METMA V International Workshop on Spatio-Temporal Modelling
Santiago de Compostela
30 Jun - 2 Jul 2010

Spatio-temporal study of Scottish rivers using wavelet analysis

M. Franco-Villoria[1,*], M. Scott[1] , T. Hoey[2] and D. Smith[3]

[1] *Department of Statistics, 15 University Gardens, University of Glasgow, G12 8QQ; mvilloria@stats.gla.ac.uk, m.scott@stats.gla.ac.uk*
[2] *Department of Geographical and Earth Sciences, University of Glasgow; Trevor.Hoey@ges.gla.ac.uk*
[3] *Department of Management, University of Glasgow; d.fischbacher-smith@lbss.gla.ac.uk*
[*] *Corresponding author*

Abstract. *Wavelet analysis is presented here a a possible method for detecting and comparing trends and periods of significant variability of non-stationary environmental time series. The results from a set of Scottish rivers suggest a difference in river flow maxima between the East and the West that has already been pointed out by previous studies.*

Keywords. *Discrete Wavelet Transform; Stationarity; Variability*

1 Introduction

1.1 Background

River flow records are an integral part of providing flood risk estimates and traditionally, classical time series models have been used. However, current changes in the environmental conditions, specifically those driven by climate change, have led investigators to look for new statistical models which account for non-stationarity and seasonal variations. In addition, there is continuing interest in spatial relationships, e.g. in Scotland, evidence of an East West difference in terms of rainfall and river flow is apparent over the last 30 years [1, 2, 6]. Climate change impacts are also expected to vary spatially and thus would be expected to result in changes in river flows. Newer statistical methodology, namely wavelets are presented here and applied to river flow data to investigate evidence of spatial differences.

1.2 Data set

Eight rivers of different catchment sizes across Scotland were selected based on data quality and spatial location (Table 1). Since the main interest lies in the extreme values the (logged) series of monthly maxima was calculated. Data (gauged daily flow) were provided by the National River Flow Archive and the Scottish Environment Protection Agency (SEPA).

River	Catchment Area (km^2)	Location	Data record
Nith(Friars Carse)	799.0	West	1/10/57 - 1/10/08
Dee(Woodend)	1370.0	East	1/10/29 - 30/9/08
Tweed(Norham)	4390.0	East	1/10/62 - 31/10/08
Ewe(Poolewe)	441.1	West	19/10/70 - 31/12/08
Lossie(Sheriffmills)	216.0	East	1/10/63 - 31/10/07
Tay(Ballathie)	4857.1	East	1/10/52 - 31/10/08
Water of Leith(Murrayfield)	107.0	East	1/1/63 - 31/12/05
Clyde(Blairston)	1704.2	West	1/10/58 - 5/11/08
Kelvin(Killermont)	335.1	West	1/10/48 - 31/12/07

Table 1: Data set.

2 Methods - Wavelet Analysis

One way of identifying the local behaviour of non-stationary time series is by wavelet analysis. By subsequently filtering the original series, we obtain sequences of results which relate to variations at different scales (frequencies). The result is a time-frequency representation of the data [4, 5]. The discrete wavelet transform (DWT) coefficients $\{W_n : n = 1,\ldots,N\}$ of a time series X of length N are calculated as $\mathbf{W} = \mathcal{W}\mathbf{X}$, where $\mathcal{W}$ is an NxN matrix constructed using the chosen filter. The original time series can then be reconstructed (known as multiresolution analysis MRA) as the sum of a number of wavelet detail components D_j and a smooth component S_J :

