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Invited Review

Buyer vendor coordination models in supply chain management

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Abstract

Coordination between two different business entities is an important way to gain competitive advantage as it lowers supply chain cost. This paper reviews literature dealing with buyer vendor coordination models that have used quantity discount as coordination mechanism under deterministic environment and classified the various models. An effort has also been made to identify critical issues and scope of future research.

Keywords: Supply chain; Coordination; Inventory; Deterministic; Quantity discount

1. Introduction

In the last two decades, both academicians as well as practitioners have shown keen interest on the subject supply chain management (SCM). Globalization of market, increased competition, reducing gap between products in terms of quality and performance are compelling the academicians and industry to rethink about how to manage business operations more efficiently and effectively. Since, scope for improvement within the organization is decreasing, the academicians and captains of industry are looking for newer alternatives of integrating the business activities beyond the organization's boundary. More specifically, they are trying to align and coordinate the business processes and activities of the channel members to improve the overall performance and effectiveness of supply chain.

A supply chain consists of a number of distinct entities (e.g. raw material supplier, manufacturer, transporter, retailers, etc.) who are responsible for converting the raw material into finished product and make them available to ultimate customers

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to satisfy their demand in time at least possible cost. In principle, all the steps from procurement of raw materials to final delivery of products can be included into a supply chain, connecting raw materials suppliers, manufacturers, distributors and finally customers. Thus, a supply chain can be viewed as a group of entities interacting to transform raw material into finished product and then final delivery of the product to the customer. The entities may be owned by one individual/organization or by several individuals/organizations. Most supply chains of today belong to the later category. Focus on individual links of the supply chain invariably leads to inefficient and high cost product/service delivery system. In the process, such a supply chain looses to supply chain that is customer focussed where the individual links orient their business processes and decisions to ensure least cost delivery of products/services to the ultimate customer. Narasimhan and Carter (1998) in their work have mentioned that a well-integrated supply chain involves coordinating the flows of materials and information between suppliers, manufacturers, and customers. Thomas and Grifin (1996) have mentioned that effective supply chain management requires planning and coordination among the various channel members including manufacturers, retailers and intermediaries if any.

Several strategies are used to align the business processes and activities of the members of a supply chain to ensure better supply chain performance in terms of cost, response time, timely supply and customer service. Supply chain coordination is concerned with the development and implementation of such strategies. There is no universal coordination strategy that will be efficient and effective for all supply chains as the performance of a coordination strategy is supply chain characteristics dependent. Various types of coordination mechanisms have been used in supply chain coordination literature such as quantity discount, credit option, buy back/return policies, quantity flexibility, commitment of purchase quantity, etc. Supply chain coordination through quantity discount has received much attention in Production/Operations Management literature only recently (Weng, 1995a,b).

Our intention here is not to cover the entire gamut of supply chain coordination mechanism literature. Since quantity discount is considered to be one of the most popular mechanisms of coordination between the business entities, this paper primarily investigates supply chain coordination models that have used quantity discount as coordination tool under deterministic environment. However, we have also included here some integrated buyer vendor models that have similar type of objective function to achieve production distribution coordination and that improves the performance of the supply chain. In this paper, the word vendor, supplier and manufacturer is used alternatively to represent the same upstream member in the supply chain who sells the item to the buyer unless specifically mentioned.

2. Notation

Subscript 1 and 2 represent vendor and buyer respectively,

D	the buyer's annual demand for the product
S_i	setup and ordering cost for the firm i, $i = \{1, 2\}$
r_i	annual inventory holding cost expressed as a percentage of the value of the item for the firm i, $i = \{1, 2\}$
Q	the buyer's order quantity
M_2	the vendor/manufacturer's gross profit on sales expressed as a percentage
d_k	discount per unit offered by the manufacturer
R_2	the manufacturer's production rate in unit per year
P_0	the buyer's base purchase price without quantity discount
C_2	the manufacturer's manufacturing cost per unit excluding order processing, setup, and inventory holding costs per unit

3. Background and taxonomies of review

Study of integrated inventory models can be viewed as one of the origin of supply chain coordination study from Operations Management perspective. These models mainly examine the benefits accrued in the system due to coordination in order quantities between the two parties. Earlier, Goyal and Gupta (1989) have reviewed the literature of buyer vendor coordination models. Benton and Park (1996) and Munson and Rosenblatt (1998) have also reviewed some of the papers discussed here under different context. We have mainly considered here the literature of channel coordination/supply chain coordination models that have operations approach. Operations approach mainly concentrates on the operating cost of the channel. Operating cost is considered as a function of retailer's/buyer's order quantity where a fixed retail price is assumed and this leads to a fixed final demand.

