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Characterizing spatial-temporal forest fire patterns

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Abstract. *The spatial-temporal patterns of wildfire incidence and their relationship to various meteorological, geographical and geological variables is analyzed. Such relationships may be treated as components in separable point process models for wildfire activity. We show some of the techniques for the analysis of spatial point patterns that have become available due to recent developments in point process modelling software. These developments permit convenient exploratory data analysis, model fitting, and model assessment. The discussion of these techniques is conducted jointly with and in the context of some preliminary analyses of a collection of data sets which are of considerable interest in their own right. These data sets consist of the complete records of wildfires which occurred in Castilla La Mancha (a central Spanish region) during the years 1998-2007.*

Keywords. *Covariates; Inhomogeneous K-function; Intensity; Point process; Wildfires.*

1 Introduction and data set

Spatial point pattern data occur frequently in a wide variety of scientific disciplines, including seismology, ecology, forestry, geography, spatial epidemiology, and material science, see e.g [2, 5]. For an overview of spatial point processes, we refer to [5, 9]. We analyze the spatio-temporal patterns produced by wildfire incidences in Castilla La Mancha, located in the middle of the Iberian peninsula, and occupying the greater part of the Submeseta Sur, the vast plain composing the southern part of the Meseta Central. The region is bordered by mountain landscapes: the southern slopes of Sistema Central in the north, Sistema Ibérico in the northeast, and Sierra Morena and Montes de Toledo in the south. Castilla La Mancha is the third largest of Spain's autonomous regions, with a surface area of 79463 square kilometres (30681 sq mi), representing 15.7% of the Spanish national territory. The total number of fires recorded in the study area during the period 1998-2007 is 8488. In addition to the locations of the fire centroids, we consider the following covariates and marks: (a) forest and vegetation types, (b) climatological characteristics (temperature, humidity,...), (c) altitude, slope and orientation, (d) type of wildfire, (e) area burned, and (f) time of occurrence. Figure 1 shows the set of recorded fires and the map of elevations.

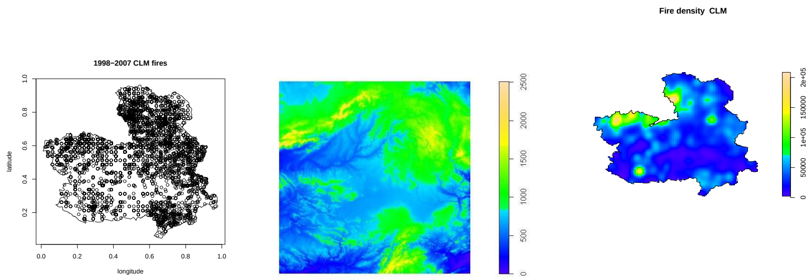

Figure 1: *Left*: Wildfires in Castilla La Mancha in the period 1998-2007; *Middle*: DEM image; *Right*: Density estimation of the intensity function for the whole period 1998-2007

2 Statistical methods

A convenient, and conventional, starting point for the analysis of a spatial point patterns is to test for complete spatial randomness (CSR), under which the data are a realization of a homogeneous Poisson process. A homogeneous Poisson process is a point process which satisfies two conditions: the number of events in any planar region A follows a Poisson distribution with mean $\lambda|A|$, where $|\Delta|$ denotes area and λ is the intensity or mean number of events per unit area; and the numbers of events in disjoint regions are independent. It follows that, conditional on the number of events in any region A, the locations of the events form an independent random sample from the uniform distribution on A. With this, a regular pattern is one in which events are more evenly spaced throughout A than would be expected under CSR, and typically arises through some form of inhibitory dependence between events. Conversely, an aggregated pattern is one in which events tend to occur in closely spaced groups.

Informally, the first-order and second-order properties of a point process ([9]) can be described by the (first-order) intensity function $\lambda(x)$, which is the point process analogue of the mean function for a real-valued stochastic process, and the second-order intensity function $\lambda_2(x,y)$, which is analogous to the covariance function of a real-valued process. The pair correlation function is defined as $g(x,y) = \lambda_2(x,y)/\lambda(x)\lambda(y)$. The process is stationary and isotropic if its statistical properties do not change under translation and rotation, respectively. If we now assume that the process is stationary and isotropic, the intensity function reduces to a constant, λ, equal to the expected number of events per unit area. In the nonstationary case, they are functions of the distance between points, or in the general case of the individual locations.