$$X = \mathcal{W}^T\mathbf{W} = \sum_{j=1}^{J} D_j + S_J \qquad (1)$$

where J is the level of decomposition. D_J (wavelet detail) is a time series related to variations in X at scale τ_j=2^{j-1} and S_J (wavelet smooth) is a time series associated with scales $\tau_{j+1} = 2^j$ and higher and can interpreted as the trend. The DWT has some restrictions in terms of sample size(it has to be a power of two), filter choice and starting point of the time series. The maximum overlap discrete wavelet transform (MODWT) provides a MRA of the time series without the restrictions of the DWT (although the computational cost is higher). The time-dependent wavelet variability at a particular scale τ_j provides a measure of the variability of the time series at each scale τ_j and time t and can be estimated as:

$$\hat{\nu}_{X,t}(\tau_j) = \frac{1}{N_S}\sum_{u=-(N_S-1)/2}^{(N_S-1)/2} W_{j,t+|\nu_j^{(H)}|+u \bmod N} \qquad (2)$$

where N_S=12 and $W_{j,t+|\nu_j^{(H)}|+u \bmod N}$ is the wavelet coefficient $W_{j,t}$ circularly shifted so that it is aligned in time with the original time series [4].

3 Results

MODWT (based on an LA(8) filter [4]) with 4 levels of decomposition was applied to the monthly maxima series (normalized to the overall mean). Each of the 8 river series was decomposed as $\sum_{j=1}^{4} D_j + S_4$. D_1-D_4 reflect changes over the scales 1, 2, 4 and 8 months respectively. The smooth component S_4 reflects changes over a scale of 16 months (and higher) and can be regarded as the trend.

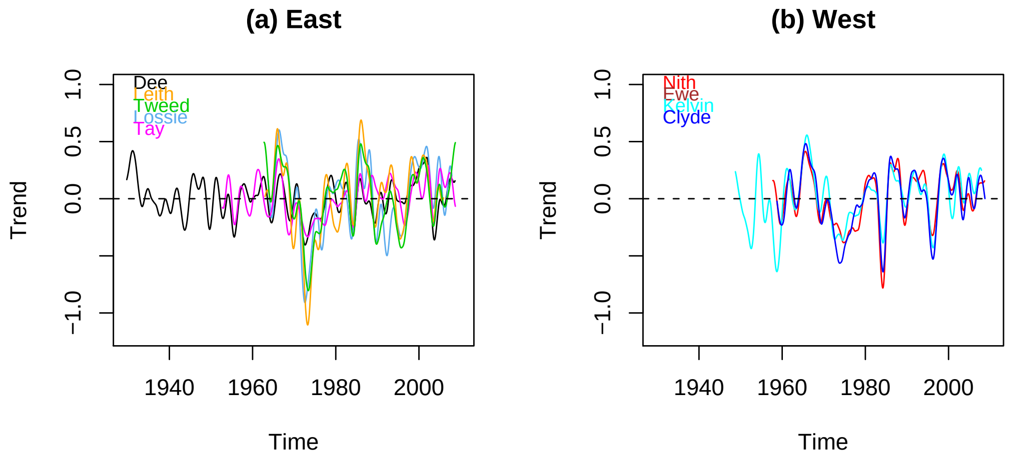

Figure 1: Trend from wavelet decomposition for (a)Eastern and (b)Western rivers.