The traditional inventory model assumes that a rational buyer would prefer to purchase his optimal order quantity (EOQ) as any deviation from this quantity would increase his total cost. The buyer's annual total cost for order quantity Q can be expressed as

$$\mathrm{TC}(Q) = P_0 D + \left(\frac{D}{Q}\right) S_1 + \left(\frac{Q}{2}\right) r_1 P_0. \qquad (1)$$

When quantity discount is not allowed, the buyer's optimal order size is given as

$Q^* = \sqrt{\frac{2DS_1}{r_1 P_0}}$. Thus, the total annual cost is given as

$$\mathrm{TC}(Q^*) = P_0 D + \sqrt{2DS_1 r_1 P_0}. \qquad (2)$$

Corresponding to buyer's order quantity Q, the vendor's/manufacturer's yearly net profit considering only order processing and setup costs for lot for lot policy can be written as

$$\mathrm{YNP}_2 = DM_2 P_0 - \left(\frac{DS_2}{Q}\right). \qquad (3)$$

The total channel cost is the sum of the individual cost component of buyer and vendor respectively and can be written as

$$\mathrm{TCC}(Q) = P_0 D + \frac{D}{Q}(S_1 + S_2) + \left(\frac{Q}{2}\right) r_1 P_0. \qquad (4)$$

Vendor's/manufacturer's order processing and setup cost per order are considered to be larger than the buyer's order processing cost per order. If a buyer adopts his EOQ as the order quantity for minimizing his total annual costs, the vendor/manufacturer incurs a significant cost penalty. Therefore, vendor/manufacturer induces the buyer through quantity discounts to order larger quantity to maximize his profits. Manufacturer can maximize his profit when the lot size is as high as infinity! When buyer purchases in larger order quantity $Q > \mathrm{EOQ}$ then there is an increase in profit for the vendor because of potential savings in order processing cost, manufacturing setup costs and transportation costs. By selling fewer but larger orders, the vendor generates lower sales cost. Also, vendor may save by seeking quantity discounts on raw materials he receives from his supplier. The increase size of order quantity or lot size ultimately helps in improving the channel profits. Increase in profits should be shared in some equitable fashion so that coordination in real sense is useful and parties in the channel shows interest to coordinate.

In this paper, we have categorized the various coordination models as follows:

(i) One can maximize the supplier's yearly net profit as shown by Eq. (3) in our general model by adopting different lot size by giving incentive to the buyer. The authors who have attempted the coordination problem from this perspective are classified here as vendor's/manufacture's perspective coordination models.

(ii) Similarly, one can minimize the total system cost with respect to coordinated lot size or the order quantity as shown by Eq. (4) and thereby improves the system savings. We have classified here those models as joint buyer and seller/manufacturer perspective coordination models.

(iii) On the otherhand, some authors have studied the buyer vendor coordination through quantity discount as a non-cooperative and cooperative game. In a non-cooperative game, each member will try to maximize his profit or minimize his cost. Thus the objective will be here to maximize Eq. (3) and minimize Eq. (1) of the general model. However, in a cooperative game, the objective will be to maximize system profit subject to the constraint that no player looses or incurs more from their non-cooperative solution. We have categorized these models as a buyer and a seller/manufacturer coordination models under game theoretic frame work.

(iv) Finally, we have kept all the coordination models that have considered multiple buyers under the separate category as single manufacturer and multiple buyers coordination models. These models are largely based on any one of the objective function of the above first three categories of models to achieve channel coordination.

In this stream of literature, most of the models have assumed that seller/manufacturer knows or can estimate the buyer's setup and holding costs. Further, EOQ assumptions are considered for the buyer. The buyer is assumed to act optimally and order the quantity leading to his lowest total cost.

4. Manufacturer's/seller's perspective coordination models

The relationship between the quantity discount and the efficiency of buyer seller transaction was studied as early as in 50's. The traditional quantity discount models provide lot size solution for the buyer under the assumption that the quantity discount schedule already exists. The role of vendor/supplier is ignored completely from whom the buyer actually purchases the item. With the assumption that vendor's have full information about the buyer's cost structure, some researchers have examined the problem from vendor's/supplier's point of view and the objective function here is to maximize the vendor's profit as shown by the Eq. (3). They have derived quantity discount-pricing schemes that induce the buyer to change its order quantity from that computed without price-discounted price. The pricing scheme is profitable as long as the discount paid to the buyer is less than the vendor's cost savings.

Crowther (1964) was first to consider quantity discount policy from supplier's point of view. The idea was of determining the difference between the profits accrued to a vendor by persuading a buyer to order the items in quantities larger than his EOQ. Following Crowther's argument, Monahan (1984) in his model suggested that a vendor could encourage his customer to increase the order quantities from EOQ by offering a price discount. With the quantity discount, the buyer will be motivated to increase the order size up to KQ^* where K is a factor by which the vendor entices the buyer's order size The amount of discount offered by the vendor compensates buyer's increased in inventory costs. For the increased order size, total cost of the buyer is given as

$$\mathrm{TC}(KQ^*) = P_1D + \sqrt{2DS_1r_1P_0}\left(1 + \frac{(K-1)^2}{2K}\right). \tag{5}$$

