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Economic Order Quantity Under Conditions of Permissible Delay in Payments

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In this paper, mathematical models have been derived for obtaining the economic order quantity for an item for which the supplier permits a fixed delay in settling the amount owed to him. An example has been solved to illustrate the method.

Key word: inventory

INTRODUCTION

In deriving the economic order quantity formula, it is tacitly assumed that the supplier must be paid for the items as soon as the items are received. However, in practice a supplier will allow a certain fixed period for settling the amount owed to him for the items supplied. Usually there is no charge if the outstanding amount is settled within the permitted fixed settlement period. Beyond this period, interest is charged.

When a supplier allows a fixed time period for settling the account, he is actually giving his customer a loan without interest during this period. During the period before the account has to be settled, the customer can sell the items and continue to accumulate revenue and earn interest instead of paying off the overdraft which is necessary if the supplier requires settlement of the account immediately after replenishment. Therefore, it makes economic sense for the customer to delay the settlement of the replenishment account up to the last moment of the permissible period allowed by the supplier. We would now develop the mathematical models for determining the economic order quantity.

MATHEMATICAL FORMULATION

D = annual demand
h = unit stock-holding cost per item per year excluding interest charges
I_c = interest charges per \$ investment in stocks per year
I_d = interest which can be earned per \$ in a year
p = unit purchase price in \$
S = cost of placing one order
t = permissible delay in settling accounts
T = time interval between successive orders
$Z(T)$ = total annual variable cost.

The following assumptions are made in deriving the model.

(1) The demand for the item is constant with time.
(2) Shortages are not allowed.
(3) During the time the account is not settled, generated sales revenue is deposited in an interest-bearing account. At the end of this period, the account is settled and we start paying for the interest charges on the items in stock.
(4) Time period is infinite.

The total annual variable cost consists of the following elements.

(1) Cost of placing orders = S/T.
(2) Cost of stock holding (excluding interest charges). The average stock equals $DT/2$ (see Figure 1), hence stock-holding cost per year is given by $DTh/2$.
(3) Cost of interest charges for the items kept in stock. As items are sold, and before the replenishment account is settled, the sales revenue is used to earn interest. When the replen-

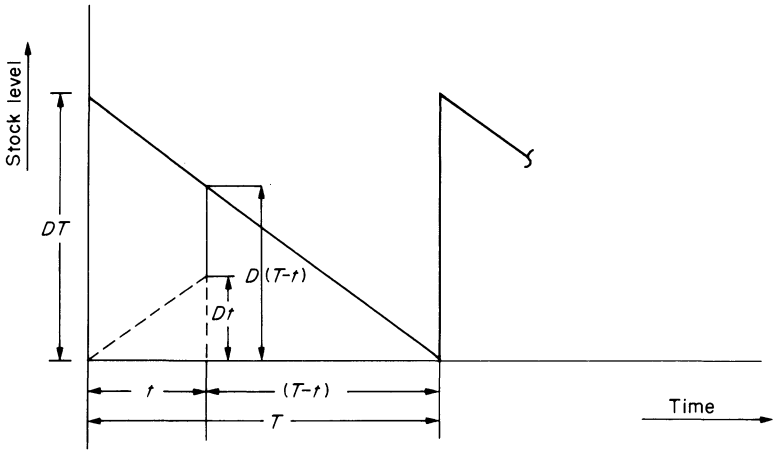

FIG. 1. *Time-weighted inventory when* $T \geqslant t$.

ishment account is settled, the situation is reversed, and effectively the items still in stock have to be financed at interest rate I_c. The stock level at the time of settling the replenishment account equals $D(T-t)$ (see Figure 1) and the interest is payable during time $(T-t)$.

$$\text{Interest payable in one cycle} = \frac{Dp(T-t)^2 I_c}{2}.$$

$$\text{Interest payable per year} = \frac{Dp(T-t)^2 I_c}{2T} = \frac{DpTI_c}{2} + \frac{Dpt^2 I_c}{2T} - DptI_c.$$

(4) Interest earned during the permissible settlement period. The maximum accumulated amount earning interest during the settlement period equals Dtp if $T \geqslant t$ (see Figure 1) or DTp if $T < t$ (see Figure 2). Hence the interest earned during the permissible settlement period for the two cases is obtained as follows:

Case I: $T \geqslant t$, *shown in Figure 1*

$$\text{Interest earned in one cycle} = \frac{Dpt^2 I_d}{2}$$

$$\text{Interest earned in one year} = \frac{Dpt^2 I_d}{2T}.$$

Case II: $T < t$, *shown in Figure 2*

$$\text{Interest earned in one cycle} = \left(\frac{DT^2 p}{2} + DTp(t-T)\right) I_d = DTpI_d\left(t - \frac{T}{2}\right)$$

$$\text{Interest earned in one year} = DpI_d\left(t - \frac{T}{2}\right).$$

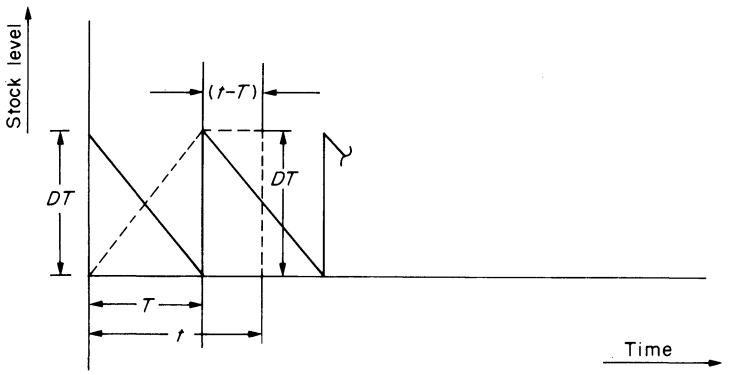

FIG. 2. *Time-weighted inventory when* $T \leqslant t$.