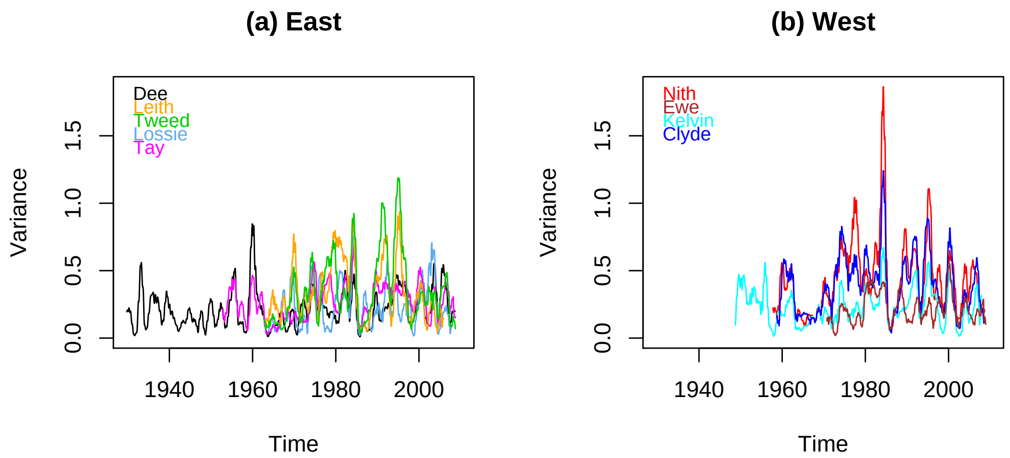

Figure 2: Seasonal variability for (a)Eastern and (b)Western rivers.

The trend series (component S_4 from wavelet decomposition), plotted on Figure 1, suggest some differences between the East and the West. There is a sharp decrease in the East around 1973 that, even though is also present in the West series, is not as low for the latter. There a second decrease around 1996 although in this case it is more prominent in the West than in the East. It is difficult to say whether there is an overall increasing or decreasing trend as the trend series are not linear; however, it looks as if there has been a slight increase in the West towards the end of the record but not in the East.

Component D_3 is of special interest, as it reflects changes over a scale of 4 months and therefore it can be regarded as the seasonal component. Also, it is the main contributor to the sample variance for all 8 rivers. A plot of this series (not shown) showed that the seasonal component is not constant from year to year and hence the time dependent wavelet variability was calculated to get a measure of fluctuations in variability. The resulting series are plotted on Figure 2. Overall, the variability is higher in the West than in the East. There is a clear indication of non-stationarity with periods of high variability alternated with periods of very low variability. In particular, there is a clear change point around mid 1986 for both eastern and western rivers, when the variability is minimum. This is in agreement with [1], which suggest a shift towards a "flood rich" period in the late 1980s. The 'cluster' of high variability just before that (from about 1977 to 1986), especially in the West, would correspond to the wettest period on record for the UK [3]. Overall, the variability seems to be greater in the West than in the East. Figure 2(a) also suggest a North to South difference, as the rivers Water of Leith and Tweed, situated in the Southeast of Scotland, seem to have higher variability than those in the Northeast (Lossie, Dee and Tay).

4 Summary and Future Work

Wavelet analysis is presented here as a powerful tool for exploring the variability of non-stationary time series as well as comparing rivers at different locations. The results suggest differences between the East and the West; however, further investigation is needed, and spatial methods will be applied in order to gain a better understanding of the spatial structure. [1] compared Scotland with the rest of Europe, finding similar results in terms of increases in annual maximum flood in Norway, southern Finland and Estonia over the last 30-50 years, suggesting that there might be a common climate cause. The North Atlantic Oscillation and the Atlantic Meridional Oscillation will be explored as potential drivers of these changes.

References

[1] Black, A.R. (1996). Major flooding and increased flood frequency in Scotland since 1988. *Physics and Chemistry of the Earth* **20(5-6)**, 463-468.

[2] Black, A.R. and Burns, C.J. (2002). Re-assessing the flood risk in Scotland. *The Science of the Total Environment* **294**, 169-184.

[3] Marsh, T.J. (1995). The 1995 drought - a water resources review in the context of the recent hydrological instability. *Hydrological Data UK Series. Institute of Hydrology, Wallingford* 25-33.

[4] Percival, D.B. and Walden, A.T. (2006). *Wavelet Methods for Time Series Analysis*. Cambridge University Press, Cambridge Series in Statistical and Probabilistic Mathematics.

[5] Torrence, C. and Compo, G.P. (1998). A Practical Guide to Wavelet Analysis. *Bulletin of the American Meteorological Society* **79**, 61-78.

[6] Werritty, A. (2002). Living with uncertainty: climate change, river flows and water resource management in Scotland. *The Science of the Total Environment* **294**, 29-40.