The increase in cost resulting from larger order size is the difference between the costs at the EOQ and costs at the order size KQ^* as given by Eq. (5). The vendor offers a price discount per unit equal to the increase in cost at buyer's side, which is given as

$$d_k = \sqrt{\frac{2S_1r_1P_0}{D}}\left\{\frac{(K-1)^2}{2K}\right\}. \tag{6}$$

Supplier's yearly net profit after giving discount amount is given as follows:

$$\mathrm{YNP}_2 = D(M_2P_0 - d_k) - \frac{D}{KQ^*}S_2. \tag{7}$$

Substituting the value of d_k in Eq. (7), maximize the supplier's profit equation YNP_2 with respect to K. The optimal value of K is obtained as

$$K^* = \sqrt{\frac{S_2}{S_1} + 1}. \tag{8}$$

From the expression of K^* in Eq. (8), one can easily say that when the value of S_2 is large, the supplier can entice the buyer to order in larger quantity and the value of K^* is independent of the amount of discount offered by the supplier. One important issue here is that when buyer is exactly compensated for increase in cost due to larger order size, buyer will be indifferent towards increasing his order quantity. Monahan developed the model considering lot-for-lot policy, an all unit quantity discount schedule with single price break.

Rosenblatt and Lee (1985) in their paper relaxed Monahan's (1984) assumption that the vendor operates on lot-for-lot policy. The implicit assumption in the model is that the vendor purchases the item from another supplier and resells it to the buyer. They have proved that the optimal order quantity of the vendor is an integer multiple (k) of buyer's order quantity KQ^*. Incorporating inventory-holding cost, the vendor's yearly net profit equation is given as

$$\mathrm{YNP}_2 = \{(P_0 - C_2) - d_k\}D - \left(\frac{DS_2}{kKQ^*}\right) - (r_2C_2)\frac{(k-1)KQ^*}{2}. \tag{9}$$

Assuming a linear discount schedule, the authors developed an algorithm to determine the optimal order quantity and the optimal discount amount that the vendor could offer to the buyer while maximizing his profit.

Lee and Rosenblatt (1986) further generalized the earlier work of Monahan (1984) by putting an additional constraint on minimum acceptable profit margin for the vendor. They allowed the vendor to purchase integer multiple (k) of buyer's order quantity (KQ) rather than lot-for-lot and maximize the vendor's yearly net profit as shown by Eq. (9) subject to the constraint on the discount amount offered to the buyer. The authors also develop an efficient algorithm to determine the values of k and K. Goyal (1987a) however, in his work, suggests that the constraint on the amount of discount offered to the buyer by Lee and Rosenblatt's (1986) seems to be unreasonable as the objective of the vendor, in all likelihood is to increase his own profits. Goyal also proposes a much simpler method in his model to determine the value of k and K compared to Lee and Rosenblatt's model. Other work to be cited here is Hwang and Kim (1986).

In all the previous models discussed above consider vendor as supplier who purchases the item from another supplier. Considering vendor as manufacturer, Banerjee (1986b) extended the earlier work of Monahan by incorporating vendor's inventory holding cost in the model. When the vendor is a manufacturer, he supplies the item to the buyer only after completion of the production run and therefore author incorporates the inventory carrying cost term in the supplier's profit equation. The supplier's yearly net profit equation is given as

$$\mathrm{YNP}_2 = D(M_2P_0 - d_k) - \frac{DS_2}{KQ^*} - \frac{DKQ^*}{2R_2}r_2C_2, \tag{10}$$

where production rate R_2 is assumed to be greater or equal than that of demand rate D. He finds out the optimal value of K^* and showed that when the production rate is infinite, the model approaches the original model of Monahan. He also shows that under certain condition the value of K^* can be less than one and in such situation vendor should entice the buyer to order a smaller quantity.

Banerjee (1986c) suggested a pricing model from the vendor's perspective. A single buyer and lot for lot production strategy are assumed in the development of the model. The objective of the model was to determine the price of an item in order to achieve a stated level of gross profit of the vendor. Goyal (1987b) suggested another model to determine the economic ordering policy of a vendor by taking into consideration the amount of quantity discount the vendor may wish to give to the buyer He assumed that the cost of holding a unit in the stock for a vendor is constant and does not depend upon the amount of discount per unit.

Joglekar (1988) in his work, extends Monahan's (1984) model, to show that an optimal production lot size policy is superior to the policy of optimal price discounts particularly when the setup cost of manufacturer is substantially larger than the ordering cost of the buyer. He shows that there is no reason as to why the vendor could not use both the optimal price discount and production lot strategy together.

Further, extending Monahan's (1984) model, Drezner and Wesolowsky (1989) have found optimum price break quantities for a given discount scheme. However, special cases in optimizing the prices (discounts) are also discussed in their work. They develop an exact optimal solution to the problem maximizing the supplier's profit function.

Weng and Wong (1993) formulate an all unit quantity discount model. They consider the supplier's profitability with quantity discount policy. The characteristic of the model is that it can be used for the analysis of the supplier's profit maximization or the supplier's increased profit share analysis i.e. the model allows the supplier to determine a series of optimal quantity policies based on the incentive required by the buyer. This helps the negotiation process between the supplier and the buyer.