Note that the interest earned should be subtracted from other variable costs in order to get the total variable cost per year.

Now we shall develop the mathematical models for determining economic order quantities for the two cases.

DETERMINATION OF ECONOMIC ORDER QUANTITY WHEN $T \geqslant t$

In this case the total variables cost per year is given by

$$Z(T) = \frac{S}{T} + \frac{DTh}{2} + \frac{DTpI_c}{2} + \frac{Dpt^2I_c}{2T} - DptI_c - \frac{Dpt^2I_d}{2T}$$

or

$$Z(T) = \frac{(2S + Dpt^2(I_c - I_c))}{2T} + \frac{DT}{2}(h + pI_c) - DptI_c. \tag{1}$$

In order to minimize $Z(T)$, set $[\mathrm{d}Z(T)/\mathrm{d}T] = 0$ and obtain the economic value of $T = T_1^*$

$$T_1^* = \sqrt{\frac{(2S + Dpt^2(I_c - I_c))}{D(h + pI_c)}}. \tag{2}$$

It may be pointed out that the effect of use of money, which means $t > 0$ and $(I_c - I_d) \geqslant 0$ for normal conditions, is to increase the apparent order placement cost. The economic order quantity $Q(T_1^*)$ and the minimum annual variable cost $Z(T_1^*)$ are given by

$$Q(T_1^*) = DT_1^* = \sqrt{\frac{D(2S + Dpt^2(I_c - I_d))}{(h + pI_c)}} \tag{3}$$

and

$$Z(T_1^*) = \sqrt{D(2S + Dpt^2(I_c - I_d))\,(h + pI_c)} - DptI_c. \tag{3a}$$

As a result of permissible delay in settlement of the replenishment account, the order quantity as obtained by (3) is generally higher than the order quantity given by the classical economic order quantity model given by

$$Q^* = \sqrt{2DS/(h + pI_c)}. \tag{3b}$$

The extent of such a change in the order quantity depends entirely on the parameters of the problem situation. When $I_c = I_d$, then the economic order quantity given by (3) is exactly equal to the order quantity obtained from the classical economic order quantity model. On the other hand, the highest value of $Q(T_1^*)$ is obtained when the funds cannot be invested ($I_d = 0$). Under such conditions, the economic order is given by

$$\sqrt{\frac{D(2S + Dpt^2I_c)}{(h + pI_c)}}.$$

In the rare case when $I_c - I_d \leqslant 0$, the economic order quantity obtained from (3) is lower than the economic order quantity obtained from the classical economic order quantity model.

DETERMINATION OF ECONOMIC ORDER QUANTITY WHEN $T < t$

In this case, no interest charges are paid for the items kept in stock. The total variable cost in this case is given by

$$Z(T) = \frac{S}{T} + \frac{DTh}{2} - DpI_d\left(t - \frac{T}{2}\right)$$

$$Z(T) = \frac{S}{T} + \frac{DT}{2}(h + pI_d) - DptI_d. \tag{4}$$

Hence the economic order interval and the order quantity are given by

$$T_2^* = \sqrt{\frac{2S}{D(h + pI_d)}} \tag{5}$$

and

$$Q(T_2^*) = DT_2^* = \sqrt{\frac{2SD}{(h + pI_d)}}. \tag{6}$$

Note that for $I_c \geqslant I_d$ $Q(T_2^*) \geqslant Q^*$.

It may be pointed out that the total annual variable cost, $Z(t)$, at $T = t$ is obtained on substituting $T = t$ in (1) or in (4).

$$Z(t) = \frac{S}{t} + \frac{Dth}{2} - \frac{DptI_d}{2} \tag{7}$$

and

$$Q(t) = Dt. \tag{8}$$

In order to obtain the economic operating policy, follow these steps:

Step 1: Determine T_1^* from (2). If $T_1^* \geqslant t$, obtain $Z(T_1^*)$ from (1).
Step 2: Determine T_2^* from (5). If $T_2^* < t$, evaluate $Z(T_2^*)$ from (4).
Step 3: If $T_1^* < t$ and $T_2^* \geqslant t$, then evaluate $Z(t)$ from (7).
Step 4: Compare $Z(T_1^*)$, $Z(T_2^*)$ and $Z(t)$. Select the replenishment interval and the order quantity associated with the least annual cost value evaluated in steps 1 and 2 or 3.

CONCLUDING REMARKS

As a result of permissible delay in settling the replenishment account, the economic replenishment interval and order quantity generally increases marginally, although the annual cost decreases considerably. The saving in cost as a result of permissible delay in settling the replenishment account largely comes from the ability to delay payment without paying any interest. As a result of increased order quantity under conditions of permissible delay in payments, we need to order less often. The economic order quantity model given in this paper is an extension of the classical economic order quantity model and hence can be used in a similar manner for determining ordering policy for items having probabilistic demand.

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