley value. J. M. Alonso-Meijide, B. V. Casas-Méndez, A. M. González-Rueda, S. M. Lorenzo-Freire.

11-05 Share functions for cooperative games with levels structure of cooperation. M. Álvarez-Mozos, R. van den Brink, G. van der Laan, O. Tejada.

2012

12-01 A goodness of fit test for the functional linear model with scalar response. E. García–Portugués, W. González–Manteiga, M. Febrero-Bande.

12-02 Bootstrap independence test for functional linear models. W. González-Manteiga, G. González-Rodríguez, A. Martínez-Calvo, E. García-Portugués

12-03 Allocation rules for games with optimistic aspirations. L. Carpente, B. Casas-Méndez, I. García-Jurado, A. van den Nouweland

12-04 Kernel density estimation for directional-linear data. E. García-Portugués, R.M. Crujeiras, W. González-Manteiga

2013

13-01 Extended Additive Regression for Analysing LPUE Indices in Fishery Research. V. Mamouridis, N. Klein, T. Kneib, C- Cadarso Suárez, F. Maynou.

2014

14-01 On the core of an airport game and the properties of its center. J. González-Díaz, M. A. Mirás Calvo, C. Quinteiro Sandomingo, E. Sánchez Rodríguez.

14-02 Gas loss allocation in gas transmission networks. G. Bergantiños, J. González-Díaz, Á. M. González-Rueda, M. P. Fernández de Córdoba

14-03 Assignment problems in wildfire suppression: a case study on control of flight resources. J. Rodríguez-Veiga, I. Gómez-Costa, M. J. Ginzo-Villamayor, B. Casas-Méndez

Previous issues (2001 – 2003):
http://eio.usc.es/reports.php