5. Joint buyer and seller/manufacturer coordination models

Some authors have used quantity discount as a coordination mechanism to maximize the joint profit of the buyer and the vendor. The objective function here in all likelihood is to minimize the total channel cost as shown by Eq. (4). The models here provide some explicit mechanism for division of surplus generated in the channel due to coordination. Like the seller's perspective model, here also it is assumed that seller's have full information about buyer's cost structure.

Dolan (1978) formalizes Crowther's argument by specifying quantity discount as a mechanism for inducing the buyer to purchase in quantities minimizing the system cost. The idea of joint optimization for buyer and vendor was initiated by Goyal (1976) and later reinforced by Bannerjee (1986a). The objective of Goyal's model was to minimize total relevant cost for both the vendor and the buyer for the order quantity Q. He assumed that manufacturer does not produce the item and in fact purchases it from another supplier. Moreover, he assumed that inventory holding costs are independent of the price of the item.

Bannerjee (1986a) formulated a joint economic lot size (JELS) model for a buyer and a vendor system where the vendor has a finite production rate. He determines the JELS Q^* by differentiating the total system cost equation with respect to Q.

$$\mathrm{TC}(Q) = \frac{D}{Q}(S_1 + S_2) + \frac{Q}{2} r\left(P_0 + \frac{D}{R_2} C_2\right), \tag{11}$$

$$Q^* = \sqrt{\frac{2D(S_1 + S_2)}{r\left(P_0 + \frac{D}{R_2} C_2\right)}}. \tag{12}$$

The assumption they consider is that a production setup is incurred every time when an order is placed. He finds that without quantity discount, the buyer incurs loss, but the supplier gets benefit if JELS is adopted rather than buyer's EOQ. He developed the two bounds of discounts that allow the joint benefit to both the parties if the buyer increases the order quantity from EOQ to the JELS quantity. When discount amount is fixed at lower bound, all the benefits go to the supplier and the buyer is indifferent where as when amount of discount is set at maximum level, all benefits shift to the buyer and the supplier is indifferent. While suggesting equal distribution of the gains from Joint Economic Ordering, Bannerjee (1986a) mentioned that question of pricing and lot-sizing decisions are settled through negotiations between the buyer and the seller. Later on, we will see how some authors have incorporated in their model the bargaining power of the channel members in fixing the order quantity and amount of discounts.

Goyal (1988) extended Bannerjee's (1986a) model by relaxing the lot-for-lot production assumption and showed that the economic production quantity of vendor could be an integer multiple of the buyer's purchase quantity. The author has shown that manufacturing a batch, which is made up of n integral number of equal shipments generally produced a lower cost solution. He assumed that whole lot is produced before the first shipment is made to the customer.

Dada and Srikanth (1987) in their paper formulate a model that provides economic incentives to the seller to offer a quantity discount-pricing scheme beneficial to both buyer and seller. They characterized a range of order size and price for which savings can be realized by the buyer and the seller. The authors also mentioned the mechanism for sharing the savings, where the buyer makes an annual lump sum payment to the seller. It is similar to the one proposed by Jeuland and Shugan (1983).

Chakrabarty and Martin (1988) in their work have examined a quantity discount-pricing model considering periodic review. The model assumes that buyer and the supplier have a common order interval time so that supplier's finished goods are directly shipped to the buyer. They assume infinite replenishment rate of the supplier. The model determines the optimal order interval time by minimizing total cost function constrained by proportion of savings to be shared by the parties. They have proposed an approach to share the annual savings in a pre-specified manner. Further, they extend the problem to multi-buyers case also. Later on Chakrabarty and Martin (1989) extend their earlier work by considering decreasing demand function.

Following the work of Monahan (1984) and Lee and Rosenblatt (1986), Arcelus and Srinivasan (1989) have considered both buyer and vendor as maximizer of their return on average investments in inventory. The end result of the work is a range of possibilities within which the parties may negotiate for the desired quantity and discount levels.

Kim and Hwang (1989) in their paper have examined the effects of price and order size on the inventory related cost of a customer and the profit of a supplier. They consider three scenarios: (i) when seller maximizes his profit, (ii) when buyer minimizes his cost and (iii) when the buyer and the seller minimize their joint cost. They have shown that supplier can encourage the buyer to order larger quantity by providing discount as long as the amount of discount for the order quantity benefits both the parties.

Anupindi and Akella (1993) have presented optimal ordering policies for a single buyer with multiple vendors and thereby this work is different from the earlier models. They have developed three models and for each model they have presented an optimal ordering policy. The buyer's policies are: (i) order nothing when the inventory level is above an upper bound, (ii) order from one vendor when the inventory level is between a lower and an upper bound and (iii) order from both vendors when the inventory level is below the lower bound.

Weng (1995a) has extended Jeuland and Shugan's (1983) model. The focus of his model is on determining how to implement a mechanism that divides the additional profit generated through coordination. Under the assumptions that the buyer will receive a fixed fraction of the incremental profit, the author has shown that a quantity discount for the buyer along with the franchise fee paid to the supplier is sufficient to induce the buyer to make decisions that lead to joint profit maximization. The assumption considered in the model that annual demand increases in response to a price reduction, is a more realistic than constant demand assumption of the earlier model. Weng (1995b) in his paper considers both all unit and incremental quantity discount policy under price sensitive demand condition. He observes the benefit for both the buyer and supplier by maximizing the supplier's profit equation and the joint profit equation respectively. He has shown that optimal quantity discount policy and incremental discount policy is equivalent in benefiting both the supplier and the buyer.