If the temporal argument is of interest, we may consider data in the form of a realization of a spatio-temporal point process within a finite spatio-temporal region. For many areas of application, a suitable point process model must be able to accommodate spatial and/or temporal inhomogeneity. In this setting, a natural starting point for analysis of spatio-temporal point process data is to investigate the nature of any stochastic interactions among the points of the process after adjusting for spatial and/or temporal inhomogeneity.

[9] introduced the K-function as a tool for data-analysis. One of its advantages over the pair correlation function is that it can be interpreted as a scaled expectation of an observable quantity. Specifically, let $E(s)$ denote the expected number of further events within distance s of an arbitrary event. Then, $K(s) = \lambda^{-1}E(s)$. [3] extended to two dimensional processes a method for nonparametric estimation of the intensity function of an inhomogeneous Poisson process that had been proposed in the one-dimensional case by [4]. [6] extended [9] definition of a reduced second moment measure, or K-function, to include a class of nonstationary processes.

2.1 Modeling forest fires

Forest fires can be regarded as spatio-temporal point patterns and thus space-time statistical tools can be of help in analyzing the behavior of fires ([6, 8]). Additionally, forest fires represent a problem of considerable social importance worldwide. Investigation of the intensity is one of the first steps in analysing a point pattern (Figure 1). We are in particular interested in testing whether the point pattern intensity depends on a covariate, and thus we fit an inhomogeneous Poisson process model with an intensity function which is log-linear on an observed covariate. In order to model the dependence of a point pattern on a spatial covariate, there are several requirements. First, the covariate must be a quantity $Z(u)$ observable (in principle) at each location u in the window (e.g. altitude, slope, orientation). Such covariates may be continuous valued or factors (e.g. vegetation type). Second, the values $Z(x_i)$ of Z at each point of the data point pattern must be available. Thirdly, the values $Z(u)$ at some other points u in the window must be available. In addition to covariates there are other variables measured only at fire locations which are considered as marks. For example, fires are classified into two or more different types depending on the field size burned or on the cause starting the fire. In such cases, we may consider multitype point patterns (Figure 2). The inhomogeneous K-function depends on the first-order intensity.

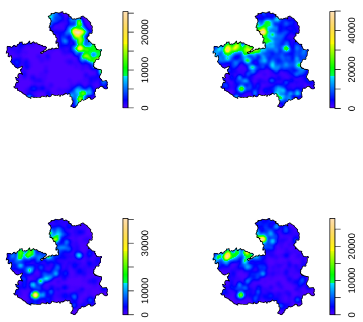

Figure 2: Forest fires with marks defining the four different causes in Castilla La Mancha

Accounting for the degree of inhomogeneity may force a variation in the form of the K-function. Thus treatment of the homogeneous function may lead to misinterpretation of the results. This is highlighted in Figure 3.

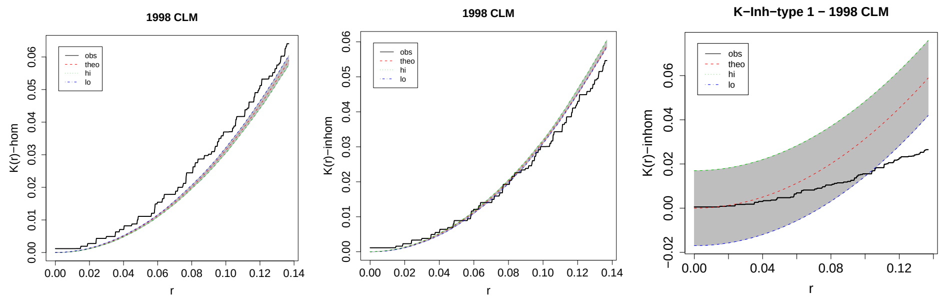

Figure 3: *Left*: Homogeneous K-function and envelopes under CSR for 1998; *Middle*: Inhomogeneous K-function for 1998; *Right*: Inhomogeneous K-function for 1998 and fire cause of type 1

Having all the previous descriptive information at hand, we propose several spatial point process models, univariate and multivariate taking into account the corresponding covariates and marks. Previous to finally consider the space-time structure, we first test for separability and then we propose a model for the space-time first-order intensity function. Cox processes are very useful for modelling aggregated point patterns, in particular in the spatial case where the two main classes of models are log-Gaussian Cox processes and shot-noise Cox processes. For spatio-temporal point pattern data, spatio-temporal log-Gaussian Cox process models have recently found different applications. Here we study and apply spatio-temporal shot-noise Cox point process models in the line suggested by [8] to model the space-time first-order intensity function.

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