Lu (1995) presented an integrated inventory model, where he worked out an optimal solution to a vendor and a buyer problem. He presented a heuristic solution for the single vendor multi-buyers case under some assumptions. Goyal (1995) in his work on integrated buyer vendor model where manufacturer produces an integer multiple of buyer's order size considered that first shipment of item to be made to the buyer before whole lot is produced. He incorporated a policy in which size of successive shipments from manufacturer to customer within a production cycle increases by a factor equal to the ratio of production rate to the demand rate.

Hill (1997) showed that neither the equal shipment size policy (Goyal, 1988) nor the increasing shipment size policy (Goyal, 1995) is always optimal. Rather, these are two extremes on a continuum. Hill provided a generalized policy for finding the value of the factor by which to increase the shipment sizes. Hill (1999) combining increasing shipment size policy of Goyal (1995) and an equal shipment size policy derived a globally optimal batching and shipment policy for the single vendor and single buyer integrated problem. This policy of Hill gives a lower total channel cost as compared to the earlier policies and improves the performance of the channel. Other related literatures that can be mentioned here are Goyal and Nebebe (2000), Hoque and Goyal (2000), Bylka (2003). These models can also be viewed from production distribution coordination perspective.

Viswanathan (1998) in his paper has compared two supply policies for an integrated vendor buyer inventory model. In first policy, the vendor produces a batch and supply to the buyer in number of equal shipment size at constant interval. The second policy is to supply the production batch to the buyer in increasing shipment size. He identified problem parameters under which the equal shipment size policy and increasing shipment size policy is optimal. The author has observed that neither of the two policies dominates the other for all problem parameters. The second policy attempts to shift inventory to the buyer as quickly as possible. This type of strategy works better if the holding cost for the buyer is not much higher than that for the vendor.

Aderohunmu et al. (1995) in their study have shown that when a vendor and a buyer follow a cooperative batching policy and share cost information along with other information in time, significant cost savings can be achieved to the advantage of both the parties and improves the performance of the channel. The result of this work gives evidence of the importance of timely and honest cost information exchange between the parties in the JIT environment.

Recently, Taylor (2002) has introduced channel rebate as coordination mechanism for the supply chain. He has considered two forms of rebate. They are linear rebate and target rebate. In linear rebate case, rebate is paid for each unit sold and target rebate is paid for each unit sold beyond a specified target level. The author has shown that when demand is not influenced by sales effort, a properly designed target rebate achieves channel coordination. Author has distinguished rebate from a reduction in manufacturer's wholesale price by mentioning that reduction in price caused by the rebate is only realized if the item is sold to an end user. Since, it is also a price reduction technique for coordination, so we have included the paper here.

5.1. Three level coordination models

The models that we have discussed above, mainly focuses on the integration of two members. But a supply chain is not ended up with only two members and its scope is much larger. Some authors have tried to integrate the raw materials supplier into the earlier buyer vendor models to make it a three-stage supply chain. Banerjee and Kim (1995) presented their model from an integrated standpoint of the buyer, the manufacturer, and the raw materials supplier in a JIT environment.

Munson and Rosenblatt (2001) have extended the two level supply chain to a three level supply chain by considering a supplier (who is supplying raw materials to manufacturer), a manufacturer and a retailer and they explored the benefit of using quantity discount on both ends of the supply chain to decrease cost. Like the earlier scenario, manufacturer's production lot size is an integer multiple of the buyer's order quantity and the manufacturer orders an integer multiple of his production lot size to the raw materials supplier. They have shown that by quantity discount mechanism; company can coordinate its purchasing and production functions. This creates an integrated plan that dictates order and production quantities throughout a three firm channel. They have considered manufacture as the dominant member in the channel who takes the lead role in coordinating the channel.

Yang and Wee (2001) in their paper have also considered integration of producer, distributors, and retailers a three-stage supply chain. They have developed an economic ordering policy under constant demand for the arborescent (i.e. a tree like) inventory model structure. They have shown that the integrated approach results in a significant cost reduction compared to that of independent decision making by each individual entity of the supply chain. The model however, has not considered how the increase in cost at retailer level is to be compensated due to implementation of the integrated policy.

Recently, Khouja (2003a) has also considered three stage supply chain of tree like inventory model structure. He has considered three coordination mechanisms between the members of the supply chain and has shown that some of the coordination mechanisms can lead to significant reduction in total cost. The author however, has not considered the distribution of savings between the different members of the supply chain.

Khouja (2003b) in this paper also studied coordination of the entire supply chain from raw materials to customer considering single and multiple components. They consider components scheduling decisions at each stage in which manufacturing occurs and its impact on the holding cost. They have shown that complete synchronization in the chain leads to loss of some members of the supply chain. They provide an algorithm for optimal synchronization of supply chain and incentive alignment along the supply chain.

6. A buyer and a seller/manufacturer coordination models under game theoretic framework

Some authors have viewed the buyer vendor coordination problem through quantity discount mechanism as a two-person game. They can be formulated as non-zero sum game having elements of both conflict and cooperation. In a non-cooperative game playing independently, the intention of the players (vendor and the buyer) is to maximize their individual gain. The objective function for this game from the general model can be written as

$$\text{Minimize} \quad \mathrm{TC} = P_0 D + \frac{DS_1}{Q} + \frac{Q}{2} r_1 P_0, \tag{13}$$

$$\text{Maximize} \quad \mathrm{YNP}_2 = DM_2 P_0 - \frac{DS_2}{Q}. \tag{14}$$

Generally, the solution to the non-cooperative game can be obtained by using established equilibrium concept. Different types of game models have different solution concept. In the Stackelberg game, the player who holds more powerful position is called the leader and enforces his strategy on the other and the other player who reacts to the leader decision is called the follower. The solution obtained to this game is the Stackelberg equilibrium solution.

On the otherhand in a cooperative game both buyer and seller would consider maximizing system profit subject to buyer's total annual cost at cooperation should be less than or at most equal to those at non-cooperation. Similarly, seller's total annual profit at cooperation should be greater than or at least equal to those at non-cooperation. The objective function for this game from the general model can be written as

$$\begin{aligned} \text{Max} \quad & \lambda \mathrm{YNP}_2 - (1-\lambda)\mathrm{TC} \qquad (15)\\ \text{Subject to} \quad & \mathrm{TC} \leqslant \mathrm{TC}^* \\ & \mathrm{YNP}_2 \geqslant \mathrm{YNP}_2^*, \end{aligned}$$

where TC^* and YNP_2^* represents the cost and profit of buyer and seller before cooperation. Depending upon the bargaining power of the seller and the buyer, the value of λ varies between 0 and 1. In the cooperative game a group of strategies is called a pareto efficient point when at least one player will be better off and no player will be worse off from the initial condition. In the decentralized supply chain where the members belong to two different firms, the method of bargaining and negotiated solution which is dynamic in nature may result better coordination in the supply chain as compared to static coordinated solution in a centralized supply chain.

Kohli and Park (1989) formulate a cooperative game and examined the negotiation process between the seller and the buyer when they bargain for the order quantity and the average unit price. They assume that both the buyer and the seller know their own and each other's holding and ordering costs. Their focus is on a subset of Pareto efficient schedules for order quantity and quantity discounts. The procedures for determination of an optimal order quantity and an optimal quantity discount are independent. A particular optimal schedule is determined by an explicit bargaining power assigned to each member.

Abad (1994) in his work formulated the problem of buyer vendor coordination as a two person cooperative game and developed the Pareto efficient and Nash bargaining solutions. They have presented two pricing schemes for coordination between a vendor and multiple buyers case. Chiang et al. (1994) have also formulated quantity discount problem under game theory framework considering both cooperative and non-cooperative game models. They have shown the benefits resulting from cooperation between the buyer and the seller. Chirsty and Grout (1994) have prescribed a model to safeguards the relationship between buyer and supplier in a supply chain using principle of game theory and transaction cost economics.

Further, Li et al. (1995, 1996) have also developed buyer seller cooperation model assuming buyer is in a monopolistic market for the product in a constant demand situation under game theory framework. Comparing the cooperative and non-cooperative models, they have shown that system profit is higher at cooperation than at non-cooperation and the wholesale price of the seller to the buyer is lower at cooperation than at non-cooperation. The authors also have shown that quantity discount approach is an effective mechanism for achieving system cooperation.

Viswanathan and Wang (2003) have examined single vendor and single buyer distribution channel coordination considering quantity discount and volume discount. In volume discount, price break points are based on the total dollar volume of business across all products purchased from the vendor. The demand faced by the retailer is considered deterministic but price elastic. The vendor and the retailer act independently and rationally and each party wants to maximize their profit. Formulating the problem as stackelberg game, they have shown that effectiveness of volume discount coordination mechanism is more when sensitivity of demand to price changes is higher and effectiveness of quantity discount is higher with lower price sensitive of demand. Further their numerical study shows that perfect coordination is achieved when both the discount mechanisms are incorporated.

Relaxing the assumption that supplier has complete information about the buyer's cost structure; Corbett and de Groote (2000) consider a supplier and a buyer model where the buyer's holding cost information is not known to the supplier. They consider that supplier has a prior distribution of holding cost of the buyer and the problem is formulated as direct revelation game. The supplier asks the buyer to announce the value of holding cost and on that basis supplier determines the lot size and the discount amount. The objective is to determine the lot size and discount amount that minimizes supplier's expected cost subject to the incentive compatibility constraint that the buyer will always reveal the true information about the holding cost. They have shown that optimal lot size and discount amount both are decreasing function, which can be interpreted as a quantity discount-pricing scheme. They derive optimal quantity discount scheme to coordinate between the seller and buyer under asymmetric information.

7. A manufacturer and multiple buyers coordination models

To differentiate single manufacturer multiple buyers literature is a difficult task since in many cases many authors after developing the model for single buyer case have extended it for multiple buyers and these models are mostly confined to homogeneous group of buyers. Therefore, drawing a clear demarcation line between these two literatures is somewhat arbitrary. However, here an effort is made to give a brief review of the works that have exclusively dealt with multiple heterogeneous buyers' case.

Lal and Staelin (1984) initially studying one vendor and one group of homogeneous buyers have extended the study to heterogeneous group of buyers for determination of optimal pricing policy. His heterogeneous groups differ in size, holding cost, order cost and demand rates. They are varying between the groups but not within a group. They have not obtained a close form solution. The solution to the heterogeneous group of buyers is offered by considering continuous approximation of a discrete quantity discount schedule.

Kim and Hwang (1988) have studied a supplier supplying a product to multiple retailers. They have shown that supplier's profit increases without incurring any additional cost to the retailers, resulting from the reduced number of setups. Drezner and Wesolowsky (1989) also dealt with multi-buyers case and they have offered a method for solving the problem when the vendor offers a single quantity discount schedule to all the buyers.

On the otherhand, Joglekar and Tharthare (1990) in their study on one supplier and many buyers case have mentioned that cooperation between a buyer and a supplier is anti ethical to the free enterprise concept and they put forward their argument in favor of allowing each party to adopt its own independently derived optimal replenishment policy. Further, they have mentioned that above policy is not only being consistent with free market principles but also economically more desirable.

Banerjee and Burton (1994) in their paper, assuming a deterministic situation and vendor's demand rate as approximately constant, have shown that in multiple buyers case, classical economic lot size model may not be able to truly reflect the exact scenario due to discrete vendor inventory depletion. They have observed that even under deterministic situation, in the absence of an adequate production reorder point policy, stock-out may occur. They have considered a common replenishment cycle based coordinated inventory model and have shown that it is superior to individual optimization approach in the multi-buyers case.

Relaxing the assumption of Banerjee and Burton's (1994) model of vendors demand rate as approximately constant, Bylka (1999) in his study on multi-buyers case has considered buyer and vendor demand as periodic sequence and each buyer uses his own replenishment policy. An optimal vendor production schedule is determined by the author. Banerjee et al. (2003) have made simulation study to see the effect of lateral shipment in a two level supply chain where single supplier is at higher level and multiple buyers are at lower level. They have shown that their proposed lateral shipment policy performs better than a policy where there are no such shipments under certain conditions.

With the assumption that vendor follows lot-for-lot policy, Viswanathan and Piplani (2001) has shown that a vendor could implement the common replenishment period mechanism by offering price discounts to buyers in a one vendor, multi-buyer supply chain for a product. Under the proposed strategy, the vendor specifies the common replenishment period for all the buyers, which is a cost increasing alternative for the buyers. Therefore, the vendor offers a price discount to encourage the buyer to accept this strategy. The price discount must be such that it compensates buyers for any increase in inventory costs and possibly provide additional savings. The important assumption in the model is that buyers cost and demand parameters are known and therefore, the vendor can anticipate the buyer's reaction.

Misra (2004) has extended the above model of Viswanathan and Piplani (2001) by considering selective discount policy. He has studied the common replenishment period mechanism by allowing some buyers to participate in the coordination scheme where they get discount for ordering larger quantity whereas other buyers continue to order at earlier fashion without going for discount. They have found that in some situations, it might be beneficial to segment the buyers by offering multiple common replenishment periods.

Gurnani (2001) has also studied quantity discount-pricing models with different ordering structure in a system consisting of a single supplier and heterogeneous buyers. This work is an extension of the earlier work of Zahir and Sarker (1991) where authors have considered a price dependent demand function for multiple regional wholesalers who are served by a single manufacturer. Gurnani has considered order coordination, order consolidation, and multi-tier ordering hierarchy. He has shown that for identical buyers, order coordination leads to reduction in system cost. For heterogeneous buyers, they have determined the sufficiency condition when the coordination will be preferable.

Chen et al. (2001) in their paper of one supplier and many retailers have shown that same optimum level of channel wide profit can be achieved in decentralized system as that of centralized system only if coordination is achieved via periodically charged, fixed fees, and a non-traditional discount-pricing scheme. Under such scheme, the discount given to the retailer is the sum of the three discount components based on the retailers annual sales volume, order quantity and order frequency. Further, they have shown that no traditional discount scheme based on order quantities only suffices to optimize channel wide profit when there are multiple non-identical retailers.

Woo et al. (2001) have studied an integrated inventory system where a single vendor purchases and processes raw materials in order to deliver it to multiple retailers. With the objective that both vendor and buyers are willing to invest to reduce joint ordering cost, authors have developed an analytical model to derive optimal investment amount and replenishment decisions for both vendor and buyers. They have shown that the vendor and all buyers can obtain benefit directly from costs savings. Klastorin et al. (2002) have studied coordination between a firm supplying a product to many retailers facing a static demand. The authors propose a policy in which the manufacturer out sources production and offers a price discount to retailers to coordinate the timing of their orders with its own.

Boyaci and Gallego (2002) in their work have analyzed coordination issues in a supply chain consisting of one wholesaler and one or more retailers under deterministic price sensitive customer demand. They have focused on inventory and pricing policies that jointly maximize channel profit. They have shown that an optimal policy can be implemented cooperatively by an inventory consignment agreement. Also, it is worth mentioning that the above policy is capable of distributing the gains of channel coordination without requiring side payments.

Wang (2002) has presented an analysis for a supplier's quantity discount decision for heterogeneous buyers. Author has mentioned that for single buyer and group of homogeneous buyers, a single price break is sufficient for the supplier to maximize his quantity discount gain but for multiple heterogeneous buyers single price break point is not sufficient. He has analyzed supplier's quantity discount decision by using common discrete all unit quantity discount schedule to all the buyers.

8. Insight and limitations

From the study of the above models, it is seen that this stream of literature describes the supply chain in a highly aggregated level and often considers only two decision makers. The important insight provided by the above literature is that there is an increase in profit for the manufacturer when the buyer purchases more than his EOQ. It is based primarily on the facts that (i) Manufacturer's setup cost is much higher compared to the buyer's ordering cost, and (ii) Manufacturer may use a production cycle which is an integer multiple of ordering cycle of the buyer.

- Excepting a few, majority of the models are developed considering deterministic demand, zero lead-time, no stockouts. Holding cost of buyer is considered to be independent of purchase price.
- With a few exception, rest of the models are developed where supplier offers all unit quantity discount with a single price break point. Further, the manufacturer is assumed to have two ways of acquiring the item either by outside purchasing or manufacturing the item subject to specific production capacity.
- Most of the models simplify the purchasing/production system to one product and one machine.
- Many of the models fail to specify how the incremental savings to the manufacturer can be passed onto the buyer. Some authors have mentioned about equal splitting of the surplus, whereas some have suggested splitting the surplus according to their investment. Most of the models are silent about conflict resolution between the supply chain partners e.g. division of surplus between buyers and suppliers. Such new problem may call for the use of game theoretic negotiation model.
- Most of the models assume that a supply chain partner has complete information (including cost, demand, lead time, etc.) about the other partner. This is considered to be major limitations of these models. In a decentralized supply chain, hardly will be the situation where complete information will be available with the parties. Coordination under limited information sharing is an important issue of concern to be studied for the decentralized supply chain.
- Single vendor multiple buyers' literature is still in its infancy state. Particularly, how a supplier should develop a quantity discount schedule when dealing with many buyers with different

demand and cost structure is not known. Thus, mechanism for additional profit sharing between vendor and multiple heterogeneous buyers is an important issue that needs investigation.

- In single vendor multiple buyers' literature very little work is available considering vendor as a manufacturer producing the items to supply multiple heterogeneous buyers. Under such situation, how to tackle the discrete vendor inventory depletion into the model is an area that requires further study.

9. Conclusions and future directions of research

The results of conceptual buyer vendor coordination models in terms of supply chain are very optimistic as coordination leads to savings in the system and ultimately improves the performance of the supply chain. However, the buyer vendor coordination models from supply chain management perspective are not free from limitations like other models in the literature. In terms of supply chain, these models have considered only a small part of the entire supply chain. Extension of these models to coordinate raw material procurement to distribution coordination can be considered as well-deserved future research work. Application of operations research technique to develop a comprehensive supply chain coordination model is more time demanding.

Further, most of the models discussed above consider simplified purchasing/production system. It is limited to one product and one machine only and thereby fails to capture the essence of real supply chain. Also many of the proposed models have considered deterministic demand situation but a situation of demand varying with time or price of the product is considered to be more realistic one. Similarly, deterministic lead-time or lead-time considered to be zero does not represent the actual physical system. Incorporation of transportation lead-time in the model is important since logistics play a bigger role in the efficient management of supply chain. In many models, sellers are assumed to be local monopolist in the market and the buyer is assigned to the supplier. However, in a market in the presence of many suppliers, they have to compete to capture the buyer. Future coordination model can incorporate this aspect also.

Various proposed models demonstrated that due to coordination between the parties in the supply chain, surplus is generated. But cooperation is feasible only when both the parties get their due share of profit. As stated in many of the models, pre-determined static division of surplus generated due to coordination through side payment may not always be an acceptable proposition for the members of a decentralized supply chain. In the literature, very few models are available (e.g. Kohli and Park, 1989; Ertogral and Wu, 2001) that have prescribed negotiation and bargaining for division of surplus between the parties. For a win-win situation, both the parties of a supply chain must participate in the division of profit exercise. A dynamic division of surplus amongst the channel members that involves the partners in decision-making may be an approach with investigation. Finally, since different parties are involved in the coordination process, to make coordination successful, faith between the parties and true revelation of information is necessary which model builder should take into consideration in their model in future. This review hopefully will be able to give some insights and new research issues in direction of supply chain management.

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